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Analysis on Centrifugal Load Effect in FG Hollow Sphere Subjected to Magnetic Field

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ABSTRACT
This paper presents the effect of centrifugal load in functionally graded (FG) hollow sphere subjected to uniform magnetic field. Analytical solution for stresses and perturbation of the magnetic field vector are determined using the direct method and the power series method. The material stiffness, the magnetic permeability and the density vary continuously across the thickness direction according to the power law functions of radial directions. Magnetic field results in decreasing the radial displacement, the radial and shear stresses due to centrifugal load and has a negligible effect on circumferential displacement and also small effect compared with the other quantities on the circumferential stress due to centrifugal load. Increasing the angular velocity results in increasing the all above quantities due to magnetic field. With increasing the power law indices the radial displacement, the shear and circumferential stresses due to centrifugal load and magnetic field all are decreased and the radial stress due to centrifugal load and magnetic field increased.

Keywords: Centrifugal load; FG; Hollow cylinder; Magnetic field

1 INTRODUCTION

FUNCTIONALLY graded materials (FGMs) are heterogeneous and advanced materials in which the elastic and thermal properties vary gradually and continuously from one surface to another. FGMs cause to decrease the thermal stresses and hence, very useful in nuclear, aircraft, and space engineering applications. The application of this issue is in geophysics, seismology, plasma-physics, magnetic storage elements, magnetic structural elements, measurement techniques of magneto-elasticity. Lutz and Zimmerman [1] solved the problem of uniform heating of a spherical body, whose elastic modulus and thermal expansion coefficient vary linearly with radius. Tanigawa et al. [2] reported the derivation of systems of fundamental equations for a three-dimensional thermoelastic field with non-homogonous material properties and its application to a semi infinite body. Eslami et al. [3] presented a general solution for one-dimensional steady-state thermal and mechanical stresses in a hollow thick sphere made of FGM. The material properties, except Poisson's ratio are assumed to vary along the radius r according to a power law function. Poultangari et al. [4] presented an analytical method to obtain the solution for the two-dimensional steady state thermal and mechanical stresses in a hollow thick sphere made of FGM using Legendre polynomials. The material properties are assumed to vary through the thickness according to the power law functions. Lee [5] considered the problem of three-dimensional axisymmetric quasi-static coupled magneto-thermoelasticity for the laminated circular conical shells subjected to magnetic and temperature fields. Laplace transform and finite different methods are used to analyze the problem. He obtained solutions for the temperature and thermal deformation distributions in a transient and steady state. Dai and Fu [6] considered the magneto-thermoelastic problem of FGM hollow structures subjected to mechanical loads. The material stiffness, the thermal expansion coefficient and the

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magnetic permeability are assumed to obey the simple power law variation through the structures' wall thickness. The aim of their research was to understand the effect of composition on magneto-thermoelastic stresses and to design optimum FGM hollow cylinders and hollow spheres.

Wang and Dong [7] presented theoretical methods for analyzing magneto-thermoelastic responses and perturbation of the magnetic field vector in a conducting non-homogeneous thermoelastic cylinder subjected to thermal shock. By making use of finite Hankle integral transforms, the analytical expressions are obtained for magneto-thermo-dynamic stress and perturbation response of an axial magnetic field vector in a non-homogeneous cylinder. Dai and Wang [8] presented an analytical method to solve the problem for the dynamic stress-focusing and centered-effect of perturbation of the magnetic field vector in orthotropic cylinders under thermal and mechanical shock loads. Analytical expressions for the dynamic stresses and the perturbation of the magnetic field vector are obtained by means of finite Hankel transforms and Laplace transforms. Misra et al. [9] considered the problem of a half-space under the influence of an external primary magnetic field and an elevated temperature field arising out of a ramp-type heating of the surface. It is found that the stress distribution and the secondary magnetic field are almost independent of thermal relaxation time, but are significantly dependent on the mechanical relaxation time. Massalas [10] deals with the phenomenological description of the magneto-thermoelastic interactions in ferromagnetic material in the framework of the generalized theory of thermoelasticity proposed by Green and Laws. The material is supposed to be homogeneous, anisotropic and elastic undergoing large deformations. The analysis is based on the thermodynamic laws of quasi-magneto-statics.

Misra et al. [11] presented a solution for the induced temperature and stress fields in an infinite transversely isotropic solid continuum with a cylindrical hole using the integral transform. The solid medium is considered to be exposed to a magnetic field and the cavity surface is assumed to be subjected to a ramp-type heating. Green and Lindsay model is used to account for finite velocity of heat conduction. Paul and Narasimhan [12] studied the problem of axisymmetric axial stress wave generation in a thermoelastic circular cylindrical bar in the presence of an applied magnetic field. It is assumed that the surface of the cylinder is free from mechanical loadings and thermal radiation. A general solution is obtained by perturbation technique and is annihilated to a particular case, where in the applied magnetic field is constant in space and time. Maruszewsk [13] presented the nonlinear magneto-thermoelastic equations in soft ferromagnetic and elastic bodies. The symmetry of couplings in these equations is also investigated. Chen and Lee [14] worked on magneto-thermoelasticity by introducing two displacement and two stress functions. The governing equations of the linear theory of magneto-electro-thermo-elasticity with transverse isotropy are simplified. The material non-homogeneity along the axis of symmetry can be taken into account and an approximate laminate model is employed to facilitate deriving analytical solutions.

Tianhu et al. [15] reported the theory of generalized thermoelasticity, based on the theory of Lord and Shulman with one relaxation time, used to study the electro-magneto-thermoelastic interactions in a semi-infinite perfectly conducting solid subjected to a thermal shock on its surface, when the solid and its adjoining vacuum is subjected to a uniform axial magnetic field. He used Laplace transform. The Maxwell's equations are formulated and the generalized electro-magneto-thermoelastic coupled governing equations are established. Sharma and Pal [16] investigated the propagation of magnetic-thermoelastic plane wave in an initially unstressed, homogeneous isotropic conducting plate under uniform static magnetic field. The generalized theory of thermoelasticity is employed by assuming the electrical behaviour as quasi-static and the mechanical behaviour as dynamic. At short wavelength limits, the secular equations for symmetric and skew-symmetric modes reduced to Rayleigh surface wave frequency equation, because a finite thickness plate in such situation behaves like a semi-infinite medium.

In this work, the effect of centrifugal load of FG hollow sphere subjected to magnetic field is considered. Analytical solution for stresses and perturbation of the magnetic field vector are determined using the infinitesimal theory of magneto-elasticity. The material stiffness, the magnetic permeability and the density vary continuously across the thickness direction according to power law functions of radial direction.

2 STRESS ANALYSIS

The spherical coordinate (r,θ,ϕ) are considered along the radial, circumferential and direction. Fig. 1 shows FG sphere subjected to centrifugal force and magnetic field. Let u and v, be the displacement components in the radial and circumferential directions. Thus strain-displacement relations are:

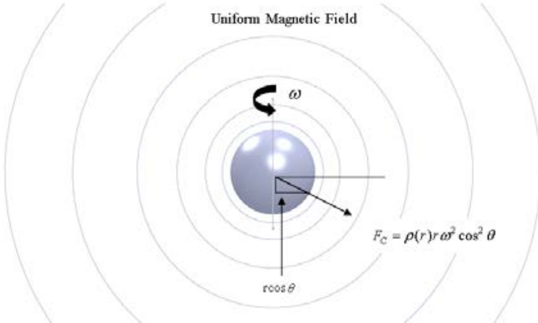

Fig. 1
FG sphere due to centrifugal force and uniform magnetic field.

$$
\begin{aligned}
\varepsilon_{rr} &= u_{,r} \\
\varepsilon_{\theta\theta} &= \frac{1}{r}(u + v_{,\theta}) \\
\varepsilon_{\varphi\varphi} &= \frac{1}{r}(u + v\cot\theta) \\
\varepsilon_{r\theta} &= \frac{1}{2}\left(\frac{u_{,\theta}}{r} + v_{,r} - \frac{v}{r}\right)
\end{aligned} \tag{1}
$$

The Hooke's law for two-dimensional hollow sphere can be written as:

$$
\begin{aligned}
\sigma_{rr} &= \frac{E(r)}{(1+\nu)(1-2\nu)}[(1-\nu)\varepsilon_{rr} + \nu\varepsilon_{\theta\theta} + \nu\varepsilon_{\varphi\varphi}] \\
\sigma_{\theta\theta} &= \frac{E(r)}{(1+\nu)(1-2\nu)}[\nu\varepsilon_{rr} + (1-\nu)\varepsilon_{\theta\theta} + \nu\varepsilon_{\varphi\varphi}] \\
\sigma_{\varphi\varphi} &= \frac{E(r)}{(1+\nu)(1-2\nu)}[\nu\varepsilon_{rr} + \nu\varepsilon_{\theta\theta} + (1-\nu)\varepsilon_{\varphi\varphi}] \\
\sigma_{r\theta} &= \frac{E(r)}{(1+\nu)}\varepsilon_{r\theta}
\end{aligned} \tag{2}
$$

Assuming that the magnetic permeability, μ, of the FG hollow sphere is equal to the magnetic permeability of the medium around it, and also the medium is non-ferromagnetic and non-ferroelectric and ignoring the Thompson effect, the simplified Maxwell's equations of electrodynamics for a perfectly conducting elastic medium are:

$$
\vec{J} = \nabla \times \vec{h}\,, \quad \vec{h} = \nabla \times (\vec{U} \times \vec{H}), \quad div\,\vec{h} = 0\,, \quad \tau_{\varphi} = \mu(r)(\vec{J} \times \vec{H}) \tag{3}
$$

Applying an initial magnetic field vector $\vec{H} = (0,0,H_{\varphi})$ in spherical coordinates (r,θ,ϕ) to Eq. (3), yields to:

$$
\begin{aligned}
&\vec{U} = (u,v,0), \quad \vec{h} = (0,0,h_{\varphi}), \quad h_{\varphi} = -H_{\varphi}\left(u_{,r} + \frac{2u}{r} + \frac{1}{r}v_{,\theta} + \frac{\cot\theta}{r}v\right) \\
&\tau_{\varphi} = \mu(r)(\vec{J} \times \vec{H}) = \mu(r)H_{\varphi}^{2}\left(u_{,r} + \frac{2u}{r} + \frac{1}{r}v_{,\theta} + \frac{\cot\theta}{r}v\right)
\end{aligned} \tag{4}
$$

The equilibrium equations of FG hollow sphere, with respect to centrifugal load are:

$$
\begin{aligned}
&\sigma_{rr,r} + \frac{1}{r}(\sigma_{r\theta,\theta} + 2\sigma_{rr} - \sigma_{\theta\theta} - \sigma_{\varphi\varphi} + \sigma_{r\theta}\cot\theta) + \frac{\partial\tau_{\varphi}}{\partial r} + \rho(r) r\omega^{2}\cos^{2}\theta = 0 \\
&\sigma_{r\theta,r} + \frac{1}{r}(\sigma_{\theta\theta,\theta} + (\sigma_{\theta\theta} - \sigma_{\varphi\varphi})\cot\theta + 3\sigma_{r\theta}) + \frac{1}{r}\frac{\partial\tau_{\varphi}}{\partial\theta} = 0
\end{aligned} \tag{5}
$$

The material stiffness and the magnetic permeability of the sphere are assumed to be graded along the thickness direction according to the power law function as [17-21]:

$$E(r) = E_0(\frac{r}{a})^{m_1}, \mu(r) = \mu_0(\frac{r}{a})^{m_2}, \rho(r) = \rho_0(\frac{r}{a})^{m_3} \tag{6}$$

where E_0, μ_0, ρ_0 are the material parameters and m_1, m_2, m_3 are the power law indices. Using Eqs. (1) to (6), Navier equations in terms of radial and circumferential displacements are as follows:

$$\begin{aligned}
&u_{,rr} + (m_1+2)\frac{1}{r}u_{,r} + 2(\frac{m_1\nu}{1-\nu}-1)\frac{1}{r^2}u + (\frac{1-2\nu}{2-2\nu})\frac{1}{r^2}u_{,\theta\theta} + (\frac{1-2\nu}{2-2\nu})\frac{\cot\theta}{r^2}u_{,\theta} + (\frac{1}{2-2\nu})\frac{1}{r}v_{,r\theta} \\
&+(\frac{m_1\nu}{1-\nu} - \frac{3-4\nu}{2-2\nu})\frac{1}{r^2}v_{,\theta} + (\frac{1}{2-2\nu})\frac{\cot\theta}{r}v_{,r} + (\frac{m_1\nu}{1-\nu} - \frac{3-4\nu}{2-2\nu})\frac{\cot\theta}{r^2}v \\
&+\frac{H_\phi{}^2\mu_0(1+\nu)(1-2\nu)}{E_0(1-\nu)}r^{m_2-m_1}(u_{,rr} + \frac{2}{r}u_{,r} - \frac{2}{r^2}u + \frac{1}{r}v_{,r\theta} - \frac{1}{r^2}v_{,\theta} + \frac{\cot\theta}{r}v_{,r} - \frac{\cot\theta}{r^2}v) \\
&+\frac{\rho_0(1+\nu)(1-2\nu)}{E_0(1-\nu)}r^{m_3-m_1+1}\omega^2\cos^2\theta = 0 \\
&v_{,rr} + (m_1+2)\frac{1}{r}v_{,r} - (m_1 + (\frac{2-2\nu}{1-2\nu})(1+\cot^2\theta))\frac{1}{r^2}v + (\frac{2-2\nu}{1-2\nu})\frac{1}{r^2}v_{,\theta\theta} + (\frac{2-2\nu}{1-2\nu})\frac{\cot\theta}{r^2}v_{,\theta} + (\frac{1}{1-2\nu})\frac{1}{r}u_{,r\theta} \\
&+(m_1 + \frac{4-4\nu}{1-2\nu})\frac{1}{r^2}u_{,\theta} + \frac{2H_\phi{}^2\mu_0(1+\nu)}{E_0}r^{m_2-m_1}(\frac{1}{r}u_{,r\theta} + \frac{2}{r^2}u_{,\theta} + \frac{1}{r^2}v_{,\theta\theta} + \frac{\cot\theta}{r^2}v_{,\theta} - \frac{(1+\cot^2\theta)}{r^2}v) = 0
\end{aligned} \tag{7}$$

In this study if power law index of the elasticity modulus and magnetic permeability of functionally graded sphere are not equal. For solving, the Navier equations using the Frobenius series is needed. But, it leads to more unnecessary complication. Thus, just for preventing this action, it is assumed that the two power law indices, m_1 and m_2 are equal $(m_1 = m_2 = m_3)$. Therefore, the solutions of Navier's equations are:

$$\begin{aligned}
u(r,\theta) &= \sum_{n=0}^{+\infty} u_n(r)P_n(\cos\theta) \\
v(r,\theta) &= \sum_{n=0}^{+\infty} v_n(r)\sin\theta P_n'(\cos\theta)
\end{aligned} \tag{8}$$

where $P_n(\cos\theta)$ is Legendre serie and $P_n'(\cos\theta)$ is differentiation of Legendre serie. Using Eqs. (8) and by substituting into the Navier equations, yields the followings:

$$\begin{aligned}
&u_n''(1+A) + (m+2+2A)\frac{1}{r}u_n' + [2(\frac{\nu m}{1-\nu} - 1 - A) - n(n+1)(\frac{1-2\nu}{2-2\nu})]\frac{1}{r^2}u_n \\
&\qquad + n(n+1)(\frac{1}{2-2\nu} + A)\frac{1}{r}v_n' + n(n+1)(\frac{\nu m}{1-\nu} - \frac{3-4\nu}{2-2\nu} - A)\frac{1}{r^2}v_n = 0 \\
&v_n'' + (m+2)\frac{1}{r}v_n' - [n(n+1)(\frac{2-2\nu}{1-2\nu}) + m + Bn(n+1)]\frac{1}{r^2}v_n \\
&\qquad - (\frac{1}{1-2\nu} + B)\frac{1}{r}u_n' - (m + \frac{4-4\nu}{1-2\nu} + 2B)\frac{1}{r^2}u_n = 0
\end{aligned} \tag{9}$$

where

$$A = \frac{H_\phi{}^2\mu_0(1+\nu)(1-2\nu)}{E_0(1-\nu)}, B = \frac{2H_\phi{}^2\mu_0(1+\nu)}{E_0}, S = \frac{\rho_0(1+\nu)(1-2\nu)}{E_0(1-\nu)} \tag{10}$$

and the symbol $(')$ denotes derivative with respect to *r*. The solutions of Eqs. (9) are:

$$u_n^g(r) = Cr^{\mu}, v_n^g(r) = Dr^{\mu} \tag{11}$$

Substituting Eqs. (11) into Eqs. (9), yields:

$$\begin{aligned}
&C\{\mu(\mu-1)(1+A)+(m+2+2A)\mu+\frac{2m\nu}{1-\nu}-2-2A-n(n+1)(\frac{1-2\nu}{1-\nu})\}+\{n(n+1)(\frac{1}{2-2\nu}+A)\mu \\
&\qquad +n(n+1)(\frac{m\nu}{1-\nu}-\frac{3-4\nu}{2-2\nu}-A)\}D=0 \\
&D\{\mu(\mu-1)+(m+2)\mu-n(n+1)\frac{2-2\nu}{1-2\nu}-m-Bn(n+1)\}+C\{-\mu(\frac{1}{1-2\nu}+B)-(m+\frac{4-4\nu}{1-2\nu}+2B)\}=0
\end{aligned} \tag{12}$$

Eqs. (12) are a system of algebraic equations that for obtaining their nontrivial solution, their determinant should be equal to zero and their four roots are evaluated as follows:

$$\begin{aligned}
&\{\mu(\mu-1)(1+A)+(m+2+2A)\mu+\frac{2m\nu}{1-\nu}-2-2A-n(n+1)(\frac{1-2\nu}{1-\nu})\}\{\mu(\mu-1)+(m+2)\mu \\
&-n(n+1)(\frac{2-2\nu}{1-2\nu})-m-Bn(n+1)\}+\{n(n+1)(\frac{1}{2-2\nu}+A)\mu+ \\
&n(n+1)(\frac{m\nu}{1-\nu}-\frac{3-4\nu}{2-2\nu}-A)\}\{\mu(\frac{1}{1-2\nu}+B)+m+\frac{4-4\nu}{1-2\nu}+2B\}=0
\end{aligned} \tag{13}$$

Therefore,

$$u_n^g(r) = \sum_{j=1}^{4} C_{nj} r^{\mu_{nj}}, v_n^g(r) = \sum_{j=1}^{4} N_{nj} C_{nj} r^{\mu_{nj}} \tag{14}$$

where

$$N_{nj} = -\frac{\mu(\mu-1)(1+A)+(m+2+2A)\mu+\frac{2m\nu}{1-\nu}-2-2A-n(n+1)(\frac{1-2\nu}{2-2\nu})}{n(n+1)[\mu(\frac{1}{2-2\nu}+A)+\frac{m\nu}{1-\nu}-\frac{3-4\nu}{2-2\nu}-A]} \qquad j=(1,\ldots,4), \quad n\neq 0 \tag{15}$$

The decoupled case for $n=0$ must be considered:

$$\begin{aligned}
&u_{rr}(1+A)+(m+2+2A)\frac{1}{r}u_r+2(\frac{\nu m}{1-\nu}-1-A)\frac{1}{r^2}u=0 \\
&(\frac{1}{1-2\nu}+B)\frac{1}{r}u_r+(m+\frac{4-4\nu}{1-2\nu}+2B)\frac{1}{r^2}u=0
\end{aligned} \tag{16}$$

The general solution in this case is as follows:

$$u_0^g(r) = \sum_{i=1}^{2} a_{0i} r^{\eta_i} \tag{17}$$

$$\eta_{1,2} = \frac{1}{2} \pm \sqrt{\frac{1}{4}+\frac{(1-2\nu)(m+2+2A)(m+\frac{4-4\nu}{1-2\nu}+2B)}{(1+A)(1+B(1-2\nu))}-2(\frac{m\nu}{(1+A)(1-\nu)}-1)} \tag{18}$$

Particular solutions are as follows:

$$u_n^p(r) = E_n r^{m_3-m+3}, \quad v_n^p(r) = F_n r^{m_3-m+3} \tag{19}$$

The constants E_n and F_n are evaluated from the following equations

$$E_n = \frac{d_4 d_5 - d_2 d_6}{d_1 d_4 - d_2 d_3}, \quad F_n = \frac{d_1 d_6 - d_3 d_5}{d_1 d_4 - d_2 d_3} \tag{20}$$

where d_1 to d_6 are given in the appendix. Therefore, the solutions of Navier equations are obtained as follows:

$$\begin{aligned} u(r,\theta) &= \sum_{n=1}^{+\infty}\{\sum_{j=1}^{4} C_{nj} r^{\mu_{nj}}\} P_n(\cos\theta) + \sum_{i=1}^{2} a_{0i} r^{\eta_i} + \sum_{n=0}^{+\infty}\{E_n r^{m_3-m+3}\} P_n(\cos\theta) \\ v(r,\theta) &= \sum_{n=0}^{+\infty}\{\sum_{j=1}^{4} N_{nj} C_{nj} r^{\mu_{nj}} + F_n r^{m_3-m+3}\} \sin\theta P_n'(\cos\theta) \end{aligned} \tag{21}$$

Substituting Eqs. (21) into Eq. (1), yields to:

$$\begin{aligned} \varepsilon_{rr} &= \sum_{n=1}^{+\infty}\{\sum_{j=1}^{4} \mu_{nj} C_{nj} r^{\mu_{nj}-1}\} P_n(\cos\theta) + \sum_{i=1}^{2} \eta_i a_{0i} r^{\eta_i-1} + \sum_{n=0}^{+\infty}\{(m_3 - m + 3) E_n r^{m_3-m+2}\} P_n(\cos\theta) \\ \varepsilon_{\theta\theta} &= \sum_{n=1}^{+\infty}\{\sum_{j=1}^{4} C_{nj} r^{\mu_{nj}-1}\} P_n(\cos\theta) + \sum_{i=1}^{2} a_{0i} r^{\eta_i-1} + \sum_{n=0}^{+\infty}\{E_n r^{m_3-m+2}\} P_n(\cos\theta) + \sum_{n=0}^{+\infty}\{\sum_{j=1}^{4} N_{nj} C_{nj} r^{\mu_{nj}-1} + F_n r^{m_3-m+2}\}[n(n+1) P_n(\cos\theta) - \cos\theta P_n'(\cos \\ \varepsilon_{\varphi\varphi} &= \sum_{n=1}^{+\infty}\{\sum_{j=1}^{4} C_{nj} r^{\mu_{nj}-1}\} P_n(\cos\theta) + \sum_{i=1}^{2} a_{0i} r^{\eta_i-1} + \sum_{n=0}^{+\infty}\{E_n r^{m_3-m+2}\} P_n(\cos\theta) + \sum_{n=0}^{+\infty}\{\sum_{j=1}^{4} N_{nj} C_{nj} r^{\mu_{nj}-1} + F_n r^{m_3-m+2}\} \cos\theta P_n'(\cos\theta) \\ \varepsilon_{r\theta} &= \frac{1}{2}\{\sum_{n=1}^{+\infty}\{\sum_{j=1}^{4} -C_{nj} r^{\mu_{nj}-1}\} \sin\theta P_n'(\cos\theta) + \sum_{n=0}^{+\infty}\{-E_n r^{m_3-m+2}\} \sin\theta P_n'(\cos\theta) \\ &+ \sum_{n=0}^{+\infty}\{(\mu_{nj} - 1) N_{nj} C_{nj} r^{\mu_{nj}-1} + (m_3 - m + 2) F_n r^{m_3-m+2}\} \sin\theta P_n'(\cos\theta)\} \end{aligned} \tag{22}$$

Substituting Eqs. (22) into Eqs. (2) result in stresses and the perturbation of the magnetic field vector is as follows:

$$\begin{aligned} h_\phi &= -H_\phi \times \{\sum_{n=1}^{+\infty}\{\sum_{j=1}^{4} (\mu_{nj} + 2) C_{nj} r^{\mu_{nj}-1}\} P_n(\cos\theta) + \sum_{i=1}^{2} (\eta_i + 2) a_{0i} r^{\eta_i-1} + \sum_{n=0}^{+\infty}\{(m_3 - m + 5) E_n r^{m_3-m+2}\} P_n(\cos\theta) \\ &+ n(n+1) \sum_{n=0}^{+\infty}\{\sum_{j=1}^{4} N_{nj} C_{nj} r^{\mu_{nj}-1} + F_n r^{m_3-m+2}\} P_n(\cos\theta)\} \end{aligned} \tag{23}$$

To determine the displacements and stresses, four boundary conditions are required to evaluate the four unknown constants C_{n1} to C_{n4}, a_{01} and a_{02}. The four boundary conditions may be selected from the list of boundary conditions given in Eq. (24). This procedure is done by expanding the given boundary conditions into the Legendre series. These constants are calculated by solving the system of algebraic equations formed by four boundary conditions in the following expressions:

$$\begin{aligned} &u(a,\theta) = g_1(\theta), u(b,\theta) = g_2(\theta) \qquad \sigma_{rr}(a,\theta) = g_5(\theta), \sigma_{rr}(b,\theta) = g_6(\theta) \\ &v(a,\theta) = g_3(\theta), v(b,\theta) = g_4(\theta) \qquad \sigma_{r\theta}(a,\theta) = g_7(\theta), \sigma_{r\theta}(b,\theta) = g_8(\theta) \end{aligned} \tag{24}$$

where $g_i(\theta), (i = 1,...,8)$, are known boundary conditions functions.

3 RESULTS AND DISCUSSION

Now consider an FG hollow sphere of inner radius (metal constituent) *a*=1 m and outer radius (ceramic constituent) *b*=1.2 m. The Poisson's ratio is assumed to be constant and taken as 0.3, material properties and evaluated power law indices used for the analysis are given in Table 1 [22] and $H_\phi = 2.23\times10^9 (A/m)$ and $\mu_0 = 4\pi\times10^{-7}(H/m)$ [6]. Mechanical boundary conditions are assumed to be traction free at the inside and outside of the sphere just for surveying the centrifugal load and magnetic field effects. As the first example, consider the effect of magnetic field on the displacements and stresses due to centrifugal load with angular velocity 50 (rad/s) at $\theta = \pi/3$. Figs. 2-4 show the effect of magnetic field on the radial displacement, radial and shear stresses due to centrifugal load. The variation of radial displacement, radial and shear stresses with magnetic field is smaller in magnitude than the radial displacement, radial and shear stresses without magnetic field. Figs. 5 and 6 show the effect of magnetic field on the circumferential displacement and circumferential stress due to centrifugal load. As seen, the circumferential displacement with magnetic field is almost the same as circumferential displacement without magnetic field and circumferential stress with magnetic field illustrates some discrepancy respect to circumferential stress without magnetic field.

Table. 1
Material properties of FGM

Metal	Ceramic	Power law indices
$E = 66.2$ GPa	$E = 117$ GPa	$m_1 = 3.1236$
$\rho = 4410$ kg/m^3	$\rho = 5600$ kg/m^3	$m_3 = 1.3103$

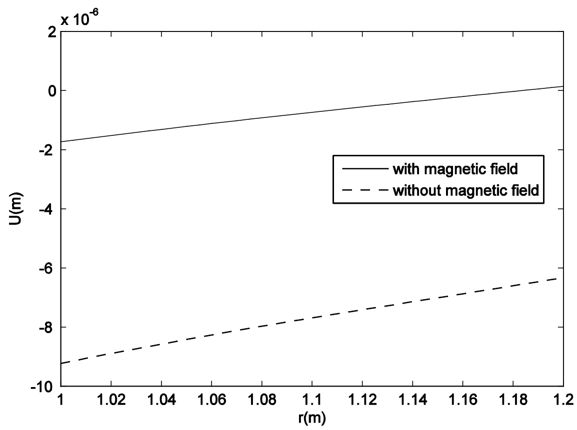

Fig. 2
Radial displacement due to centrifugal load at $\theta = \pi/3$ and $\omega = 50$ (rad/s) (example 1).

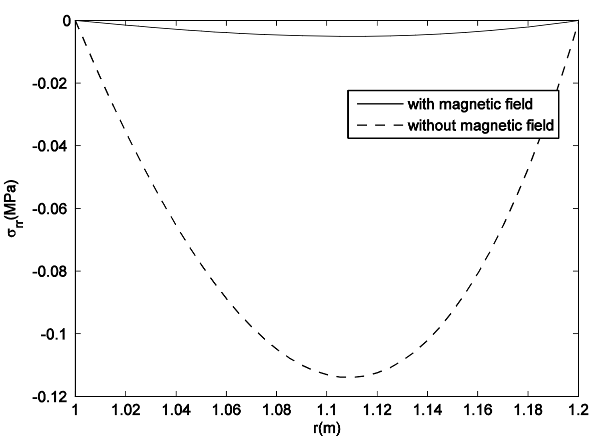

Fig. 3
Radial stress due to centrifugal load at $\theta = \pi/3$ and $\omega = 50$ (rad/s) (example 1).

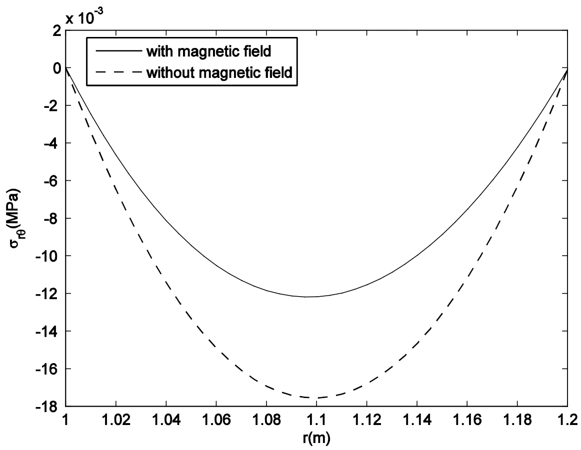

Fig. 4
Shear stress due to centrifugal load at $\theta = \pi / 3$ and $\omega = 50$ (rad/s) (example 1).

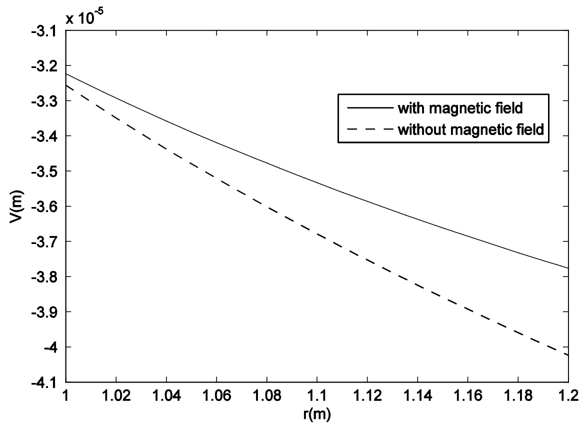

Fig. 5
Circumferential displacement due to centrifugal load at $\theta = \pi / 3$ and $\omega = 50$ (rad/s) (example 1).

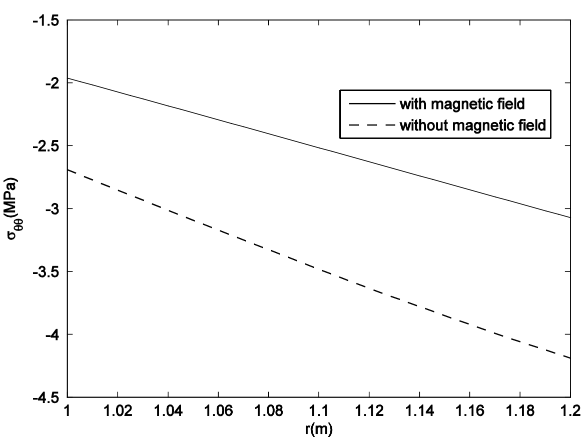

Fig. 6
Circumferential stress due to centrifugal load at $\theta = \pi / 3$ and $\omega = 50$ (rad/s) (example 1).

Now, consider the effect of various angular velocity of centrifugal load from 20 (rad/s) to 100 (rad/s) at $\theta = \pi / 3$ on the displacements and stresses due to magnetic field. Figs. 7-11 show the variation of centrifugal load on the radial and circumferential displacements, radial, shear and circumferential stresses all due to magnetic field. As seen, with increasing the angular velocity the radial and circumferential displacements, radial, shear and circumferential stresses due to magnetic field all are increased.

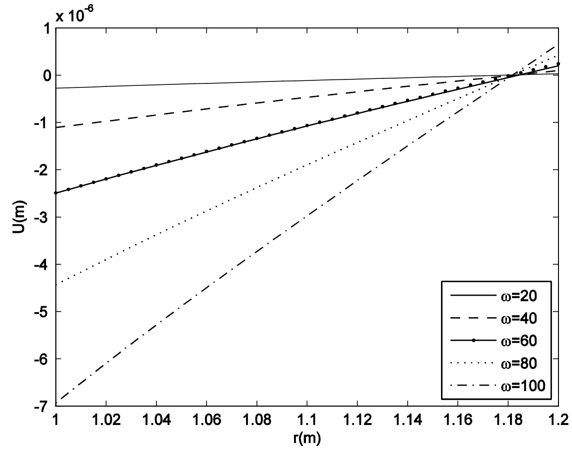

Fig. 7
Radial displacement due to magnetic field and various angular velocities at $\theta = \pi / 3$ (example 2).

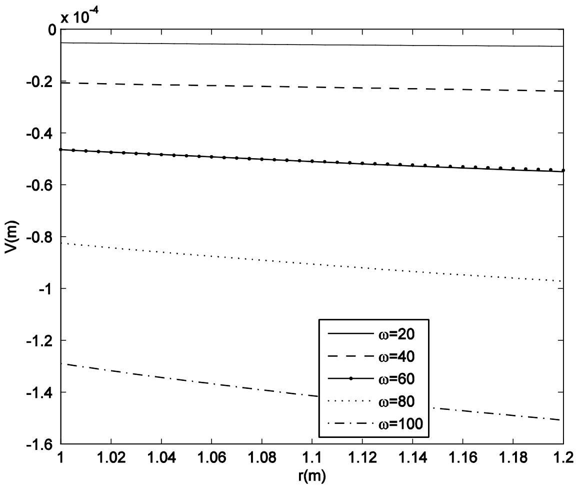

Fig. 8
Circumferential displacement due to magnetic field and various angular velocities at $\theta = \pi / 3$ (example 2).

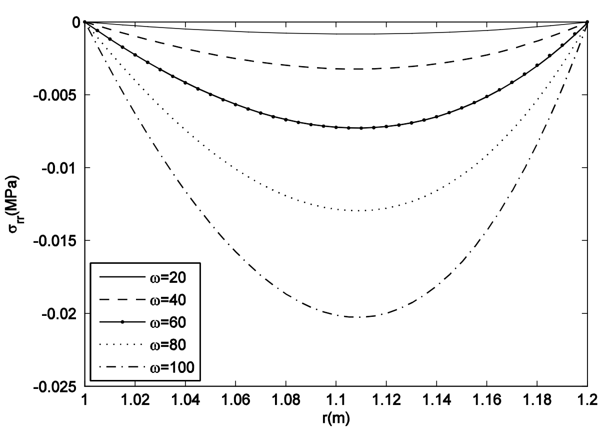

Fig. 9
Radial stress due to magnetic field and various angular velocities at $\theta = \pi / 3$ (example 2).

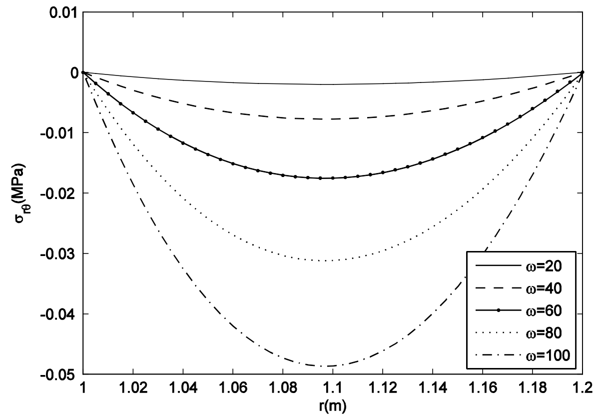

Fig. 10
Shear stress due to magnetic field and various angular velocities at $\theta = \pi/3$ (example 2).

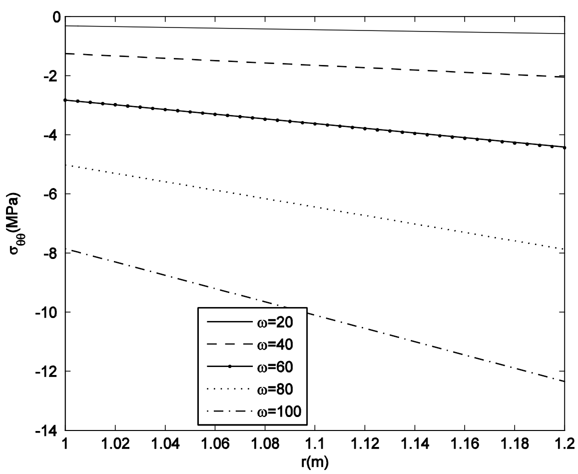

Fig. 11
Circumferential stress due to magnetic field and various angular velocities at $\theta = \pi/3$ (example 2).

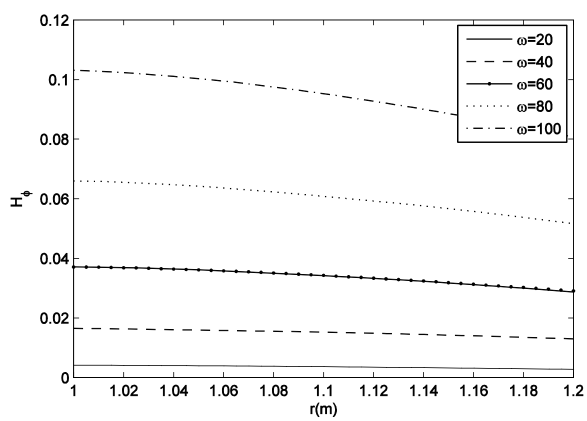

Fig. 12
Perturbation due to various angular velocities at $\theta = \pi/3$ (example 2).

Fig. 12 shows the variation of centrifugal load on the perturbation. As expected, with increasing the angular velocity the perturbation get increased. As the last example, consider the effect of variation of power law indices on displacements and stresses due to centrifugal load with angular velocity of 50 (rad/s) at $\theta = \pi/3$ and magnetic field and also on the perturbation due to centrifugal load. In order to study the effect of power law indices on the behavior of the FG hollow sphere at the presence of magnetic field and centrifugal load, the power indices of material properties are considered to be identical as $m_1 = m_2 = m_3 = m$. Figs. 13-16 show the radial displacement and the radial, shear and circumferential stresses all due to centrifugal load and magnetic field with various power law indices. As seen, radial displacement due to centrifugal load and magnetic field decreased because of increasing the gravity with increasing the power law indices. Radial stress due to centrifugal load and magnetic field increased because the radial displacement gets decreased. Shear and circumferential stresses due to centrifugal load and magnetic field decreased.

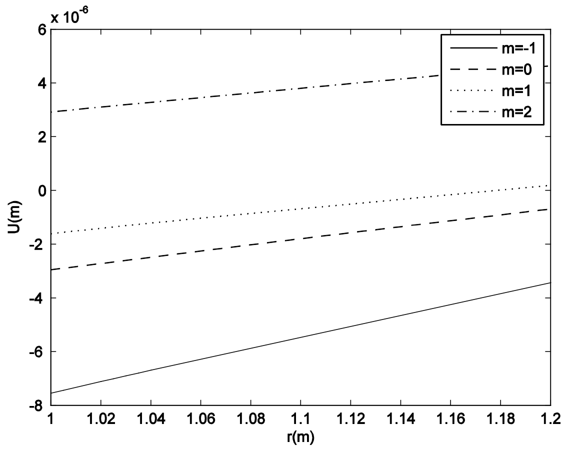

Fig. 13
Radial displacement due to centrifugal load and magnetic fieldalong the thickness of FG sphere with various power law indices at $\theta = \pi / 3$ and $\omega = 50$ (rad/s) (example 3).

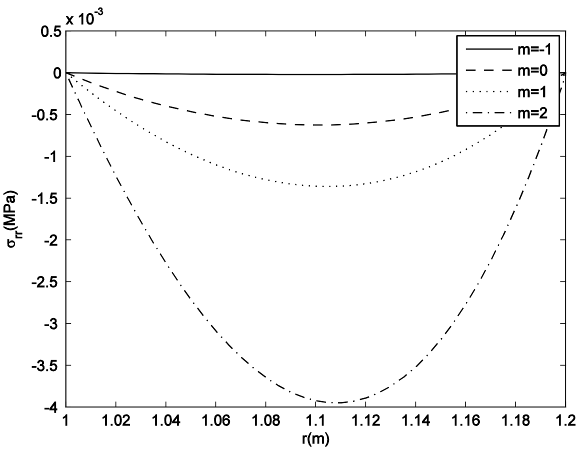

Fig. 14
Radial stress due to centrifugal load and magnetic field along the thickness of FG sphere with various power law indices at $\theta = \pi / 3$ and $\omega = 50$ (rad/s) (example 3).

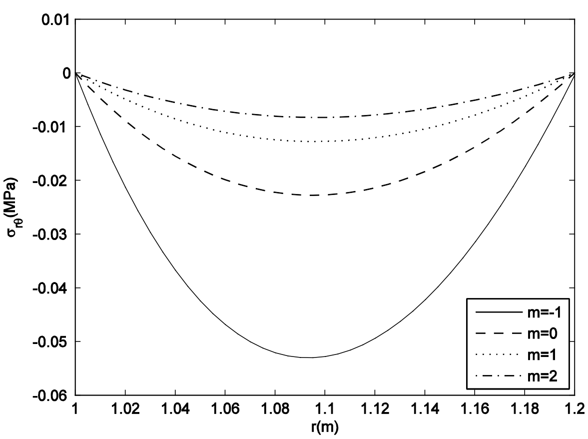

Fig. 15
Shear stress due to centrifugal load and magnetic fieldalong the thickness of FG sphere with various power law indices at $\theta = \pi / 3$ and $\omega = 50$ (rad/s) (example 3).

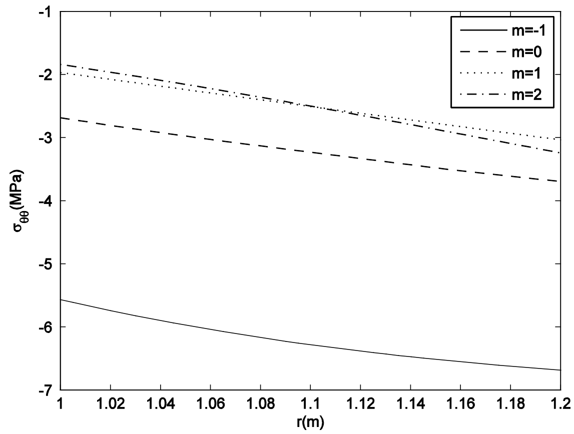

Fig. 16
Circumferential stress due to centrifugal load and magnetic field along the thickness of FG sphere with various power law indices at $\theta = \pi/3$ and $\omega = 50$ (rad/s) (example 3).

4 CONCLUSION

In this paper, the analytical solution for evaluating the effect of centrifugal load on FG hollow sphere under magnetic field is studied. Analytical solution for stresses and perturbation are determined using power series method. The material stiffness, the magnetic permeability and the density vary continuously across the thickness direction according to the power functions of radial directions. Magnetic field results in decreasing the radial displacement, the radial and shear stresses due to centrifugal load and has a negligible effect on circumferential displacement and also small effect compared with the other quantities on circumferential stress due to centrifugal load. Increases of the angular velocity results in increasing the all above quantities due to magnetic field. With increasing the power law indices the radial displacement, the shear and circumferential stresses due to centrifugal load and magnetic field all are decreased and the radial stress due to centrifugal load and magnetic field increased.

5 ACKNOWLEDGMENT

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6 APPENDIX A

$$
\begin{aligned}
d_1 &= (m_3 - m + 3)(m_3 - m + 2)(1 + A) + (m_3 - m + 3)(m + 2 + 2A) + 2(\frac{\nu m}{1-\nu} - 1 - A) - n(n+1)(\frac{1-2\nu}{2-2\nu}) \\
d_2 &= (m_3 - m + 3)n(n+1)(\frac{1}{2-2\nu} + A) + n(n+1)(\frac{\nu m}{1-\nu} - \frac{3-4\nu}{2-2\nu} - A) \\
d_3 &= -(m_3 - m + 3)(\frac{1}{1-2\nu} + B) - (m + \frac{4-4\nu}{1-2\nu} + 2B) \\
d_4 &= (m_3 - m + 3)(m_3 - m + 2) + (m_3 - m + 3)(m + 2) - [n(n+1)(\frac{2-2\nu}{1-2\nu}) + m + Bn(n+1)] \\
d_5 &= -\frac{2n+1}{2} S\omega^2 \int_0^{\pi} \cos^2\theta P_n(\cos\theta)\sin\theta \, \mathrm{d}\theta \\
d_6 &= 0
\end{aligned}
\tag{A.1}
$$

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