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An Exact Solution for Lord-Shulman Generalized Coupled Thermoporoelasticity in Spherical Coordinates

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ABSTRACT
In this paper, the generalized coupled thermoporoelasticity model of hollow and solid spheres under radial symmetric loading condition (*r*, *t*) is considered. A full analytical method is used and an exact unique solution of the generalized coupled equations is presented. The thermal, mechanical and pressure boundary conditions, the body force, the heat source and the injected volume rate per unit volume of a distribute water source are considered in the most general forms and where no limiting assumption is used. This generality allows simulate varieties of applicable problems. At the end, numerical results are presented and compared with classic theory of thermoporoelasticity.

Keywords: Coupled Thermoporoelasticity; Lord-Shulman; Hollow Sphere; Exact solution

1 INTRODUCTION

THE classical theory of thermoelasticity is based on the conventional heat conduction equation. The conventional heat conduction theory assumes that the thermal disturbances propagate at infinite speeds. This prediction is unrealistic from a physical point of view, particularly in simulations like those involving very short transient duration, sudden high heat flux situations, and/or for very low temperatures near the absolute zero [1]. Thus, some modified dynamic thermoelastic models are proposed to analyze the problems with the second sound effects, such as the Lord-Shulman (LS) [2], the Green-Lindsay [3], and the Green-Naghdi [4] theories. These nonclassical theories are referred to as the generalized thermoelasticity theories with finite thermal wave speed, or thermoelasticity with the second sound effect. For the generalized thermoporoelasticity problems, coupled thermal and poro-mechanical processes play an important role in a number of problems of interest in the geomechanics such as stability of boreholes and permeability enhancement in geothermal reservoirs. A thermoporoelastic approach combines the theory of heat conduction with poroelastic constitutive equations and coupling the temperature field with the stresses and pore pressure.

There are a limited numbers of papers that present the closed-form or analytical solution for the coupled porothermoelasticity problems. Youssef [5] derived the governing equations, which describe the behavior of thermoelastic porous medium in the context of the theory of generalized thermoelasticity with one relaxation time (Lord-Shulman). Bai [6] investigated the response of saturated porous media subjected to local thermal loading on the surface of semi-infinite space. He used the numerical integral methods for calculating the unsteady temperature, pore pressure and displacement fields. This author also studied the fluctuation responses of saturated porous media subjected to cyclic thermal loading [7]. In the mentioned paper, an analytical solution was deduced which was proposed by using the Laplace transform and the Gauss-Legendre method and Laplace transform inversion. Droujinine [8] investigated dispersion and attenuation of body waves in a wide range of materials representing realistic rock structures. He used the time-domain asymptotic ray theory to a new generalized coordinate-free wave

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equation with an arbitrary tensor relaxation function. Using Laplace transform and numerical Laplace transform inversion, Bai and Li [9] found a solution for cylindrical cavity in saturated thermoporoelastic medium.a

The number of papers that present the closed-form or analytical solution for the coupled thermoelasticity problems is also limited. Hetnarski [10] found the solution of the coupled thermoelasticity in the form of a series function. Hetnarski and Ignaczak presented a study of the one-dimensional thermoelastic waves produced by an instantaneous plane source of heat in homogeneous isotropic infinite and semi-infinite bodies of the Green-Lindsay type [11]. Also, these authors presented an analysis for laser-induced waves propagating in an absorbing thermoelastic semi-space of the Green-Lindsay theory [12]. Georgiadis and Lykotrafitis obtained a three-dimensional transient thermoelastic solution for Rayleigh-type disturbances propagating on the surface of a half-space [13]. Wagner [14] presented the fundamental matrix of a system of partial differential operators that governs the diffusion of heat and the strains in elastic media. This method can be used to predict the temperature distribution and the strains by an instantaneous point heat, point source of heat, or by a suddenly applied delta force.

A full analytical method is used here to obtain the response of the governing equations and an exact solution is presented. The method of solution is based on the Fourier's expansion and eigenfunction methods, which are traditional and routine methods in solving the partial differential equations. Since the coefficients of equations are not functions of the time variable (t), an exponential form is considered for the general solution matched with the physical wave properties of thermal and mechanical waves. For the particular solution, that is the response to mechanical and thermal shocks, the eigenfuncion method and Laplace transformation is used. This work is following the previous works for coupled problems [15-18].

2 GOVERNING EQUATIONS

A hollow porous sphere with inner and outer radius ri and ro, respectively, made of isotropic material subjected to radial-symmetric mechanical, thermal and pressure shocks is considered.

The Navier coupled thermoelastic equation in spherical coordinate is [16]

$$\begin{aligned} &u,_{rr} + \frac{2}{r}u,_r - \frac{2}{r^2}u - \alpha\frac{(1+\nu)(1-2\nu)}{(1-\nu)E}p,_r - \beta\frac{(1+\nu)(1-2\nu)}{(1-\nu)E}T,_r \\ &\qquad -\rho\frac{(1+\nu)(1-2\nu)}{(1-\nu)E}\ddot{u} = -\frac{(1+\nu)(1-2\nu)}{(1-\nu)E}F(r,t) \end{aligned} \tag{1}$$

Heat conduction equation based on LS theory is obtained as [4]

$$\begin{aligned} &T,_{rr} + \frac{2}{r}T,_r - Z\frac{T_\circ}{K}(\dot{T} + t_0\ddot{T}) + Y\frac{T_\circ}{K}\dot{p} \\ &\qquad -\beta\frac{T_\circ}{K}\left(t_0\left(\ddot{u},_r + \frac{2}{r}\ddot{u}\right) + \dot{u},_r + \frac{2}{r}\dot{u}\right) = -\frac{1}{K}Q(r,t) \end{aligned} \tag{2}$$

According to Darcy's law and continuity condition of seepage, the equation of mass conservation can be written as [16]

$$p,_{rr} + \frac{2}{r}p,_r - \alpha_p\frac{\gamma_w}{k}\dot{p} - Y\frac{\gamma_w}{k}\dot{T} - \alpha\frac{\gamma_w}{k}(\dot{u},_r + \frac{2}{r}\dot{u}) = -\frac{\gamma_w}{k}W(r,t) \tag{3}$$

where (,) denotes partial derivative, u is the displacement component in the radial direction, p is the pore pressure, ρ is bulk mass density, $\alpha = 1 - C_s / C$ is the Biot's coefficient, t_0 is relaxation time, $C_s = 3(1-2\nu_s)E_s$ is the coefficient of volumetric compression of the solid grains, with E_s and ν_s being the elastic modulus and Poisson's ratio of solid grains and $C = 3(1-2\nu)E$ is the coefficient of volumetric compression of solid skeleton, with E and ν being the elastic modulus and Poisson's ratio of solid skeleton, $T_\circ$ is initial reference temperature, $\beta = 3\alpha_s / C$ is the thermal expansion factor, α_s is the coefficient of linear thermal expansion of solid grains, $Y = 3(n\alpha_w + (\alpha - n)\alpha_s)$ and $\alpha_p = n(C_w - C_s) + \alpha C_s$ are coupling parameters, α_w and C_w are the coefficients of linear thermal expansion and volumetric compression of pure water, n is the porosity, k is the hydraulic conductivity, γ_w is the unit of pore water and $Z = \frac{((1-n)\rho_s c_s + n\rho_w c_w)}{T_\circ} - 3\beta\alpha_s$ is coupling parameter, ρ_w and ρ_s are the densities of pore water and solid grains and c_w and c_s are the heat capacities of pore water and solid grains and K is the coefficient of heat conductivity. Here, F(r, t), Q(r, t) and W(r, t) are the body force, heat generation and the injected volume rate per unit volume of a distribute water source, respectively. The mechanical, thermal and pressure boundary conditions are

$$\begin{aligned}
&C_{11}u(r_i,t) + C_{12}u_{,r}(r_i,t) + C_{13}T(r_i,t) + C_{14}p(r_i,t) = f_1(t)\\
&C_{21}u(r_o,t) + C_{22}u_{,r}(r_o,t) + C_{23}T(r_o,t) + C_{24}p(r_o,t) = f_2(t)\\
&C_{31}T(r_i,t) + C_{32}T_{,r}(r_i,t) = f_3(t)\\
&C_{41}T(r_o,t) + C_{42}T_{,r}(r_o,t) = f_4(t)\\
&C_{51}p(r_i,t) = f_5(t)\\
&C_{61}p(r_o,t) = f_6(t)
\end{aligned} \tag{4}$$

where C_{ij} are the mechanical, thermal and pressure coefficients, and by assigning different values for them, different types of mechanical, thermal, and pressure boundary conditions may be obtained. These boundary conditions include the displacement, strain, stress(for the first and second boundary conditions), specified temperature, convection, heat flux condition (for the third and forth boundary conditions), and pressure (for the fifth and sixth boundary conditions). $f_1(r)$ to $f_6(r)$ are arbitrary functions which show mechanical, thermal and pressure shocks, respectively. The initial boundary conditions are assumed in the following general form

$$\begin{aligned}
&u(r,0) = f_7(r) \qquad u_{,t}(r,0) = f_8(r)\\
&T(r,0) = f_9(r) \qquad T_{,t}(r,0) = f_{10}(r)\\
&p(r,0) = f_{11}(r)
\end{aligned} \tag{5}$$

where $f_7(r)$ to $f_{11}(r)$ are arbitrary function which show initial distributions of displacement, temperature and pressure, respectively.

3 SOLUTION

The Eqs. (1)-(3) are the system of non-homogeneous partial differential equations with non-constant coefficients (functions of radius variable *r* only) has general and particular solutions.

3.1 General solution with homogeneous boundary conditions

Since the coefficients of these equations are independent of time variable (t), the exponential function form of time variable may be assumed for the general solution as

$$u(r,t) = [U^*(r)]e^{\lambda t} \qquad T(r,t) = [\theta^*(r)]e^{\lambda t} \qquad p(r,t) = [P^*(r)]e^{\lambda t} \tag{6}$$

Substituting Eq. (6) into homogeneous parts of Eqs. (1) to (3), yields

$$U^{*''}+\frac{2}{r}U^{*'}-\frac{2}{r^2}U^*+d_1P^{*'}+d_2\theta^{*'}+d_3\lambda^2U^*=0$$
$$\theta^{*''}+\frac{2}{r}\theta^{*'}+d_4(\lambda+t_\circ\lambda^2)\theta^*+d_5\lambda P^*+d_6(\lambda+t_\circ\lambda^2)(U^{*'}+\frac{2}{r}U^*)=0 \tag{7}$$
$$P^{*''}+\frac{2}{r}P^{*'}+d_7\lambda P^*+d_8\lambda\theta^*+d_9\lambda(U^{*'}+\frac{2}{r}U^*)=0$$

Eqs. (7) are system of ordinary differential equations, where the prime symbol (') shows differentiation with respect to the radius variable (r) and d_1 to d_9 are constant parameters given in the appendix.

3.2 Change in dependent variables

To obtain a solution for Eq. (7), the dependent variables are changed as

$$U^*(r)=r^{-\frac{1}{2}}U(r) \qquad \theta^*(r)=r^{-\frac{1}{2}}\theta(r) \qquad P^*(r)=r^{-\frac{1}{2}}P\left(r\right) \tag{8}$$

Substituting Eq. (8) into Eq. (7) gives

$$U''+\frac{1}{r}U'-\frac{9}{4}\frac{1}{r^2}U+d_3\lambda^2U-d_2\frac{1}{2}\frac{1}{r}\theta+d_2\theta'-d_1\frac{1}{2}\frac{1}{r}P+d_1P'=0$$
$$\theta''+\frac{1}{r}\theta'-\frac{1}{4}\frac{1}{r^2}\theta+d_4(\lambda+t_\circ\lambda^2)\theta+d_6(\lambda+t_\circ\lambda^2)\frac{3}{2}\frac{1}{r}U+d_6(\lambda+t_\circ\lambda^2)U'+d_5\lambda P=0 \tag{9}$$
$$P''+\frac{1}{r}P'-\frac{1}{4}\frac{1}{r^2}P+d_7\lambda P+d_8\lambda\theta+d_9\lambda\frac{3}{2}\frac{1}{r}U+d_9\lambda U'=0$$

3.3 Solution

The first solutions of U_1 , θ_1 and P_1 are considered as

$$U_1(r)=A_1J_{\frac{3}{2}}(\beta r) \qquad \theta_1(r)=B_1J_{\frac{1}{2}}(\beta r) \qquad P_1(r)=C_1J_{\frac{1}{2}}(\beta r) \tag{10}$$

Substituting Eqs. (10) into Eqs. (9) yields

$$\{(-\beta^2+\lambda^2d_3)A_1-d_2\beta B_1-d_1\beta C_1\}J_{\frac{3}{2}}(\beta r)=0$$
$$\{(\lambda+t_\circ\lambda^2)d_6\beta A_1+(-\beta^2+((\lambda+t_\circ\lambda^2)d_4)B_1+\lambda d_5C_1\}J_{\frac{1}{2}}(\beta r)=0 \tag{11}$$
$$\{\lambda d_9\beta A_1+\lambda d_8B_1+(-\beta^2+\lambda d_7)C_1\}J_{\frac{1}{2}}(\beta r)=0$$

Eqs. (11) show that U_1 , θ_1 and P_1 can be the solutions of Eqs. (9), if and only if

$$\begin{bmatrix} -\beta^2+\lambda^2d_3 & -d_2\beta & -d_1\beta \\ (\lambda+t_\circ\lambda^2)d_6\beta & -\beta^2+(\lambda+t_\circ\lambda^2)d_4 & \lambda d_5 \\ \lambda d_9\beta & \lambda d_8 & -\beta^2+\lambda d_7 \end{bmatrix}\begin{Bmatrix} A_1 \\ B_1 \\ C_1 \end{Bmatrix}=\begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \tag{12 }$$

The non-trivial solution of Eq. (12) is obtained by equating the determinant of this equation to zero as

$$\begin{aligned}&d_3d_4d_7(\lambda+t_\circ\lambda^2)\lambda^3-d_3d_5d_8\lambda^4-\beta^2d_3d_4(\lambda+t_\circ\lambda^2)\lambda^2-\beta^2d_3d_7\lambda^3\\&\quad-\beta^2d_4d_7(\lambda+t_\circ\lambda^2)\lambda+\beta^2d_5d_8\lambda^2+\beta^2d_1d_4d_9(\lambda+t_\circ\lambda^2)\lambda-\beta^2d_1d_6d_8(\lambda+t_\circ\lambda^2)\lambda\\&\quad+\beta^2d_2d_6d_7(\lambda+t_\circ\lambda^2)\lambda-\beta^2d_2d_5d_9\lambda^2+\beta^4d_3\lambda^2+\beta^4d_4(\lambda+t_\circ\lambda^2)+\beta^4d_7\lambda\\&\qquad-\beta^4d_2d_6(\lambda+t_\circ\lambda^2)-\beta^4d_1d_9\lambda-\beta^6=0\end{aligned}\tag{13}$$

Eq. (13) is the first characteristic equation. Thus, it is concluded that U_1, θ_1 and P_1 satisfy the system of equations (9) and they are the first solution of the system. The second solutions of U_2, θ_2 and P_2 are considered as

$$\begin{aligned}&U_2(r)=[A_2J_{\frac{3}{2}}(\beta r)+A_3rJ_{\frac{5}{2}}(\beta r)]\\&\theta_2(r)=[B_2J_{\frac{1}{2}}(\beta r)+B_3rJ_{\frac{3}{2}}(\beta r)]\\&P_2(r)=[C_2J_{\frac{1}{2}}(\beta r)+C_3rJ_{\frac{3}{2}}(\beta r)]\end{aligned}\tag{14}$$

Substituting Eqs. (14) to Eqs. (9) yield

$$\begin{aligned}&\left\{(\beta^2-d_3\lambda^2)A_3+C_3d_1\beta+B_3d_2\beta\right\}rJ_{\frac{1}{2}}(\beta r)\\&\quad+\left\{-A_3\beta+A_2d_3\lambda^2-A_2\beta^2-C_3d_1-B_3d_2-B_2d_2\beta-C_2d_1\beta+A_3d_3\lambda^2\frac{3}{\beta}\right\}J_{\frac{3}{2}}(\beta r)=0\\&\left\{-B_2\beta^2+2B_3\beta+B_2d_4(\lambda+t_\circ\lambda^2)+A_2d_6\beta(\lambda+t_\circ\lambda^2)+C_2d_5\lambda\right\}J_{\frac{1}{2}}(\beta r)\\&\quad+\left\{A_3d_6\beta(\lambda+t_\circ\lambda^2)+(-\beta^2+d_4(\lambda+t_\circ\lambda^2))B_3+C_3d_5\lambda\right\}rJ_{\frac{3}{2}}(\beta r)=0\\&\left\{2C_3\beta+C_2d_7\lambda+B_2d_8\lambda+A_2d_9\lambda\beta-C_2\beta^2\right\}J_{\frac{1}{2}}(\beta r)\\&\quad+\left\{+A_3d_9\lambda\beta+B_3d_8\lambda+(-\beta^2+d_7\lambda)C_3\right\}rJ_{\frac{3}{2}}(\beta r)=0\end{aligned}\tag{15}$$

The expressions for U_2, θ_2 and P_2 can be the solutions of Eqs. (9), if and only if

$$\begin{bmatrix}-\beta^2+\lambda^2d_3 & -d_2\beta & -d_1\beta\\(\lambda+t_\circ\lambda^2)d_6\beta & -\beta^2+(\lambda+t_\circ\lambda^2)d_4 & \lambda d_5\\\lambda d_9\beta & \lambda d_8 & -\beta^2+\lambda d_7\end{bmatrix}\begin{Bmatrix}A_3\\B_3\\C_3\end{Bmatrix}=\begin{Bmatrix}0\\0\\0\end{Bmatrix}\tag{16}$$

$$(d_3\lambda^2-\beta^2)A_2+\left(d_3\lambda^2\frac{3}{\beta}-\beta\right)A_3-d_2\beta B_2-d_2B_3-d_1\beta C_2-d_1C_3=0\tag{17}$$

$$d_6(\lambda+t_\circ\lambda^2)\beta A_2+(-\beta^2+d_4(\lambda+t_\circ\lambda^2))B_2+2\beta B_3+d_5\lambda C_2=0\tag{18}$$

$$d_9\lambda\beta A_2+d_8\lambda B_2+(d_7\lambda-\beta^2)C_2+2\beta C_3=0\tag{19}$$

The non-trivial solution of Eqs. (16) is obtained by equating the determinant to zero as

$$\begin{aligned}&d_3d_4d_7(\lambda+t_\circ\lambda^2)\lambda^3-d_3d_5d_8\lambda^4-\beta^2d_3d_4(\lambda+t_\circ\lambda^2)\lambda^2-\beta^2d_3d_7\lambda^3\\&\quad-\beta^2d_4d_7(\lambda+t_\circ\lambda^2)\lambda+\beta^2d_5d_8\lambda^2+\beta^2d_1d_4d_9(\lambda+t_\circ\lambda^2)\lambda-\beta^2d_1d_6d_8(\lambda+t_\circ\lambda^2)\lambda\\&\quad+\beta^2d_2d_6d_7(\lambda+t_\circ\lambda^2)\lambda-\beta^2d_2d_5d_9\lambda^2+\beta^4d_3\lambda^2+\beta^4d_4(\lambda+t_\circ\lambda^2)+\beta^4d_7\lambda\\&\qquad-\beta^4d_2d_6(\lambda+t_\circ\lambda^2)-\beta^4d_1d_9\lambda-\beta^6=0\end{aligned}\tag{20}$$

Eqs. (17) to (19) give the relations between A_2, A_3, B_2, B_3, C_2 and C_3 and they play as the balancing ratios that make Eq. (14) to be the second solution of the system of Eqs. (9). The third solution of the system of the ordinary differential equations with non-constant coefficients (9) must be considered as

$$\begin{aligned}
U_3(r) &= [A_4 J_{\frac{3}{2}}(\beta r) + A_5 r J_{\frac{5}{2}}(\beta r) + A_6 r^2 J_{\frac{7}{2}}(\beta r)] \\
\theta_3(r) &= [B_4 J_{\frac{1}{2}}\beta r) + B_5 r J_{\frac{3}{2}}(\beta r) + B_6 r^2 J_{\frac{5}{2}}(\beta r)] \\
P_3(r) &= [C_4 J_{\frac{1}{2}}(\beta r) + C_5 r J_{\frac{3}{2}}(\beta r) + C_6 r^2 J_{\frac{5}{2}}(\beta r)]
\end{aligned} \tag{21}$$

Substituting Eqs. (21) into Eq. (9) yield

$$\begin{aligned}
&\left\{ \begin{array}{l} (-\beta^2 + d_3\lambda^2)A_4 + \left(-\beta + d_3\lambda^2 \dfrac{3}{\beta}\right)A_5 + \left(-3 + d_3\lambda^2 \dfrac{15}{\beta^2}\right)A_6 \\ -B_4 d_2 \beta - d_2 B_5 - 3d_2 \dfrac{1}{\beta} B_6 - C_4 \beta d_1 - \dfrac{3}{2} C_5 d_1 - C_6 \dfrac{3}{\beta} d_1 \end{array} \right\} J_{\frac{3}{2}}(\beta r) = 0 \\
&\left\{ (\beta^2 - d_3\lambda^2)A_6 + C_6 \beta d_1 + B_6 d_2 \beta \right\} r^2 J_{\frac{3}{2}}(\beta r) = 0 \\
&\left\{ (\beta^2 - d_3\lambda^2)A_5 + \left(\beta - \frac{5}{\beta} d_3 \lambda^2\right)A_6 + B_5 d_2 \beta + d_2 B_6 + C_5 \beta d_1 + C_6 \frac{3}{2} d_1 - \frac{1}{2} C_6 d_1 \right\} r J_{\frac{1}{2}}(\beta r) = 0 \\
&\left\{ d_6(\lambda + t_\circ \lambda^2)\beta A_4 + (-\beta^2 + d_4(\lambda + t_\circ\lambda^2))B_4 + 2\beta B_5 + d_5 \lambda C_4 \right\} J_{\frac{1}{2}}(\beta r) = 0 \\
&\left\{ -d_6(\lambda + t_\circ \lambda^2)\beta A_6 + (\beta^2 - d_4(\lambda + t_\circ\lambda^2))B_6 - d_5 \lambda C_6 \right\} r^2 J_{\frac{1}{2}}(\beta r) = 0 \\
&\left\{ \begin{array}{l} d_6(\lambda + t_\circ\lambda^2)\beta A_5 + 3d_6(\lambda + t_\circ\lambda^2)A_6 + (d_4(\lambda + t_\circ\lambda^2) - \beta^2)B_5 \\ + \left(d_4 \dfrac{3}{\beta}(\lambda + t_\circ\lambda^2) + \beta\right)B_6 + d_5\lambda C_5 + d_5 \dfrac{3}{\beta}\lambda C_6 \end{array} \right\} r J_{\frac{3}{2}}(\beta r) = 0(2) \\
&\left\{ +d_9 \lambda A_4 \beta + d_8 \lambda B_4 + (-\beta^2 + d_7\lambda)C_4 + 2C_5\beta \right\} J_{\frac{1}{2}}(\beta r) = 0 \\
&\left\{ (-d_7\lambda + \beta^2)C_6 - d_8\lambda B_6 - d_9 \lambda A_6 \beta \right\} r^2 J_{\frac{1}{2}}(\beta r) = 0 \\
&\left\{ (-\beta^2 + d_7\lambda)C_5 + \left(\beta + d_7\lambda\frac{3}{\beta}\right)C_6 + d_8\lambda B_5 + d_8 \frac{3}{\beta}\lambda B_6 + d_9 \lambda\beta A_5 + 3d_9\lambda A_6 \right\} r J_{\frac{3}{2}}(\beta r) = 0
\end{aligned} \tag{22}$$

The expressions for U_3 , θ_3 and P_3 can be solutions of Eq. (9), if and only if

$$\begin{bmatrix} -\beta^2 + \lambda^2 d_3 & -d_2\beta & -d_1\beta \\ (\lambda + t_\circ\lambda^2)d_6\beta & -\beta^2 + (\lambda + t_\circ\lambda^2)d_4 & \lambda d_5 \\ \lambda d_9 \beta & \lambda d_8 & -\beta^2 + \lambda d_7 \end{bmatrix} \begin{Bmatrix} A_6 \\ B_6 \\ C_6 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \tag{23}$$

$$\begin{aligned}
&(-\beta^2 + d_3\lambda^2)A_4 + \left(-\beta + d_3\lambda^2\frac{3}{\beta}\right)A_5 + \left(-3 + d_3\lambda^2\frac{15}{\beta^2}\right)A_6 \\
&-B_4 d_2\beta - d_2 B_5 - 3d_2\frac{1}{\beta}B_6 - C_4\beta d_1 - \frac{3}{2}C_5 d_1 - C_6\frac{3}{\beta}d_1 = 0
\end{aligned} \tag{24}$$

$$(\beta^2 - d_3\lambda^2)A_5 + \left(\beta - \frac{5}{\beta}d_3\lambda^2\right)A_6 + B_5 d_2\beta + d_2 B_6 + C_5\beta d_1 + C_6\frac{3}{2}d_1 - \frac{1}{2}C_6 d_1 = 0 \tag{25}$$

$$d_6(\lambda + t_\circ\lambda^2)\beta A_4 + (-\beta^2 + d_4(\lambda + t_\circ\lambda^2))B_4 + 2\beta B_5 + d_5\lambda C_4 = 0 \tag{26}$$

$$d_6(\lambda + t_\circ\lambda^2)\beta A_5 + 3d_6(\lambda + t_\circ\lambda^2)A_6 + (d_4(\lambda + t_\circ\lambda^2) - \beta^2)B_5 + \left(d_4\frac{3}{\beta}(\lambda + t_\circ\lambda^2) + \beta\right)B_6 + d_5\lambda C_5 + d_5\frac{3}{\beta}\lambda C_6 = 0 \tag{27}$$

$$d_9\lambda A_4\beta + d_8\lambda B_4 + (-\beta^2 + d_7\lambda)C_4 + 2C_5\beta = 0 \tag{28}$$

$$(-\beta^2 + d_7\lambda)C_5 + \left(\beta + d_7\lambda\frac{3}{\beta}\right)C_6 + d_8\lambda B_5 + d_8\frac{3}{\beta}\lambda B_6 + d_9\lambda\beta A_5 + 3d_9\lambda A_6 = 0 \tag{29}$$

The non-trivial solution of Eq. (23) is obtained by equating the determinant of this equation to zero as

$$\begin{aligned} &d_3d_4d_7(\lambda + t_\circ\lambda^2)\lambda^3 - d_3d_5d_8\lambda^4 - \beta^2 d_3d_4(\lambda + t_\circ\lambda^2)\lambda^2 - \beta^2 d_3d_7\lambda^3 - \beta^2 d_4d_7(\lambda + t_\circ\lambda^2)\lambda + \beta^2 d_5d_8\lambda^2 \\ &\quad + \beta^2 d_1d_4d_9(\lambda + t_\circ\lambda^2)\lambda - \beta^2 d_1d_6d_8(\lambda + t_\circ\lambda^2)\lambda + \beta^2 d_2d_6d_7(\lambda + t_\circ\lambda^2)\lambda - \beta^2 d_2d_5d_9\lambda^2 + \beta^4 d_3\lambda^2 \\ &\quad + \beta^4 d_4(\lambda + t_\circ\lambda^2) + \beta^4 d_7\lambda - \beta^4 d_2d_6(\lambda + t_\circ\lambda^2) - \beta^4 d_1d_9\lambda - \beta^6 = 0 \end{aligned} \tag{30}$$

The characteristic equation (30) is the same as the characteristic equations (13) and (20). This equality is interesting as it prevents mathematical dilemma and complexity and a single value for the eigenvalue β simultaneously satisfies three characteristic equations (13), (20) and (30). Equations (24) to (29) gives the relations between $A_4, A_5, A_6, B_4, B_5, B_6, C_4, C_5$ and C_6. These relations play as the balancing ratios that help Eq. (21) to be the third solution of the system of Eqs. (9). The complete general solutions for the solid sphere are

$$\begin{aligned} U^g(r) &= A_1 J_{\frac{3}{2}}(\beta r) + A_3[\zeta_1 J_{\frac{3}{2}}(\beta r) + rJ_{\frac{5}{2}}(\beta r)] + A_6[\zeta_2 J_{\frac{3}{2}}(\beta r) + \zeta_3 rJ_{\frac{5}{2}}(\beta r) + r^2 J_{\frac{7}{2}}(\beta r)] \\ \theta^g(r) &= A_1\zeta_4 J_{\frac{1}{2}}(\beta r) + A_3[\zeta_5 J_{\frac{1}{2}}(\beta r) + \zeta_6 rJ_{\frac{3}{2}}(\beta r)] + A_6[\zeta_7 J_{\frac{1}{2}}(\beta r) + \zeta_8 rJ_{\frac{3}{2}}(\beta r) + \zeta_9 r^2 J_{\frac{5}{2}}(\beta r)] \\ P^g(r) &= A_1\zeta_{10} J_{\frac{1}{2}}(\beta r) + A_3[\zeta_{11} J_{\frac{1}{2}}(\beta r) + \zeta_{12} rJ_{\frac{3}{2}}(\beta r)] + A_6[\zeta_{13} J_{\frac{1}{2}}(\beta r) + \zeta_{14} rJ_{\frac{3}{2}}(\beta r) + \zeta_{15} r^2 J_{\frac{5}{2}}(\beta r)] \end{aligned} \tag{31}$$

and for hollow sphere are

$$\begin{aligned} U^g(r) &= A_1 J_{\frac{3}{2}}(\beta r) + A_3[\zeta_1 J_{\frac{3}{2}}(\beta r) + rJ_{\frac{5}{2}}(\beta r)] + A_6[\zeta_2 J_{\frac{3}{2}}(\beta r) + \zeta_3 rJ_{\frac{5}{2}}(\beta r) + r^2 J_{\frac{7}{2}}(\beta r)] \\ &\quad + \hat{A}_1 Y_{\frac{3}{2}}(\beta r) + \hat{A}_3[\zeta_1 Y_{\frac{3}{2}}(\beta r) + rY_{\frac{5}{2}}(\beta r)] + \hat{A}_6[\zeta_2 Y_{\frac{3}{2}}(\beta r) + \zeta_3 rY_{\frac{5}{2}}(\beta r) + r^2 Y_{\frac{7}{2}}(\beta r)] \\ \theta^g(r) &= A_1\zeta_4 J_{\frac{1}{2}}(\beta r) + A_3[\zeta_5 J_{\frac{1}{2}}(\beta r) + \zeta_6 rJ_{\frac{3}{2}}(\beta r)] + A_6[\zeta_7 J_{\frac{1}{2}}(\beta r) + \zeta_8 rJ_{\frac{3}{2}}(\beta r) + \zeta_9 r^2 J_{\frac{5}{2}}(\beta r)] \\ &\quad + \hat{A}_1\zeta_4 Y_{\frac{1}{2}}(\beta r) + \hat{A}_3[\zeta_5 Y_{\frac{1}{2}}(\beta r) + \zeta_6 rY_{\frac{3}{2}}(\beta r)] + \hat{A}_6[\zeta_7 Y_{\frac{1}{2}}(\beta r) + \zeta_8 rY_{\frac{3}{2}}(\beta r) + \zeta_9 r^2 Y_{\frac{5}{2}}(\beta r)] \\ P^g(r) &= A_1\zeta_{10} J_{\frac{1}{2}}(\beta r) + A_3[\zeta_{11} J_{\frac{1}{2}}(\beta r) + \zeta_{12} rJ_{\frac{3}{2}}(\beta r)] + A_6[\zeta_{13} J_{\frac{1}{2}}(\beta r) + \zeta_{14} rJ_{\frac{3}{2}}(\beta r) + \zeta_{15} r^2 J_{\frac{5}{2}}(\beta r)] \\ &\quad + \hat{A}_1\zeta_{10} Y_{\frac{1}{2}}(\beta r) + \hat{A}_3[\zeta_{11} Y_{\frac{1}{2}}(\beta r) + \zeta_{12} rY_{\frac{3}{2}}(\beta r)] + \hat{A}_6[\zeta_{13} Y_{\frac{1}{2}}(\beta r) + \zeta_{14} rY_{\frac{3}{2}}(\beta r) + \zeta_{15} r^2 Y_{\frac{5}{2}}(\beta r)] \end{aligned} \tag{32}$$

where ζ_1 to ζ_{15} are ratios obtained from Eqs. (23) to (29),(16) to (19) and(12) and are given in the appendix. Substituting U^g, θ^g and P^g in the homogeneous form of the boundary conditions (4), three linear algebraic equations are obtained. They are the coefficients depending on λ and β. Setting the determinant of the coefficients equal to zero, the second characteristic equation is obtained. Simultaneous solution of this equation and Eq. (11), results into infinite number of two eigenvalues β_n and λ_n. λ_n are eigenvalues in time domain and are mechanical-thermal-pressure natural frequencies and β_n are eigenvalues in space and determine mode shapes. Therefore, U^g, θ^g and P^g for solid sphere are rewritten as

$$\begin{aligned} U^g(r) &= A_1\left[J_{\frac{3}{2}}(\beta r)+\zeta_{16}[\zeta_1 J_{\frac{3}{2}}(\beta r)+rJ_{\frac{5}{2}}(\beta r)]+\zeta_{17}[\zeta_2 J_{\frac{3}{2}}(\beta r)+\zeta_3 rJ_{\frac{5}{2}}(\beta r)+r^2 J_{\frac{7}{2}}(\beta r)]\right] \\ \theta^g(r) &= A_1\left[\zeta_4 J_{\frac{1}{2}}(\beta r)+\zeta_{16}[\zeta_5 J_{\frac{1}{2}}(\beta r)+\zeta_6 rJ_{\frac{3}{2}}(\beta r)]+\zeta_{17}[\zeta_7 J_{\frac{1}{2}}(\beta r)+\zeta_8 rJ_{\frac{3}{2}}(\beta r)+\zeta_9 r^2 J_{\frac{5}{2}}(\beta r)]\right] \\ P^g(r) &= A_1\left[\zeta_{10} J_{\frac{1}{2}}(\beta r)+\zeta_{16}[\zeta_{11} J_{\frac{1}{2}}(\beta r)+\zeta_{12} rJ_{\frac{3}{2}}(\beta r)]+\zeta_{17}[\zeta_{13} J_{\frac{1}{2}}(\beta r)+\zeta_{14} rJ_{\frac{3}{2}}(\beta r)+\zeta_{15} r^2 J_{\frac{5}{2}}(\beta r)]\right] \end{aligned} \quad (33)$$

where ζ_{16} and ζ_{17} are presented in the appendix. Let us show the functions in the brackets of Eq. (33) by functions H_0, H_1 and H_2 as

$$\begin{aligned} H_0 &= J_{\frac{3}{2}}(\beta r)+\zeta_{16}[\zeta_1 J_{\frac{3}{2}}(\beta r)+rJ_{\frac{5}{2}}(\beta r)]+\zeta_{17}[\zeta_2 J_{\frac{3}{2}}(\beta r)+\zeta_3 rJ_{\frac{5}{2}}(\beta r)+r^2 J_{\frac{7}{2}}(\beta r)] \\ H_1 &= \zeta_4 J_{\frac{1}{2}}(\beta r)+\zeta_{16}[\zeta_5 J_{\frac{1}{2}}(\beta r)+\zeta_6 rJ_{\frac{3}{2}}(\beta r)]+\zeta_{17}[\zeta_7 J_{\frac{1}{2}}(\beta r)+\zeta_8 rJ_{\frac{3}{2}}(\beta r)+\zeta_9 r^2 J_{\frac{5}{2}}(\beta r)] \\ H_2 &= \zeta_{10} J_{\frac{1}{2}}(\beta r)+\zeta_{16}[\zeta_{11} J_{\frac{1}{2}}(\beta r)+\zeta_{12} rJ_{\frac{3}{2}}(\beta r)]+\zeta_{17}[\zeta_{13} J_{\frac{1}{2}}(\beta r)+\zeta_{14} rJ_{\frac{3}{2}}(\beta r)+\zeta_{15} r^2 J_{\frac{5}{2}}(\beta r)] \end{aligned} \quad (34)$$

According to the Sturm-Liouville theorem, these functions are orthogonal with respect to the weight function $p(r)=r$ such as

$$\int_{r_i}^{r_o} H(\beta_n r)H(\beta_m r)\ r\ \mathrm{d}r = \begin{Bmatrix} 0 & n\neq m \\ \|H(\beta_n r)\|^2 & n=m \end{Bmatrix} \quad (35)$$

where $\|H(\beta_n r)\|$ is norm of the H function and equals

$$\|H(\beta_n r)\| = \left[\int_{r_i}^{r_o} rH^2(\beta_n r)\ \mathrm{d}r\right]^{\frac{1}{2}} \quad (36)$$

Due to the orthogonality of function H, every piece-wise continuous function, such as $f(r)$, can be expanded in terms of the function H (either $H_\circ, H_1$ or H_2), and is called the H-Fourier series as

$$f(r) = \sum_{n=1}^{\infty} e_n\ \ H(\beta_n r) \quad (37)$$

where e_n equals

$$e_n = \frac{1}{\|H(\beta_n r)\|^2}\int_{r_i}^{r_o} f(r)\,H(r)\,r\,\mathrm{d}r \quad (38)$$

Using Eqs. (6), (33) and (34) the displacement and temperature distributions due to the general solution become

$$u^g(r,t)=\sum_{n=1}^{\infty}\ \{\sum_{m=1}^{5}\ a_{nm}e^{\lambda_{nm}t}\}H_0(\beta_n r)$$
$$T^g(r,t)=\sum_{n=1}^{\infty}\ \{\sum_{m=1}^{5}\ N_{nm}a_{nm}e^{\lambda_{nm}t}\}H_1(\beta_n r) \tag{39}$$
$$p^g(r,t)=\sum_{n=1}^{\infty}\ \{\sum_{m=1}^{5}\ M_{nm}a_{nm}e^{\lambda_{nm}t}\}H_2(\beta_n r)$$

where N_{mn} and M_{nm} are ratios obtained by substituting Eqs. (39) into Eq. (1) to (3). Using the initial conditions (5) and with the help of Eqs. (36), (37) and (38), four unknown constants are obtained.

3.4 Particular solution with non-homogeneous boundary conditions

The general solutions may be used as proper functions for guessing the particular solution adapted to the non-homogeneous parts of the Eqs. (1) to (3) and the non-homogeneous boundary conditions (4) as

$$u^p(r,t)=r^{-\frac{1}{2}}\sum_{n=1}^{\infty}\{\left[G_{1n}(t)J_{\frac{3}{2}}(\beta_n r)+G_{2n}(t)rJ_{\frac{5}{2}}(\beta_n r)+G_{3n}(t)r^2J_{\frac{7}{2}}(\beta_n r)\right]+r^2G_{4n}(t)\}$$
$$T^p(r,t)=r^{-\frac{1}{2}}\sum_{n=1}^{\infty}\{\left[G_{5n}(t)J_{\frac{1}{2}}(\beta_n r)+G_{6n}(t)rJ_{\frac{3}{2}}\beta_n r)+G_{7n}(t)r^2J_{\frac{5}{2}}(\beta_n r)\right]+r^2G_{8n}(t)\} \tag{40}$$
$$p^p(r,t)=r^{-\frac{1}{2}}\sum_{n=1}^{\infty}\{\left[G_{9n}(t)J_{\frac{1}{2}}(\beta_n r)+G_{10n}(t)rJ_{\frac{3}{2}}(\beta_n r)+G_{11n}(t)r^2J_{\frac{5}{2}}\beta_n r)\right]+r^2G_{12n}(t)\}$$

For the solid sphere, the second type of Bessel function *Y* is excluded. It is necessary and suitable to expand the body force $r^{\frac{1}{2}}F(r,t)$, heat source $r^{\frac{1}{2}}Q(r,t)$ and porosity function $r^{\frac{1}{2}}W(r,t)$ in *H*-Fourier expansion form as

$$r^{\frac{1}{2}}F(r,t)=\sum_{n=1}^{\infty}\ F_n(t)H_0(\beta_n r)$$
$$r^{\frac{1}{2}}Q(r,t)=\sum_{n=1}^{\infty}\ Q_n(t)H_1(\beta_n r) \tag{41}$$
$$r^{\frac{1}{2}}P(r,t)=\sum_{n=1}^{\infty}\ P_n(t)H_2(\beta_n r)$$

where $F_n(t)$, $Q_n(t)$ and $P_n(t)$ are

$$F_n(t)=\frac{1}{\left\|H_0(\beta_n r)\right\|^2}\int_{r_i}^{r_o}F(r,t)H_0(\beta_n r)r^{\frac{3}{2}}\ \mathrm{d}r$$
$$Q_n(t)=\frac{1}{\left\|H_1(\beta_n r)\right\|^2}\int_{r_i}^{r_o}Q(r,t)H_1(\beta_n r)r^{\frac{3}{2}}\ \mathrm{d}r \tag{42}$$
$$P_n(t)=\frac{1}{\left\|H_2(\beta_n r)\right\|^2}\int_{r_i}^{r_o}P(r,t)H_2(\beta_n r)r^{\frac{3}{2}}\ \mathrm{d}r$$

Substituting Eqs. (40) and (41) into non-homogeneous form of equations (1) into (3) yield

$$\left\{\begin{array}{l}-G_1(t)\beta^2+d_3\ddot{G}_1(t)-G_2(t)\beta+d_3\ddot{G}_2(t)\dfrac{3}{\beta}-3G_3(t)+d_3\ddot{G}_3(t)\dfrac{15}{\beta^2}+\left\{C_0+C_1\dfrac{3}{\beta}+C_2\dfrac{15}{\beta^2}\right\}d_{16}G_4(t)\\ +\left\{C_0+C_1\dfrac{3}{\beta}+C_2\dfrac{15}{\beta^2}\right\}d_{19}\ddot{G}_4(t)-G_5(t)d_2\beta-d_2G_6(t)-3d_2G_7(t)\dfrac{1}{\beta}+\left\{C_0+C_1\dfrac{3}{\beta}+C_2\dfrac{15}{\beta^2}\right\}d_{17}G_8(t)\\ -d_1G_9(t)\beta-d_1G_{10}(t)-3d_1G_{11}(t)\dfrac{1}{\beta}+\left\{C_0+C_1\dfrac{3}{\beta}+C_2\dfrac{15}{\beta^2}\right\}d_{18}G_{12}(t)-\left\{C_0+C_1\dfrac{3}{\beta}+C_2\dfrac{15}{\beta^2}\right\}d_{10}d_{13}F_n(t)\end{array}\right\}=0 \quad (43a)$$

$$\left\{\begin{array}{l}G_3(t)\beta^2-d_3\ddot{G}_3(t)-d_{16}C_2G_4(t)-d_{19}C_2\ddot{G}_4(t)+G_7(t)d_2\beta-d_{17}C_2G_8(t)-d_{18}C_2G_{12}(t)\\ +d_1G_{11}(t)\beta+d_{10}d_{13}C_2F_n(t)\end{array}\right\}=0 \quad (43b)$$

$$\left\{\begin{array}{l}G_2(t)\beta^2-d_3\ddot{G}_2(t)+G_3(t)\beta-d_3\ddot{G}_3(t)\dfrac{5}{\beta}+\left\{-C_1-C_2\dfrac{5}{\beta}\right\}d_{16}G_4(t)\\ +\left\{-C_1-C_2\dfrac{5}{\beta}\right\}d_{19}\ddot{G}_4(t)+d_2G_6(t)\beta+d_2G_7(t)-\left\{C_1+C_2\dfrac{5}{\beta}\right\}d_{17}G_8(t)+d_1G_{10}(t)\beta+d_1G_{11}(t)\\ -\left\{C_1+C_2\dfrac{5}{\beta}\right\}d_{18}G_{12}(t)+\left\{C_1F_n(t)+C_2\dfrac{5}{\beta}\right\}d_{10}d_{13}F_n(t)\end{array}\right\}=0 \quad (43c)$$

$$\left\{\begin{array}{l}+d_6\left\{\dot{G}_1(t)+t_\circ\ddot{G}_1(t)\right\}\beta+d_{21}E_0\dot{G}_4(t)-G_5(t)\beta^2+d_4\left\{\dot{G}_5(t)+t_\circ\ddot{G}_5(t)\right\}+2G_6(t)\beta+d_{20}E_0G_8(t)\\ +d_{22}E_0\dot{G}_8(t)+d_5\dot{G}_9(t)d_{23}E_0\dot{G}_{12}(t)-d_{11}d_{14}E_0Q_n(t)\end{array}\right\}=0 \quad (43d)$$

$$\left\{\begin{array}{l}-d_6\left\{\dot{G}_3(t)+t_\circ\ddot{G}_3(t)\right\}\beta-d_{21}E_2\dot{G}_4(t)+G_7(t)\beta^2-d_4\left\{\dot{G}_7(t)+t_\circ\ddot{G}_7(t)\right\}-d_{20}E_2G_8(t)-d_{22}E_2\dot{G}_8(t)-d_5\dot{G}_{11}(t)\\ -d_{23}E_2\dot{G}_{12}(t)+d_{11}d_{14}E_2Q_n(t)\end{array}\right\}=0 \quad (43e)$$

$$\left\{\begin{array}{l}d_6\left\{\dot{G}_2(t)+t_\circ\ddot{G}_2(t)\right\}\beta+3d_6\dot{G}_3(t)+\left\{E_1+E_2\dfrac{3}{\beta}\right\}d_{21}\dot{G}_4(t)-G_6(t)\beta^2+d_4\left\{\dot{G}_6(t)+t_\circ\ddot{G}_6(t)\right\}+G_7(t)\beta\\ +d_4\left\{\dot{G}_7(t)+t_\circ\ddot{G}_7(t)\right\}\dfrac{3}{\beta}+\left\{E_1+E_2\dfrac{3}{\beta}\right\}d_{20}G_8(t)+\left\{E_1+E_2\dfrac{3}{\beta}\right\}d_{22}\dot{G}_8(t)+d_5\dot{G}_{10}(t)+d_5\dot{G}_{11}(t)\dfrac{3}{\beta}\\ +\left\{E_1+E_2\dfrac{3}{\beta}\right\}d_{23}\dot{G}_{12}(t)-\left\{E_1+E_2\dfrac{3}{\beta}\right\}d_{11}d_{14}Q_n(t)\end{array}\right\}=0 \quad (43f)$$

$$\left\{\begin{array}{l}+\dot{G}_1(t)d_9\beta+D_0d_{25}\dot{G}_4(t)+d_8\dot{G}_5(t)-G_9(t)\beta^2+d_7\dot{G}_9(t)+D_0d_{26}\dot{G}_8(t)+2G_{10}(t)\beta+D_0d_{24}G_{12}(t)\\ +D_0d_{27}\dot{G}_{12}(t)-d_{12}d_{15}D_0W_n(t)\end{array}\right\}=0 \quad (43g)$$

$$\left\{\begin{array}{l}G_{11}(t)\beta^2-d_7\dot{G}_{11}(t)-d_8\dot{G}_7(t)-d_9\dot{G}_3(t)\beta-D_2r^2d_{25}\dot{G}_4(t)-D_2d_{26}\dot{G}_8(t)-D_2d_{24}G_{12}(t)\\ -D_2d_{27}\dot{G}_{12}(t)+d_{12}d_{15}D_2W_n(t)\end{array}\right\}=0 \quad (43h)$$

$$\left\{\begin{array}{l}+\dot{G}_2(t)\beta d_9+3\dot{G}_3(t)d_9+\left\{D_1+D_2\dfrac{3}{\beta}\right\}d_{25}\dot{G}_4(t)+d_8\dot{G}_6(t)+d_8\dot{G}_7(t)\dfrac{3}{\beta}\left\{D_1\dot{G}_8(t)+D_2\dfrac{3}{\beta}\right\}\dot{G}_8(t)d_{26}\\ -G_{10}(t)\beta^2+d_7\dot{G}_{10}(t)+G_{11}(t)\beta+d_7\dot{G}_{11}(t)\dfrac{3}{\beta}+\left\{D_1+D_2\dfrac{3}{\beta}\right\}d_{27}\dot{G}_{12}(t)+\left\{D_1+D_2\dfrac{3}{\beta}\right\}d_{24}G_{12}(t)\\ -\left\{+D_1+D_2\dfrac{3}{\beta}\right\}d_{12}d_{15}W_n(t)\end{array}\right\}=0 \quad (43i)$$

where d_{10} to d_{27} are the coefficients of the *H*-expansion and constant parameters presented in the appendix. By taking Laplace transform of Eq. (43) and using three boundary conditions of Eq. (4) (for solid sphere only second, forth and sixth boundary conditions are applicable), a system of algebraic equations is obtained and solved by Cramer's methods in the Laplace domain, where by the inverse Laplace transform the functions are transformed into the real time domain and finally $G_{1n}(t)$ to $G_{12n}(t)$ are calculated. In this process it is necessary to consider the following points:

1. The initial conditions (5) are considered only for the general solutions and the, initial conditions of $G_{1n}(t)$ to $G_{12n}(t)$ for the particular solutions are considered equal to zero.
2. Laplace transform of Eqs. (43) is in terms of polynomial function form of the Laplace parameter s (not the Bessel functions form of s). Therefore, the exact inverse Laplace transform is possible and somehow simple.
3. For the hollow Sphere it is enough to include the second type of Bessel function $Y(r)$ in a sequence of the particular solution as

$$\begin{aligned}
u^p(r,t) &= \sum_{n=1}^{\infty}\{\left[G_{1n}(t)J_{\frac{3}{2}}(\beta_n r)+G_{2n}(t)rJ_{\frac{5}{2}}(\beta_n r)+G_{3n}(t)r^2J_{\frac{7}{2}}(\beta_n r)\right] \\
&+\left[G_{4n}(t)Y_{\frac{3}{2}}(\beta_n r)+G_{5n}(t)rY_{\frac{5}{2}}(\beta_n r)+G_{6n}(t)r^2Y_{\frac{7}{2}}(\beta_n r)\right]+rG_{7n}(t)+r^2G_{8n}(t)\} \\
T^p(r,t) &= \sum_{n=1}^{\infty}\{\left[G_{9n}(t)J_{\frac{1}{2}}(\beta_n r)+G_{10n}(t)rJ_{\frac{3}{2}}\beta_n r)+G_{11n}(t)r^2J_{\frac{5}{2}}(\beta_n r)\right] \\
&\left[G_{12n}(t)Y_{\frac{1}{2}}(\beta_n r)+G_{13n}(t)rY_{\frac{3}{2}}\beta_n r)+G_{14n}(t)r^2Y_{\frac{5}{2}}(\beta_n r)\right]+rG_{15n}(t)+r^2G_{16n}(t)\} \\
p^p(r,t) &= \sum_{n=1}^{\infty}\{\left[G_{17n}(t)J_{\frac{1}{2}}(\beta_n r)+G_{18n}(t)rJ_{\frac{3}{2}}(\beta_n r)+G_{19n}(t)r^2J_{\frac{5}{2}}\beta_n r)\right] \\
&+\left[G_{20n}(t)J_{\frac{1}{2}}(\beta_n r)+G_{21n}(t)rJ_{\frac{3}{2}}(\beta_n r)+G_{23n}(t)r^2J_{\frac{5}{2}}\beta_n r)\right]+rG_{24n}(t)+r^2G_{25n}(t)\}
\end{aligned} \tag{44}$$

By substituting Eq. (44) in Eqs. (1) to (3), eighteen equations are obtained, where using the six boundary conditions (4) twenty four functions $G_{1n}(t)$ to $G_{24n}(t)$ are obtained for the hollow sphere.

4 RESULTS AND DISCUSSIONS

As an example, a solid sphere with $r_i = 0$, $r_o = 1$ m is considered. The material properties are listed in Table 1. To give clear explanation, numerical results have been considered and the radial distributions of displacement, temperature and pressure for two cases (Classic coupled theory and Lord-Shulman's theory) computed. An instantaneous hot spot $T(1,t) = 10^{-3}T_{\circ}\delta(t)$, where $\delta(t)$ is unit dirac function, is considered and the outside radius of the sphere is assumed to be fixed (u(1, t)=0). For plotting the graphs a nondimensional time $\hat{t} = Vt / r_{\circ}$ is considered where $V = \sqrt{E(1-\nu)/\rho(1+\nu)(1-2\nu)}$ is the dilatational wave velocity.

Table1
Material Parameters

Parameters	Value	Unit	Parameters	Value	Unit
$t_{\circ}$	1×10^{-5}	-	α_s	1.5×10^{-5}	1/^{o}C
E	6×10^{5}	Pa	α_w	2×10^{-4}	1/^{o}C
υ	0.3	-	c_s	0.8	J/g^{o}C
$T_{\circ}$	293	$^{\circ}$K	c_w	4.2	J/g^{o}C
K_s	2×10^{10}	Pa	ρ_s	2.6×10^{6}	g/m^3
K_w	5×10^{9}	Pa	ρ_w	1×10^{6}	g/m^3
K	5×10^{9}	W/m^{o}C	α	1	-

Figs 1-3 show the wave-front for the displacement, temperature, and pressure (Classic coupled theory and Lord-Shulman's theory). As a second example, mechanical shock wave is applied to the outside surface of the sphere given as $u(1,t)=10^{-12}u_{\circ}\delta(t)$ and the surface is assumed to be at zero temperature (T(1, t)=0). Figs. 4-6 show the wave fronts for the displacement and temperature distributions versus the nondimensional radius(Classic coupled theory and Lord-Shulman's theory). The convergence of the solutions for these examples is achieved by consideration of 1200 eigenvalues used for the H-Fourier expansion. By choosing more than this number for eigenvalues, round-off and truncation errors increases and the quality of the graphs are affected. The convergence of the solution is better for the displacement result in comparison with the temperature. The small oscillations in Figs. 3-5 are due to the convergence errors of solutions.

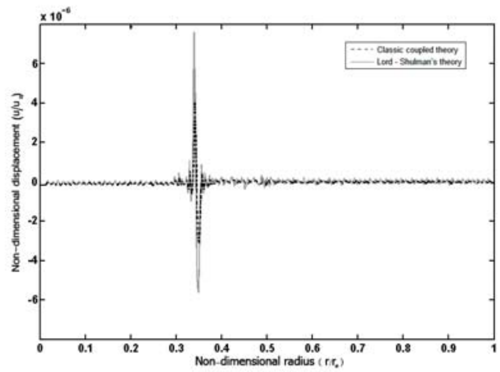

Fig. 1
Non-dimensional displacement distribution due to input T(1, t) $=10^{-3}T_0\delta(t)$ at non-dimensional time $\hat{t}=0.65$.

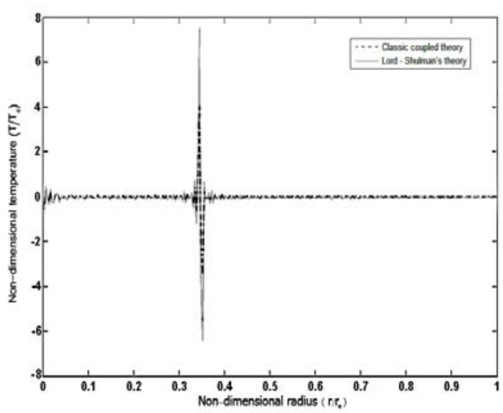

Fig. 2
Non-dimensional temperature distribution due to input T(1, t) $=10^{-3}T_0\delta(t)$ at non-dimensional time $\hat{t}=0.65$.

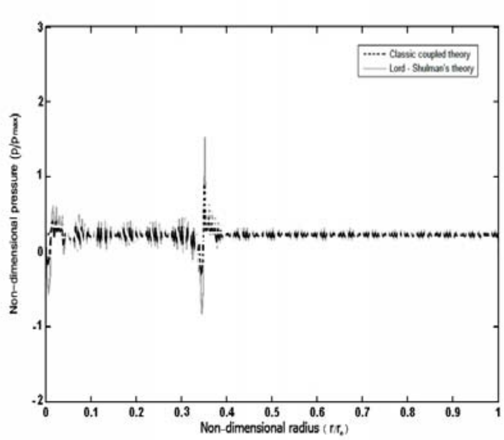

Fig. 3
Non-dimensional Pressure distribution due to input T(1, t) $=10^{-3}T_0\delta(t)$ at non-dimensional time $\hat{t}=0.65$.

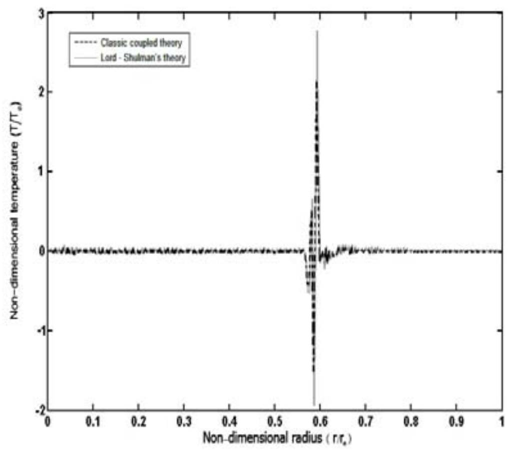

Fig. 4
Non-dimensional Displacement distribution due to input u(1, t) $=10^{-12}u_0\delta(t)$ at non-dimensional time $\hat{t}=0.4$

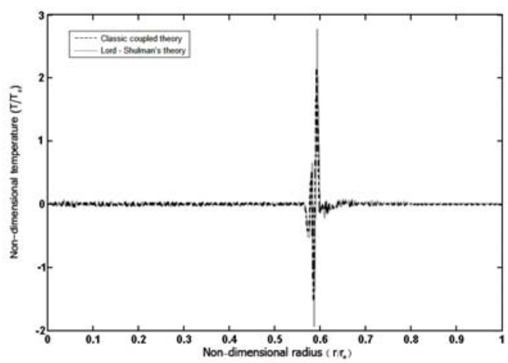

Fig. 5
Non-dimensional Temperature distribution due to input u(1, t) $=10^{-12}u_0\delta(t)$ at non-dimensional time $\hat{t}=0.4$.

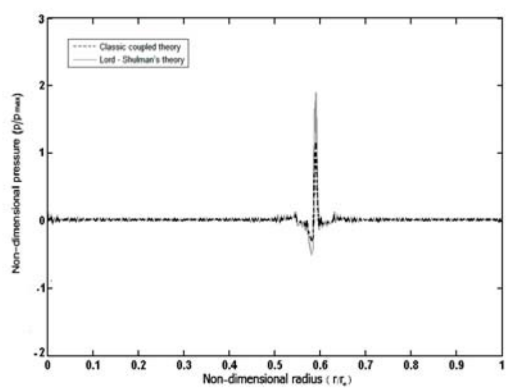

Fig. 6
Non-dimensional Pressure distribution due to input u(1, t) $=10^{-12}u_0\delta(t)$ at non-dimensional time $\hat{t}=0.4$.

5 CONCLUSIONS

In the present paper, an analytical solution for the generalized coupled thermoporoelasticity of thick spheres under radial temperature is presented. Figs (1) to (6) show relaxation time effect on variation of displacement, temperature and pressure. It is observed that the peak value of Lord-Shulman 'theory for displacement, temperature and pressure increases. The method is based on the eigenfunctions Fourier expansion, which is a classical and traditional method of solution of the typical initial and boundary value problems. The non-competetive strength of this method is its ability to reveal the fundamental mathematical and physical properties and interpretations of the problem under studying. In the coupled thermoporoelastic problem of radial-symmetric sphere, the governing equations constitute a system of partial differential equations with two independent variables, radius (r) and time (t). The traditional procedure to solve this class of problems is to eliminate the time variable using the Laplace transform. The resulting system is a set of ordinary differential equations in terms of the radius variable, whose solution falls in the Bessel functions family. This method of the analysis brings the Laplace parameter (s) in the argument of the Bessel functions, causing hardship or difficulties in carrying out the exact inverse of the Laplace transformation. As a result, the numerical inversion of the Laplace transformation is used in the papers dealing with this type of problems in literature. In the present paper, to prevent this problem, when the Laplace transform is applied to the particular solutions, it is postponed after eliminating the radius variable r by H-Fourier Expansion. Thus, the Laplace parameter (s) appears in polynomial function forms and hence the exact Laplace inversion transformation is possible.

6 ACKNOWLEDGMENT

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APPENDIX

$$d_1 = -\alpha\frac{(1+\nu)(1-2\nu)}{(1-\nu)E}, \quad d_2 = -\beta\frac{(1+\nu)(1-2\nu)}{(1-\nu)E}, \quad d_3 = -\rho\frac{(1+\nu)(1-2\nu)}{(1-\nu)E}, \quad d_4 = -Z\frac{T_\circ}{K},$$
$$d_5 = Y\frac{T_\circ}{K}, \quad d_6 = -\beta\frac{T_\circ}{K}, \quad d_7 = -\alpha_p\frac{\gamma_w}{k}\frac{1}{M}, \quad d_8 = Y\frac{\gamma_w}{k}, \quad d_9 = -\alpha\frac{\eta}{k}, \quad d_{10} = -\frac{(1+\nu)(1-2\nu)}{(1-\nu)E},$$
$$d_{11} = -\frac{1}{K}, \quad d_{12} = \frac{\eta}{K}, \quad d_{13} = \int_0^1 F(r)r\,\mathrm{d}r, \quad d_{14} = \int_0^1 G(r)r\,\mathrm{d}r, \quad d_{15} = \int_0^1 W(r)r\,\mathrm{d}r, \quad d_{16} = \frac{7}{4}\int_0^1 r\,\mathrm{d}r,$$
$$d_{17} = \frac{3}{2}d_2\int_0^1 r^2\mathrm{d}r, \quad d_{18} = \frac{3}{2}d_2\int_0^1 r^2\mathrm{d}r, \quad d_{19} = d_3\int_0^1 r^3\mathrm{d}r, \quad d_{20} = \frac{15}{4}\int_0^1 r\mathrm{d}r, \quad d_{21} = \frac{7}{2}d_6\int_0^1 r^2\mathrm{d}r,$$
$$d_{22} = d_4\int_0^1 r^3\mathrm{d}r, \quad d_{23} = d_5\int_0^1 r^3\mathrm{d}r, \quad d_{24} = \frac{15}{4}\int_0^1 r\mathrm{d}r, \quad d_{25} = \frac{7}{2}d_9\int_0^1 r^2\mathrm{d}r \quad d_{26} = d_8\int_0^1 r^3\mathrm{d}r,$$
$$d_{27} = d_7\int_0^1 r^3\mathrm{d}r \tag{A.1}$$

$$\zeta_1 = \zeta_5 = \zeta_{12} = \frac{\left\{m_4 - \frac{m_1 m_6}{m_3}\right\}}{\left\{\frac{m_2 m_6}{m_3} - m_5\right\}} \qquad \zeta_2 = \zeta_7 = \zeta_{15} = \left\{-\frac{m_1}{m_3} - \frac{m_2}{m_3}\zeta_1\right\}$$

$$\xi_3 = \frac{\left\{\left\{m_8 - m_9\frac{m_2}{m_3}\right\}\frac{\left\{m_{11}\zeta_5 - \frac{\{m_{10} - d_2\zeta_5 - d_1\zeta_7\}m_6}{m_3}\right\}}{\left\{m_5 - \frac{m_2 m_6}{m_3}\right\}} - m_{11}\zeta_7 + m_9\frac{\{m_{10} - d_2\zeta_5 - d_1\zeta_7\}}{m_3}\right\}}{\left\{m_7 - \frac{\left\{m_4 - \frac{m_1 m_6}{m_3}\right\}}{\left\{m_5 - \frac{m_2 m_6}{m_3}\right\}}m_8 - m_9\frac{m_1}{m_3}A_2 + m_9\frac{m_2}{m_3}\frac{\left\{m_4 - \frac{m_1 m_6}{m_3}\right\}}{\left\{m_5 - \frac{m_2 m_6}{m_3}\right\}}\right\}}$$

$$\xi_4 = \left\{ -\frac{\left\{ m_4 - \frac{m_1 m_6}{m_3} \right\}}{\left\{ m_5 - \frac{m_2 m_6}{m_3} \right\}} \xi_3 - \frac{\left\{ m_{11}\zeta_5 - \frac{\left\{ m_{10} - d_2\zeta_5 - d_1\zeta_7 \right\} m_6}{m_3} \right\}}{\left\{ m_5 - \frac{m_2 m_6}{m_3} \right\}} \right\}$$

$$\xi_6 = \left\{ -\frac{m_1}{m_3}\xi_3 - \frac{\left\{ m_{10} - d_2\zeta_5 - d_1\zeta_7 \right\}}{m_3} - \frac{m_2}{m_3}\xi_4 \right\}$$

$$\xi_8 = -\frac{\left\{ \begin{array}{l} \frac{\left\{ -m_3\frac{m_{11}}{m_6} - d_2 - m_{14}\frac{m_8}{m_9} \right\}}{\left\{ m_2 - m_3\frac{m_5}{m_6} \right\}\left\{ m_2 - m_3\frac{m_8}{m_9} \right\}} \left\{ \left\{ -m_{24} - m_3\frac{m_{26}}{m_9} \right\} + \left\{ -m_9\frac{m_5}{m_6} + m_8 \right\}\left\{ m_1 - m_3\frac{m_7}{m_9} \right\}\zeta_9 \right\} \\ + \frac{\left\{ -m_9\frac{m_{11}}{m_6} - m_{11}\frac{m_8}{m_9} \right\}}{\left\{ m_2 - m_3\frac{m_8}{m_9} \right\}} \left\{ -\left\{ -m_{24} - m_3\frac{m_{26}}{m_9} \right\} - \left\{ m_1 - m_3\frac{m_7}{m_9} \right\}\zeta_9 \right\} - m_{11}\frac{m_{26}}{m_9} - m_{11}\frac{m_7}{m_9}\zeta_9 \\ + \frac{\left\{ -m_9\frac{m_5}{m_6} + m_8 \right\}}{\left\{ m_2 - m_3\frac{m_5}{m_6} \right\}} \left\{ \begin{array}{l} -\left\{ m_{23} - m_{14}\frac{m_{26}}{m_9} \right\} + \frac{\left\{ m_{10} - m_{14}\frac{m_7}{m_9} \right\}}{\left\{ m_2 - m_3\frac{m_8}{m_9} \right\}}\left\{ m_1 - m_3\frac{m_7}{m_9} \right\}\zeta_9 \\ + \frac{\left\{ m_{10} - m_{14}\frac{m_7}{m_9} \right\}}{\left\{ m_2 - m_3\frac{m_8}{m_9} \right\}}\left\{ -m_{24} - m_3\frac{m_{26}}{m_9} \right\} \end{array} \right\} \end{array} \right\}}{\left\{ m_7 - m_9\frac{m_4}{m_6} \right\} - \left\{ -m_9\frac{m_5}{m_6} + m_8 \right\}\frac{\left\{ m_1 - m_3\frac{m_4}{m_6} \right\}}{\left\{ m_2 - m_3\frac{m_5}{m_6} \right\}}}$$

$$\zeta_9 = -\frac{\left\{ \left\{ +m_{25} - m_6\frac{m_{26}}{m_9} \right\} - \left\{ m_5 - m_6\frac{m_8}{m_9} \right\}\frac{\left\{ -m_{24} - m_3\frac{m_{26}}{m_9} \right\}}{\left\{ m_2 - m_3\frac{m_8}{m_9} \right\}} \right\}}{\left\{ \left\{ m_4 - m_6\frac{m_7}{m_9} \right\} - \left\{ m_5 - m_6\frac{m_8}{m_9} \right\}\frac{\left\{ m_1 - m_3\frac{m_7}{m_9} \right\}}{\left\{ m_2 - m_3\frac{m_8}{m_9} \right\}} \right\}}$$

$$\zeta_{10}=\frac{1}{\left\{m_2-m_3\dfrac{m_5}{m_6}\right\}}\left\{\begin{array}{l}-\left\{m_1-m_3\dfrac{m_4}{m_6}\right\}\xi_8+\left\{m_{10}-m_{14}\dfrac{m_7}{m_9}\right\}\dfrac{\left\{-m_{24}-m_3\dfrac{m_{26}}{m_9}\right\}}{\left\{m_2-m_3\dfrac{m_8}{m_9}\right\}}-\left\{m_{23}-m_{14}\dfrac{m_{26}}{m_9}\right\}\\ +\dfrac{\left\{m_1-m_3\dfrac{m_7}{m_9}\right\}}{\left\{m_2-m_3\dfrac{m_8}{m_9}\right\}}\left\{\left\{m_{10}-m_{14}\dfrac{m_7}{m_9}\right\}\zeta_9+\left\{-m_3\dfrac{m_{11}}{m_6}-d_2-m_{14}\dfrac{m_8}{m_9}\right\}\zeta_9\right\}\\ +\left\{-m_3\dfrac{m_{11}}{m_6}-d_2-m_{14}\dfrac{m_8}{m_9}\right\}\dfrac{\left\{-m_{24}-m_3\dfrac{m_{26}}{m_9}\right\}}{\left\{m_2-m_3\dfrac{m_8}{m_9}\right\}}\end{array}\right\}$$

$$\zeta_{11}=\left\{-\frac{\left\{m_1-m_3\dfrac{m_7}{m_9}\right\}}{\left\{m_2-m_3\dfrac{m_8}{m_9}\right\}}\zeta_9-\frac{\left\{-m_{24}-m_3\dfrac{m_{26}}{m_9}\right\}}{\left\{m_2-m_3\dfrac{m_8}{m_9}\right\}}\right\}$$

$$\zeta_{13}=\left\{-\frac{m_4}{m_6}\xi_8-\frac{m_5}{m_6}\zeta_{10}-\frac{m_{11}}{m_6}\zeta_{11}\right\}\qquad \zeta_{14}=\left\{-\frac{m_{26}}{m_9}-\frac{m_8}{m_9}\zeta_{11}-\frac{m_7}{m_9}\zeta_9\right\}$$

$$\zeta_{16}=-\frac{-J_{\frac{3}{2}}(\beta)(\zeta_7J_{\frac{1}{2}}(\beta)+\zeta_8J_{\frac{3}{2}}(\beta)+\zeta_9J_{\frac{5}{2}}(\beta))+J_0(\beta)(\zeta_2J_{\frac{3}{2}}(\beta)+\zeta_3J_{\frac{5}{2}}(\beta)+J_{\frac{7}{2}}(\beta))}{-(\zeta_1J_{\frac{3}{2}}(\beta)+J_{\frac{5}{2}}(\beta))(\zeta_7J_{\frac{1}{2}}(\beta)+\zeta_8J_{\frac{3}{2}}(\beta)+\zeta_9J_{\frac{5}{2}}(\beta))+(\zeta_5J_{\frac{1}{2}}(\beta)+\zeta_6J_{\frac{3}{2}}(\beta))(\zeta_2J_{\frac{3}{2}}(\beta)+\zeta_3J_{\frac{5}{2}}(\beta)+J_{\frac{5}{2}}(\beta))}$$

$$\zeta_{17}=\frac{J_{\frac{3}{2}}(\beta)+(\zeta_1J_{\frac{3}{2}}(\beta)+\zeta_{16}J_{\frac{5}{2}}(\beta))}{\zeta_{\frac{5}{2}}J_{\frac{3}{2}}(\beta)+\zeta_3J_{\frac{5}{2}}(\beta)+J_{\frac{5}{2}}(\beta)}a \tag{A.2}$$

$$m_{21}=3(\lambda+t_\circ\lambda^2)d_6,\qquad m_{22}=3d_9\lambda,\qquad m_{23}=m_{12}+m_{13}\zeta_{12}+m_{15}\zeta_{15},$$

$$m_{24}=m_{16}-m_{14}\zeta_{15}-\frac{1}{2}d_1\zeta_{15}+d_2\zeta_{12},\qquad m_{25}=m_{21}+m_{17}\zeta_{12}+m_{19}\zeta_{15},\qquad m_{26}=m_{18}\zeta_{15}+m_{20}\zeta_{12}+m_{22},$$

$$m_1=-\beta^2+\lambda^2d_3,\qquad m_2=-d_2\beta,\qquad m_3=-d_1\beta,\qquad m_4=(\lambda+t_\circ\lambda^2)d_6\beta,\qquad m_5=-\beta^2+(\lambda+t_\circ\lambda^2)d_4,$$

$$m_6=\lambda d_5,\qquad m_7=\beta\lambda d_9,\qquad m_8=\lambda d_8,\qquad m_9=-\beta^2+\lambda d_7,\qquad m_{10}=d_3\lambda^2\frac{3}{\beta}-\beta,\qquad m_{11}=2\beta,$$

$$m_{12}=-3+d_3\lambda^2\frac{15}{\beta^2},\qquad m_{13}=-3d_2\frac{1}{\beta},\qquad m_{14}=-\frac{3}{2}d_1,\qquad m_{15}=-\frac{3}{\beta}d_1,\qquad m_{16}=\beta-\frac{5}{\beta}d_3\lambda^2,$$

$$m_{17}=(\lambda+t_\circ\lambda^2)d_4\frac{3}{\beta}+\beta\qquad m_{18}=\beta+d_7\lambda\frac{3}{\beta},\qquad m_{19}=d_5\frac{3}{\beta}\lambda,\qquad m_{20}=d_8\frac{3}{\beta}\lambda, \tag{A.3}$$

$$C_0=1+\zeta_{16}+\zeta_{17},\qquad C_1=\zeta_1\zeta_{16}+\zeta_2\zeta_{17},\qquad C_2=\zeta_3\zeta_{17},$$

$$E_0=\zeta_4+\zeta_5\zeta_{16}+\zeta_7\zeta_{17},\qquad E_1=\zeta_6\zeta_{16}+\zeta_8\zeta_{17},\qquad E_2=\zeta_9\zeta_{17},$$

$$D_0=\zeta_{10}+\zeta_{11}\zeta_{16}+\zeta_{13}\zeta_{17},\qquad D_1=\zeta_{12}\zeta_{16}+\zeta_{14}\zeta_{17},\qquad D_2=\zeta_{15}\zeta_{17},$$

$$H_0=C_0J_{\frac{3}{2}}(\beta r)+C_1rJ_{\frac{5}{2}}(\beta r)+C_2r^2J_{\frac{7}{2}}(\beta r),\qquad H_1=E_0J_{\frac{1}{2}}(\beta r)+E_1rJ_{\frac{3}{2}}(\beta r)+E_2r^2J_{\frac{5}{2}}(\beta r),$$

$$H_2=D_0J_{\frac{1}{2}}(\beta r)+D_1rJ_{\frac{3}{2}}(\beta r)+D_2r^2J_{\frac{5}{2}}(\beta r), \tag{A.4}$$

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