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Comparison of Two Kinds of Functionally Graded Cylindrical Shells with Various Volume Fraction Laws for Vibration Analysis

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ABSTRACT

In this paper, a study on the vibration of thin cylindrical shells made of a functionally gradient material (FGM) composed of stainless steel and nickel is presented. The effects of the FGM configuration are taken into account by studying the frequencies of two FG cylindrical shells. Type I FG cylindrical shell has nickel on its inner surface and stainless steel on its outer surface and Type II FG cylindrical shell has stainless steel on its inner surface and nickel on its outer surface. The study is carried out based on third order shear deformation shell theory (TSDT). The objective is to study the natural frequencies, the influence of constituent volume fractions and the effects of configurations of the constituent materials on the frequencies. The properties are graded in the thickness direction according to the volume fraction power-law distribution. The analysis is carried out with strains-displacement relations from Love's shell theory. The governing equations are obtained using energy functional with the Rayleigh-Ritz method. Results are presented on the frequency characteristics and the influences of constituent various volume fractions for Type I and II FG cylindrical shells and simply supported boundary conditions on the frequencies.

Keywords: Vibration; Functionally gradient material; Third order shear deformation shell theory; Rayleigh-Ritz method

1 INTRODUCTION

CYLINDRICAL shells have found many applications in the industry. They are often used as load bearing structures for aircrafts, ships and buildings. The study of the vibration of cylindrical shells is an important aspect in the successful applications of the cylindrical shells. The study of the free vibrations of cylindrical shells has been carried out extensively. Arnold and Warburton [1], Ludwig and Krieg [2], Chung [3], Soedel [4], Forsberg [5], Bhimaraddi [6], Soldatos [7], Bert and Kumar [8], and Soedel [9] are among those who have studied the vibrations of cylindrical shells. The concept of functionally gradient materials (FGMs) was first introduced in 1984 by a group of materials scientists in Japan, [10, 11], as a means of preparing thermal barrier materials. Since then, FGMs have attracted much interest as heat-shielding materials. FGMs are made by combining different materials using power metallurgy methods [12]. They possess variations in constituent volume fractions that lead to continuous change in the composition, microstructure, porosity, etc. and this results in gradients in the mechanical and thermal properties [13, 14]. Studies on FGMs have been extensive but are largely confined to analysis of thermal stress and deformation [15-17]. Najafizadeh and Isvandzibaei [18] presented the vibration of functionally graded cylindrical shells based on higher order shear deformation plate theory with ring support. The advantage of FGMs is that desired mechanical properties can be tailored and this holds enormous application potential for FGMs.

In this paper, a study on the vibration of cylindrical shells composed of functionally gradient material (FGM) is presented. The considered functionally gradient material is composed of stainless steel and nickel where the volume fractions follow a power-law distribution. The objective is to study the natural frequencies, the influence of constituent volume fractions, the effects of configurations of the constituent materials on the frequencies for two

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kinds of FG cylindrical shell and the influence of simply supported boundary conditions on the frequencies. The analysis of the functionally graded cylindrical shell is carried out using third order shear deformation shell theory and solved using Rayleigh-Ritz method with obtained energy functional using an energy approach. The displacement fields employed consist of some beam eigenfunctions of vibrations that guarantee satisfaction of edge boundary conditions. Studies are carried out for functionally graded cylindrical shells with simply supported-simply supported SS-SS boundary condition.

2 FUNCTIONALLY GRADIENT MATERIALS

Functionally gradient materials (FGM) are obtained by combining two or more materials. Most of the functionally gradient materials are employed in high-temperature environments and many of the constituent materials may possess temperature-dependent properties. The material P can be expressed as a function of temperature [19], as

$$P = P_\circ (P_{-1}T^{-1} + 1 + P_1 T + P_2 T^2 + P_3 T^3) \tag{1}$$

where $P_\circ, P_{-1}, P_1, P_2$ and P_3 are the coefficients of temperature $T(\mathrm{K})$ expressed in Kelvin and are unique to the constituent materials. The material properties P of FGMs are a function of the material properties and volume fractions of the constituent material, and are expressed as

$$P = \sum_{j=1}^{k} P_j V_{fj} \tag{2}$$

where P_j and V_{fj} are, respectively, the material property and volume fraction of the constituent material j. The volume fractions of all the constituent materials should add up to one, i.e.

$$\sum_{j=1}^{k} V_{fj} = 1 \tag{3}$$

For a cylindrical shell with a uniform thickness h and a reference surface at its middle surface, the volume fraction can be written as

$$V_f = \left(\frac{z + h/2}{h} \right)^N \tag{4}$$

where N is the power-law exponent, $0 \le N \le \infty$. For a functionally gradient material with two constituent materials, the Young's modulus E, Poisson ratio ν and the mass density ρ can be expressed as

$$E = (E_1 - E_2)\left(\frac{2z + h}{2h} \right)^N + E_2 \tag{5}$$

$$\nu = (\nu_1 - \nu_2)\left(\frac{2z + h}{2h} \right)^N + \nu_2 \tag{6}$$

$$\rho = (\rho_1 - \rho_2)\left(\frac{2z + h}{2h} \right)^N + \rho_2 \tag{7}$$

From these equations, when $z = -h/2$, $E = E_2$, $\nu = \nu_2$, and $\rho = \rho_2$, and when $z = -h/2, E = E_1$, $\nu = \nu_1$, and $\rho = \rho_1$. The material properties vary continuously from material 2 at the inner surface of the cylindrical shell to material 1 at the outer surface of the cylindrical shell.

3 STRAINS-DISPLACEMENT RELATIONSHIPS

The strain-displacement relationships for a thin shell are [9],

$$\in_{11}=\frac{1}{A_1\left(1+\frac{\alpha_3}{R_1}\right)}\left[\frac{\partial U_1}{\partial\alpha_1}+\frac{U_2}{A_2}\frac{\partial A_1}{\partial\alpha_2}+U_3\frac{A_1}{R_1}\right] \tag{8}$$

$$\in_{22}\ \frac{1}{A_2\left(1+\frac{\alpha_3}{R_2}\right)}\left[\frac{\partial U_2}{\partial\alpha_2}+\frac{U_1}{A_1}\frac{\partial A_2}{\partial\alpha_1}+U_3\frac{A_2}{R_2}\right] \tag{9}$$

$$\in_{33}=\frac{\partial U_3}{\partial\alpha_3} \tag{10}$$

$$\in_{12}=\frac{A_1\left(1+\frac{\alpha_3}{R_1}\right)}{A_2\left(1+\frac{\alpha_3}{R_2}\right)}\frac{\partial}{\partial\alpha_2}\left(\frac{U_1}{A_1\left(1+\frac{\alpha_3}{R_1}\right)}\right)+\frac{A_2\left(1+\frac{\alpha_3}{R_2}\right)}{A_1\left(1+\frac{\alpha_3}{R_1}\right)}\frac{\partial}{\partial\alpha_1}\left(\frac{U_2}{A_2\left(1+\frac{\alpha_3}{R_2}\right)}\right) \tag{11}$$

$$\in_{13}=A_1\left(1+\frac{\alpha_3}{R_1}\right)\frac{\partial}{\partial\alpha_3}\left(\frac{U_1}{A_1\left(1+\frac{\alpha_3}{R_1}\right)}\right)+\frac{1}{A_1\left(1+\frac{\alpha_3}{R_1}\right)}\frac{\partial U_3}{\partial\alpha_1} \tag{12}$$

$$\in_{23}=A_2\left(1+\frac{\alpha_3}{R_2}\right)\frac{\partial}{\partial\alpha_3}\left(\frac{U_2}{A_2\left(1+\frac{\alpha_3}{R_2}\right)}\right)+\frac{1}{A_2\left(1+\frac{\alpha_3}{R_2}\right)}\frac{\partial U_3}{\partial\alpha_2} \tag{13}$$

where A_1 and A_2 are the fundamental form parameters or Lame parameters, U_1, U_2 and U_3 are the displacement at any point ($\alpha_1,\alpha_2,\alpha_3$), R_1 and R_2 are the radius of curvature related to α_1, α_2 and α_3 respectively. The third-order theory of Reddy used in the present study is based on the following displacement field

$$\begin{aligned}
U_1&=u_1(\alpha_1,\alpha_2)+\alpha_3.\phi_1(\alpha_1,\alpha_2)+\alpha_3^2.\psi_1(\alpha_1,\alpha_2)+\alpha_3^3.\beta_1(\alpha_1,\alpha_2)\\
U_2&=u_2(\alpha_1,\alpha_2)+\alpha_3.\phi_2(\alpha_1,\alpha_2)+\alpha_3^2.\psi_2(\alpha_1,\alpha_2)+\alpha_3^3.\beta_2(\alpha_1,\alpha_2)\\
U_3&=u_3(\alpha_1,\alpha_2)
\end{aligned} \tag{14}$$

These equations can be reduced by satisfying the stress-free conditions on the top and bottom faces of the laminates, which are equivalent to $\in_{13}=\in_{23}=0$ at $z=\pm\frac{h}{2}$ Thus

$$\begin{aligned}
U_1&=u_1(\alpha_1,\alpha_2)+\alpha_3.\phi_1(\alpha_1,\alpha_2)-C_1.\alpha_3^3(-\frac{u_1}{R_1}+\phi_1+\frac{\partial u_3}{A_1\partial\alpha_1})\\
U_2&=u_2(\alpha_1,\alpha_2)+\alpha_3.\phi_2(\alpha_1,\alpha_2)-C_1.\alpha_3^3(-\frac{u_2}{R_2}+\phi_2+\frac{\partial u_3}{A_2\partial\alpha_2})\\
U_3&=u_3(\alpha_1,\alpha_2)
\end{aligned} \tag{15}$$

where $C_1 = \dfrac{4}{3h^2}$. Substituting Eq. (15) into nonlinear strain-displacement relation (8) - (13) we get

$$\begin{Bmatrix} \in_{11} \\ \in_{22} \\ \in_{12} \end{Bmatrix} = \begin{Bmatrix} \in_{11}^0 \\ \in_{22}^0 \\ \in_{12}^0 \end{Bmatrix} + \alpha_3 \begin{Bmatrix} k_{11} \\ k_{22} \\ k_{12} \end{Bmatrix} + \alpha_3^3 \begin{Bmatrix} k'_{11} \\ k'_{22} \\ k'_{12} \end{Bmatrix} \tag{16}$$

$$\begin{Bmatrix} \in_{13} \\ \in_{23} \end{Bmatrix} = \begin{Bmatrix} \gamma_{13}^0 \\ \gamma_{23}^0 \end{Bmatrix} + \alpha_3^2 \begin{Bmatrix} \gamma_{13}^2 \\ \gamma_{23}^2 \end{Bmatrix} + \alpha_3^3 \begin{Bmatrix} \gamma_{13}^3 \\ \gamma_{23}^3 \end{Bmatrix} \tag{17}$$

4 FORMULATIONS

Consider a cylindrical shell which as shown in Fig.1. R is the radius, L is the length and h is the thickness. The reference surface is chosen to be the middle surface of the cylindrical shell where an orthogonal coordinate system x, θ, z is fixed. The deformations of the shell with reference to this coordinate system are denoted by U_1, U_2, and U_3 in the x, θ, and z directions, respectively. For a thin cylindrical shell, plane stress condition can be assumed. The constitutive relation for a thin cylindrical shell is consequently given by the two-dimensional Hook's law as

$$\{\sigma\} = [Q]\{\varepsilon\} \tag{18}$$

where $\{\sigma\}$ is the stress vector, $\{\varepsilon\}$ is the strain vector and $[Q]$ is the reduced stiffness matrix. The stress vector for plane stress condition is

$$\{\sigma\}^T = \{\sigma_{11} \quad \sigma_{22} \quad \sigma_{12} \quad \sigma_{13} \quad \sigma_{23}\} \tag{19}$$

where σ_{11} is the stress in x direction, σ_{22} the stress in the θ direction and σ_{12} is the shear stress on the $x\theta$ plane and σ_{13} is the shear stress on the xz plane and σ_{23} is the shear stress on the θz plane. The strain vector is defined as

$$\{\varepsilon\}^T = \{\varepsilon_{11} \quad \varepsilon_{22} \quad \varepsilon_{12} \quad \varepsilon_{13} \quad \varepsilon_{23}\} \tag{20}$$

where ε_{11} is the strain in x direction, ε_{22} the strain in the θ direction and ε_{12} is the shear strain on the $x\theta$ plane and ε_{13} is the shear strain on the xz plane and ε_{23} is the shear strain on the θz plane. The reduced stiffness $[Q]$ matrix is given as

$$[Q] = \begin{bmatrix} Q_{11} & Q_{12} & 0 & 0 & 0 \\ Q_{12} & Q_{22} & 0 & 0 & 0 \\ 0 & 0 & Q_{44} & 0 & 0 \\ 0 & 0 & 0 & Q_{55} & 0 \\ 0 & 0 & 0 & 0 & Q_{66} \end{bmatrix} \tag{21}$$

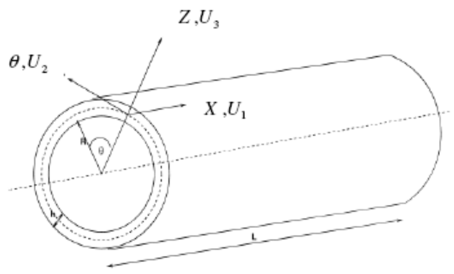

Fig. 1
Geometry of a cylindrical shell.

For an isotropic cylindrical shell, the reduced stiffness Q_{ij} (i , j=1, 2 and 6) are defined as

$$Q_{11} = Q_{22} = \frac{E}{1-\nu^2} \tag{22}$$

$$Q_{12} = \frac{\nu E}{1-\nu^2} \tag{23}$$

$$Q_{44} = Q_{55} = Q_{66} = \frac{E}{2(1+\nu)} \tag{24}$$

where E is the Young's modulus and ν is Poisson's ratio. For a thin cylindrical shell, the force and moment results are defined as

$$\begin{Bmatrix} N_{11} \\ N_{22} \\ N_{12} \end{Bmatrix} = \int_{-h/2}^{h/2} \begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{Bmatrix} \mathrm{d}\alpha_3, \qquad \begin{Bmatrix} M_{11} \\ M_{22} \\ M_{12} \end{Bmatrix} = \int_{-h/2}^{h/2} \begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{Bmatrix} \alpha_3^3 \,\mathrm{d}\alpha_3 \tag{25}$$

$$\begin{Bmatrix} P_{11} \\ P_{22} \\ P_{12} \end{Bmatrix} = \int_{-h/2}^{h/2} \begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{Bmatrix} \alpha_3^3 \,\mathrm{d}\alpha_3, \qquad \begin{Bmatrix} P_{13} \\ P_{23} \end{Bmatrix} = \int_{-h/2}^{h/2} \begin{Bmatrix} \sigma_{13} \\ \sigma_{23} \end{Bmatrix} \alpha_3^3 \,\mathrm{d}\alpha_3 \tag{26}$$

$$\begin{Bmatrix} Q_{13} \\ Q_{23} \end{Bmatrix} = \int_{-h/2}^{h/2} \begin{Bmatrix} \sigma_{13} \\ \sigma_{23} \end{Bmatrix} \mathrm{d}\alpha_3, \qquad \begin{Bmatrix} R_{13} \\ R_{23} \end{Bmatrix} = \int_{-h/2}^{h/2} \begin{Bmatrix} \sigma_{13} \\ \sigma_{23} \end{Bmatrix} {\alpha_3}^2 \,\mathrm{d}\alpha_3 \tag{27}$$

The constitutive equation is obtained as

$$\{N\} = [S]\,\{\varepsilon\} \tag{28}$$

where $\{N\}$ and $\{\varepsilon\}$ are defined as

$$\{N\}^T = \{N_{11}\ N_{22}\ N_{12}\ M_{11}\ M_{22}\ M_{12}\ P_{11}\ P_{22}\ P_{12}\ P_{13}\ P_{23}\ Q_{13}\ Q_{23}\ R_{13}\ R_{23}\} \tag{29}$$

$$\{\varepsilon\}^T = \{{\varepsilon_{11}}^0\ {\varepsilon_{22}}^0\ {\varepsilon_{12}}^0\ k_{11}\ k_{22}\ k_{12}\ k'_{11}\ k'_{22}\ k'_{12}\ {\gamma_{23}}^0\ {\gamma_{13}}^0\ {\gamma_{23}}^2\ {\gamma_{13}}^2\ {\gamma_{23}}^3\ {\gamma_{13}}^3\} \tag{30}$$

and $[S]$ is defined as

$$[S]=\begin{pmatrix} [A] & [B] & [E] \\ [B] & [D] & [F] \\ [E] & [F] & [H] \\ [E'] & [G] & [H'] \\ [A'] & [D'] & [E'] \\ [D'] & [F'] & [G] \end{pmatrix} \tag{31}$$

where A, B, E, D, F, H, and G are the extensional, coupling and bending stiffness matrices and Q_{ij} are functions of z for functionally gradient materials. Here A_{ij} denote the extensional stiffness, D_{ij} the bending stiffness, B_{ij} the bending-extensional coupling stiffness and $E_{ij}, F_{ij}, G_{ij}, H_{ij}$ are the extensional, bending, coupling, and higher-order stiffness. Defining

$$\{A_{ij}, B_{ij}, D_{ij}, E_{ij}, F_{ij}, G_{ij}, H_{ij}\} = \int_{-h/2}^{h/2} Q_{ij}\{1, \alpha_3, \alpha_3^2, \alpha_3^3, \alpha_3^4, \alpha_3^5, \alpha_3^6\}\,\mathrm{d}\alpha_3 \tag{32}$$

The strain energy and kinetic energy of a cylindrical shell can be defined as

$$U=\frac{1}{2}\iiint \{\varepsilon\}^T\{\sigma\}\,\mathrm{d}V \tag{33}$$

$$T=\frac{1}{2}\iiint \rho\left[\left(\frac{\partial u_1}{\partial t}\right)^2+\left(\frac{\partial u_2}{\partial t}\right)^2+\left(\frac{\partial u_3}{\partial t}\right)^2+\left(\frac{\partial \phi_1}{\partial t}\right)^2+\left(\frac{\partial \phi_2}{\partial t}\right)^2\right]\mathrm{d}V \tag{34}$$

where ρ is the mass density, $\{\varepsilon\}$ is the strain vector and $\{\sigma\}$ is the stress vector. By substituting from Eq. (18), the strain and kinetic energies can be written as

$$U=\frac{1}{2}\int_{\circ}^{L}\int_{\circ}^{2\pi}\{\varepsilon\}^T[S]\{\varepsilon\}\,R\,\mathrm{d}\theta\,\mathrm{d}x \tag{35}$$

$$T=\frac{1}{2}\int_{\circ}^{L}\int_{\circ}^{2\pi}\rho_T\left[\left(\frac{\partial u_1}{\partial t}\right)^2+\left(\frac{\partial u_2}{\partial t}\right)^2+\left(\frac{\partial u_3}{\partial t}\right)^2+\left(\frac{\partial \phi_1}{\partial t}\right)^2+\left(\frac{\partial \phi_2}{\partial t}\right)^2\right]R\,\mathrm{d}\theta\,\mathrm{d}x \tag{36}$$

where $\{\varepsilon\}$ is the strain vector defined in Eq. (30) and $[S]$ is the stiffness matrix defined in relation (31). The parameter ρ_T is the density per unit length defined as

$$\rho_T=\int_{-h/2}^{h/2}\rho\,\mathrm{d}z \tag{37}$$

The displacement fields for a cylindrical shell can be written as

$$\begin{aligned} u_1 &= \overline{A}\frac{\partial\phi(x)}{\partial x}\cos(n\theta)\cos(\omega t) \\ u_2 &= \overline{B}\,\phi(x)\sin(n\theta)\cos(\omega t) \\ u_3 &= \overline{C}\,\phi(x)\cos(n\theta)\cos(\omega t) \\ \phi_1 &= \overline{D}\frac{\partial\phi(x)}{\partial x}\cos(n\theta)\cos(\omega t) \\ \phi_2 &= \overline{E}\,\phi(x)\sin(n\theta)\cos(\omega t) \end{aligned} \tag{38}$$

where, $\overline{A}$, $\overline{B}$, $\overline{C}$, $\overline{D}$ and $\overline{E}$ are the constants denoting the amplitudes of the vibrations in the x, θ and z directions, $\phi(x)$ is the axial function that satisfies the geometric boundary conditions, n denotes the number of circumferential waves in the mode shape and ω is the natural angular frequency of the vibration. The axial function $\phi(x)$ is chosen as the beam function as

$$\phi(x) = \alpha_1 \cosh\left(\frac{\lambda_m x}{L}\right) + \alpha_2 \cos\left(\frac{\lambda_m x}{L}\right) - \zeta_m \left(\alpha_3 \sinh\left(\frac{\lambda_m x}{L}\right) + \alpha_4 \sin\left(\frac{\lambda_m x}{L}\right)\right) \tag{39}$$

where $\alpha_i\ (i = 1,...,4)$ are some constants with value 0 or 1 chosen according to the boundary conditions. λ_m are the roots of some transcendental equations and ζ_m are some parameters dependent on λ_m. The $\alpha_i\ (i = 1,...,4)$, the transcendental equations and the parameters ζ_m for the simply supported boundary condition considered. The geometric boundary condition for simply supported boundary condition can be expressed mathematically in terms of $\phi(x)$ as

$$\phi(x) = \phi''(x) = 0 \tag{40}$$

To determine the natural frequencies, the Rayleigh-Ritz method is used. The energy functional Π is defined by the Lagrangian function as

$$\Pi = T_{\max} - U_{\max} \tag{41}$$

Substituting Eq. (38) into Eqs. (35) and (36) and minimizing the energy functional Π with respect to the unknown coefficients are as follows

$$\frac{\partial \Pi}{\partial \overline{A}} = \frac{\partial \Pi}{\partial \overline{B}} = \frac{\partial \Pi}{\partial \overline{C}} = \frac{\partial \Pi}{\partial \overline{D}} = \frac{\partial \Pi}{\partial \overline{E}} = 0 \tag{42}$$

In Eq. (41), $T_{\max}$ and $U_{\max}$ are the maximum kinetic energy and strain energy, respectively. In Eq. (42), the five governing eigenvalue equations can be obtained. These five governing eigenvalue equation can be expressed in matrix form as

$$\begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} \\ C_{21} & C_{22} & C_{23} & C_{24} & C_{25} \\ C_{31} & C_{32} & C_{33} & C_{34} & C_{35} \\ C_{41} & C_{42} & C_{43} & C_{44} & C_{45} \\ C_{51} & C_{52} & C_{53} & C_{54} & C_{55} \end{bmatrix} \begin{Bmatrix} \overline{A} \\ \overline{B} \\ \overline{C} \\ \overline{D} \\ \overline{E} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \tag{43}$$

The eigenvalue equations are solved by imposing the non-trivial solutions condition and equating the determinant of the characteristic matrix $[C_{ij}]$ to zero. Expanding this determinant, a polynomial in even powers of ω is obtained

$$\beta_\circ \omega^{10} + \beta_1 \omega^8 + \beta_2 \omega^6 + \beta_3 \omega^4 + \beta_4 \omega^2 + \beta_5 = 0 \tag{44}$$

where $\beta_i (i = 0,1,2,3,4,5)$ are some constants. Eq. (44) is solved and five positive and five negative roots are obtained. The five positive roots obtained are the natural angular frequencies of the cylindrical shell in the x, θ, and z directions. The smallest of the five roots is the natural angular frequency studied in the present study.

5 RESULTS AND DISCUSSION

In this paper, studies are presented on the vibration of simply supported functionally graded (FG) cylindrical shell. The functionally gradient material (FGM) considered is composed of stainless steel and nickel and its properties are graded in the thickness direction according to the volume fraction power-law distribution. The influence of constituent volume fractions is studied by varying the volume fractions of the stainless steel and nickel. This is carried out by varying the value of the power law exponent N . The effects of the FGM configuration are investigated by studying the frequencies of two FG cylindrical shells. Type I FG cylindrical shell and Type II FG cylindrical shell, Type I FG cylindrical shell has nickel on its inner surface and stainless steel on its outer surface and Type II FG cylindrical shell has stainless steel on its inner surface and nickel on its outer surface. The material properties for stainless steel and nickel, calculated at $T = 300$ K , are presented in Table 1. To validate the present analysis, results for cylindrical shells are compared with Chung [3]. The comparisons show that the present results agreed well with those in the literature. Tables 3 and 4 show the variations of the volume fractions V_f of nickel and stainless steel, respectively, in the thickness direction z for Type I FG cylindrical shell. In the Table 1 the volume fraction of Stainless Steel V_{fss} increased from 0 at $z = -0.5h$ to 1 at $z = 0.5h$ and in the Table 2 the volume fraction for nickel V_{fN} decreased from 1 at $z = -0.5h$ to 0 at $z = 0.5h$ and the material properties on the inner surface of the Type I FG cylindrical shell are those of nickel and on the outer surface are those of stainless steel.

Table 1
Material properties at T=300 K

Coefficients	Stainless Steel			Nickel		
	E (N m^{-2})	ν	ρ (kg m^{-3})	E (N m^{-2})	ν	ρ (kg m^{-3})
P_0	201.04×10^{9}	0.3262	8166	223.95×10^{9}	0.3100	8900
P_{-1}	0	0	0	0	0	0
P_1	3.079×10^{-4}	-2.002×10^{-4}	0	-2.794×10^{-4}	0	0
P_2	-6.534×10^{-7}	3.797×10^{-7}	0	-3.998×10^{-9}	0	0
P_3	0	0	0	0	0	0
	2.07788×10^{11}	0.317756	8166	2.05098×10^{11}	0.3100	8900

Table 2
Comparison of frequency (rad / s) parameter $\Omega = \omega R\sqrt{((1-\nu^2)\rho)/E}$ for a clamped-clamped isotropic cylindrical shell

L/R	R/h	n	ω	Ω	
				Chung [3]	Present
10	500	4	327.5406	0.01508	0.0154656
10	20	2	1254.2173	0.05787	0.0592211
2	20	3	1380.366	0.3117	0.235887

Table 3
Variations of the volume fractions V_{fss} , in the thickness direction z for a Type I FG cylindrical shell

z	V_{fss}					
	N=0.5	N=0.7	N=1	N=2	N=5	N=15
-0.5h	0	0	0	0	0	0
-0.4h	0.3162	0.1995	0.1	0.01	0.00001	$10^{-15}\times1$
-0.3h	0.4472	0.3241	0.2	0.04	0.00032	$10^{-11}\times3.27$
-0.2h	0.5477	0.4305	0.3	0.09	0.00243	$10^{-8}\times1.43$
-0.1h	0.6324	0.5265	0.4	0.16	0.01024	0.00000107
0	0.707	0.6155	0.5	0.25	0.03125	0.00003051
0.1h	0.7745	0.6993	0.6	0.36	0.07776	0.0004701
0.2h	0.8366	0.7790	0.7	0.49	0.1680	0.004747
0.3h	0.8944	0.8553	0.8	0.64	0.3276	0.03518
0.4h	0.9486	0.9289	0.9	0.81	0.5904	0.20589
0.5h	1	1	1	1	1	1

At z away from $z=0.5h$, the rate of decrease of V_{fN} for *N*<1 is high compared to *N*>1, and at z closer to $z=0.5h$, the rate of decrease of V_{fN} for *N*>1 is much higher than for *N*<1. On the hand for V_{fss}, at z away from $z=0.5h$, the rate of increase for *N*<1 is high compared to *N*>1, and at z closer to $z=0.5h$, the rate of increase of V_{fss} for *N*>1 is much higher than for *N*<1. Also, when V_{fN} is high, V_{fss} is low, and vice versa. At any z, the sum of V_{fN} and V_{fss} is 1. For the Type II FG cylindrical shell the behaviors of V_{fN} and V_{fss} are opposite to that the Type I FG cylindrical shell. In this section variations of natural frequencies with the circumferential wave number *n* for two Types functional graded cylindrical shells with different volume fractions are presented.

Table 4
Variations of the volume fractions V_{fN}, in the thickness direction z for a Type I FG cylindrical shell

z	V_{fN}					
	N=0.5	*N=0.7*	*N=1*	*N=2*	*N=5*	*N=15*
-0.5h	1	1	1	1	1	1
-0.4h	0.6837	0.8004	0.9	0.99	0.9999	1
-0.3h	0.5527	0.6758	0.8	0.96	0.9996	1
-0.2h	0.4522	0.5694	0.7	0.91	0.9975	0.9999
-0.1h	0.3675	0.4734	0.6	0.84	0.9897	0.9999
0	0.2928	0.3844	0.5	0.75	0.9687	0.9999
0.1h	0.2254	0.3006	0.4	0.64	0.9222	0.9995
0.2h	0.1633	0.2209	0.3	0.51	0.8319	0.9952
0.3h	0.1055	0.1449	0.2	0.36	0.6723	0.9648
0.4h	0.0513	0.0710	0.1	0.19	0.4095	0.7941
0.5h	0	0	0	0	0	0

Table 5
Variations of natural frequencies with the circumferential wave number *n* for a Type I FG cylindrical shell *m*=1, *h*/*R*=0.002, *L*/*R*=20

n	*f*(*Hz*)					
	N=0.5	*N=0.7*	*N=1*	*N=2*	*N=5*	*N=15*
1	13.319	13.267	13.209	13.101	12.996	12.930
2	4.514	4.496	4.476	4.440	4.4046	4.382
3	4.190	4.173	4.156	4.123	4.0914	4.070
4	7.101	7.074	7.044	6.989	6.9357	6.899
5	11.345	11.301	11.254	11.166	11.080	11.022
6	16.609	16.545	16.475	16.348	16.222	16.137
7	22.848	22.760	22.664	22.489	22.315	22.199
8	29.052	29.937	29.811	29.580	29.351	29.198
9	38.219	38.072	37.912	37.618	37.328	37.133
10	47.347	47.166	46.967	46.604	46.244	46.002

Table 6
Variations of natural frequencies with the circumferential wave number *n* for a Type II FG cylindrical shell *m*=1, *h*/*R*=0.002, *L*/*R*=20

n	*f*(Hz)					
	N=0.5	*N=0.7*	*N=1*	*N=2*	*N=5*	*N=15*
1	13.321	13.269	13.211	13.103	12.997	12.932
2	32.679	32.553	32.416	32.165	31.917	31.751
3	91.388	91.037	90.652	89.951	89.258	88.795
4	174.998	174.326	173.590	172.246	170.920	170.035
5	282.855	281.769	280.578	278.407	276.263	274.832
6	414.818	413.225	411.480	408.295	405.151	403.053
7	570.840	568.648	566.246	561.863	557.537	554.650
8	750.900	748.016	750.853	739.091	733.400	729.603
9	954.987	951.319	947.300	939.968	932.731	927.902
10	1183.096	1178.552	1173.573	1164.489	1155.52	1149.542

Table 7
Variations of natural frequencies at different *L*/*R* ratios for a Type I FG cylindrical shell *m*=1, *h*/*R*=0.002

L/*R*	*f*(Hz)					
	N=0.5	*N=0.7*	*N=1*	*N=2*	*N=5*	*N=15*
0.2	432.864	430.195	428.345	424.948	421.582	419.430
0.5	172.483	171.816	171.072	169.724	168.389	167.520
1	85.837	85.505	85.138	84.462	83.795	83.366
2	42.630	42.465	42.283	41.948	41.617	41.404
5	16.628	16.563	16.492	16.361	16.231	16.149
10	8.456	8.423	8.387	8.321	8.256	8.213
20	4.190	4.174	4.156	4.123	4.091	4.070
50	1.466	1.460	1.454	1.442	1.431	1.424
100	0.550	0.547	0.545	0.541	0.5367	0.5340

Table 8
Variations of natural frequencies at different *L*/*R* ratios for a Type II FG cylindrical shell *m*=1, *h*/*R*=0.002

L/*R*	*f*(Hz)					
	N=0.5	*N=0.7*	*N=1*	*N=2*	*N=5*	*N=15*
0.2	424.314	425.934	427.752	431.226	434.793	437.152
0.5	169.469	170.103	170.834	172.217	173.641	174.588
1	84.335	84.659	85.019	85.708	86.417	86.887
2	41.883	42.043	42.223	42.656	42.917	43.150
5	16.339	16.401	16.471	16.605	16.743	16.834
10	8.305	8.336	8.371	8.439	8.508	8.555
20	4.114	4.130	4.147	4.181	4.215	4.238
50	1.439	1.445	1.451	1.462	1.474	1.4830
100	0.541	0.5431	0.5455	0.5501	0.5547	0.5577

Tables 5 and 6 show variations of natural frequencies for Type I FG cylindrical shell and Type II FG cylindrical shell. The influence of the constituent volume fraction on the frequencies for Type I and II FG cylindrical shells has been found to be different. For the Type I FG cylindrical shells, the natural frequencies decreased when N increased, and for the Type II FG cylindrical shells, the natural frequencies increased when N increased. In Types I and II FG cylindrical shells, the natural frequencies for all values of N lie between those for a stainless steel and nickel cylindrical shells. For $N < 1$, the natural frequencies for Type I FG cylindrical shells are higher than for Type II FG cylindrical shells and for $N > 1$, the natural frequencies for Type II FG cylindrical shells are higher than Type I FG cylindrical shells. Tables 7 and 8 show variations of natural frequencies at different *L*/*R* ratios for Type I and Type II FG cylindrical shells. In Type I FG cylindrical shell, the natural frequencies decreased when N increased, and for the Type II FG cylindrical shells, the natural frequencies increased when N increased.

6 CONCLSIONS

A study on the vibration of functionally graded (FG) cylindrical shell composed of stainless steel and nickel has been presented. The study was carried out for two types of functionally graded cylindrical shells where the configurations of the constituent materials in the functionally graded cylindrical shells are different. One is termed as a Type I FG cylindrical shell and has properties that vary continuously from nickel on its inner surface to stainless steel on its outer surface. The other is termed as a Type II FG cylindrical shell and has properties that vary continuously from stainless on its inner surface to nickel on its outer surface. The analysis of the functionally graded cylindrical shell is carried out using third order shear deformation shell theory and solved using Rayleigh-Ritz method with energy functional, obtained using an energy approach. Studies were made on the natural frequencies, the influence of constituent volume fractions, the effects of configurations of the constituent materials on the frequencies for two kinds of FG cylindrical shell and the influence of boundary conditions simply support on the frequencies. The study showed that the constituent volume fractions and the configurations of the constituent materials affect the natural frequencies. However, the functionally graded cylindrical shells exhibit interesting frequency characteristics when the constituent volume fractions are varied. This is done by varying the power law exponent N. The influence of the constituent volume fraction on the frequencies for Type I and II FG cylindrical shells has been found to be different. For the Type I FG cylindrical shells, the natural frequencies decreased when N increased, and for the Type II FG cylindrical shells, the natural frequencies increased when N decreased. In Types I and II FG cylindrical shells, the natural frequencies for all values of N lie between those for a stainless steel and nickel cylindrical shells. For $N < 1$, the natural frequencies for Type I FG cylindrical shells are higher than for Type II FG cylindrical shells and for $N > 1$, the natural frequencies for Type II FG cylindrical shells are higher than Type I FG cylindrical shells. Thus, the constituent volume fractions and the configurations of the constituent materials affect the natural frequencies.

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