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Boundary Value Problems in Generalized Thermodiffusive Elastic Medium

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ABSTRACT

In the present study, the boundary value problems in generalized thermodiffusive elastic medium has been investigated as a result of inclined load. The inclined load is assumed to be a linear combination of normal load and tangential load. Laplace transform with respect to time variable and Fourier transform with respect to space variable are applied to solve the problem. As an application of the approach, distributed sources and moving force have been taken. Expressions of displacement, stresses, temperature and concentration in the transformed domain are obtained by introducing potential functions. The numerical inversion technique is used to obtain the solution in the physical domain. Graphical representation due to the response of different sources and use of angle of inclination are shown. Some particular cases are also deduced.

Keywords: Generalized thermodiffusion; Inclined load; Distributed sources; Moving force; Concentration.

1 INTRODUCTION

DANILOUSKAYA [1] was the first to solve a problem in the theory of elasticity with non-uniform heat known as theory of uncoupled elasticity. In this theory, the temperature change is governed by a partial differential equation which doesn't contain any elastic terms. Later on, many attempts were made to remove the shortcomings of this theory. Thermoelasticity theories, which admit a finite speed for thermal signals, have been receiving a lot of attention for the past four decades. In contrast to the conventional coupled thermoelasticity theory based on a parabolic heat equation Biot [2] which predicts an infinite speed for the propagation of heat, these theories involve a hyperbolic heat equation and are referred to as generalized thermoelasticity theories.

Considered in [3] is a wave-type heat equation by postulating a new law of heat conduction (the Maxwell-Catlaneo equation) to replace the classical Fourier law. Because the heat equation of this theory is of wave type, it automatically ensures finite speeds of propagation of heat and elastic waves. The remaining governing equations for this theory, namely, the equations of motions and constitutive relations remain the same as those for the coupled and the uncoupled theories. Diffusion can be defined as the random walk, of an ensemble of particles, from regions of great concentration to regions of lower concentration. There is now a great deal of interest in the study of this phenomenon, due to its many applications in geophysics and industrial applications. In integrated circuit fabrication, diffusion is used to introduce "dopants" in controlled amounts into the semiconductor substrate. In particular, diffusion is used to form the base and emitter in bipolar transistors, form integrated resistors, form the source/drain regions in MOS transistors and dope poly-silicon gates in MOS transistors.

Nowacki [4-7] developed the theory of coupled themoelastic diffusion. This implies infinite speeds of propagation of theromelastic waves. Olesiak and Pyryev [8] discussed a coupled quasi-stationary problem of

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thermodiffusion for an elastic cylinder. They studied the influence of cross effects. The thermal excitation results in an additional to mass concentration and the mass concentration generates the additional filed of temperature. Sherief et. al [9] developed the theory of generalized thermoelastic diffusion with one relaxation time, which allows the finite speed of propagation of waves. Sherief and Shaleh [10] investigated a half space problem in the theory of generalized thermoelastic diffusion with one relaxation time. Singh [11, 12] investigated the reflection of P and SV waves at the free surface of generalized thermoelastic diffusion. Aouadi [13-17] investigated the different types of problems in thermoelastic diffusion. Sharma et.al [18, 19] and Kumar and Kansal [20] study various types of problem in thermoelastic diffusion. Kumar and Rani [21], Kumar and Ailawalia [22] and Kumar and Gupta [23] investigated various problems in different medium due to inclined load. Recently, Sharma et al. [24, 25] and Sherief and El-Maghraby [26] discussed different source problems in generalized thermoelastic diffusion. The deformation at any point of the medium is useful to analyze the deformation field around mining tremors and drilling into the crust of the earth. It can also contribute to the theoretical consideration of the seismic and volcanic sources since it can account for the deformation field in the entire volume surrounding the source region.

The present investigation seeks to determine the components of stress, temperature distribution and concentration due to distributed and moving sources due to inclined load in generalized thermodiffusive elastic medium. The results of the present problem may be applied to a wide class of geographical problems involving temperature shape and concentration. Physical applications are found in the mechanical engineering, geophysical and indusrial activities.

2 BASIC EQUATIONS

Following Sherief et al. [9], the governing equations for an isotropic homogeneous elastic solid generalized thermodiffusive elastic solid in the absence of body forces, diffusive mass and heat sources include:

The constitutive relations:

$$t_{ij} = 2\mu e_{ij} + \delta_{ij}(\lambda e_{kk} - \beta_1 T - \beta_2 C) \tag{1}$$

$$P = -\beta_2 e_{kk} + bC - aT \tag{2}$$

The equations of motion:

$$\mu u_{i,jj} + (\lambda+\mu)u_{j,ij} - \beta_1 T_{,i} - \beta_2 C_{,i} = \rho \ddot{u}_i \tag{3}$$

The equation of heat conduction:

$$\rho C_E(\dot{T} + \tau_0 \ddot{T}) + \beta_1 T_0(\dot{e} + \tau_0 \ddot{e}) + aT_0(\dot{C} + \tau_0 \ddot{C}) = KT_{,ii} \tag{4}$$

Equation of mass diffusion:

$$D\beta_2 e_{,ii} + DaT_{,ii} + (\dot{C} + \tau^0 \ddot{C}) - DbC_{,ii} = 0 \tag{5}$$

where

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \qquad (i, j = 1,2,3)$$

$$\beta_1 = (3\lambda + 2\mu)\alpha_t, \qquad \beta_2 = (3\lambda + 2\mu)\alpha_c$$

where λ and μ are Lame's constants, α_t is coefficient of linear thermal expansion, α_c is coefficient of linear diffusion expansion, $T = T - T_0$, where T is absolute temperature and T_0 is temperature of the medium in the natural state assummed to be such that $|T/T_0| < 1$, t_{ij} is component of the stress tensor, u_i is component of the displacement vector, ρ is density, e_{ij} is components of the strain tensor, $e = e_{kk}$, P is chemical potential per unit mass, C is concentration, C_E is specific heat at constant strain and K is coefficient of thermal conductivity , D is thermoelastic diffusion constant. τ_0 is thermal relaxation time, τ^0 is diffusion relaxation time, a and b- constants, the superposed dots denote derivatives with respect to time. δ_{ij} is the Kronecker delta.

3 FORMULATION AND SOLUTION OF THE PROBLEM

Consider an isotropic, homogeneous generalized thermodiffusive elastic medium in the undeformed state at temperature T_0. The rectangular cartesian co-ordinate system (x_1, x_2, x_3) having origin on the surface $x_3 = 0$ with x_3- axis pointing normally into the medium is introduced. Suppose that an inclined line load , per unit length, is acting on the x_2-axis and its inclination with the x_3-axis is δ in Fig. 1 as shown in Appendix A. For two dimensional problem, we take

$$\vec{u} = (u_1, 0, u_3) \tag{6}$$

The initial and regularity conditions are given by

$$\begin{aligned}
&u_1(x_1, x_3, 0) = 0 = \dot{u}_1(x_1, x_3, 0), \quad u_3(x_1, x_3, 0) = 0 = \dot{u}_3(x_1, x_3, 0),\\
&T(x_1, x_3, 0) = 0 = \dot{T}(x_1, x_3, 0), \quad C(x_1, x_3, 0) = 0 = \dot{C}(x_1, x_3, 0),\\
&P(x_1, x_3, 0) = 0 = \dot{P}(x_1, x_3, 0) \quad \text{for} \quad x_3 \ge 0, -\infty < x_1 < \infty\\
&u_1(x_1, x_3, t) = u_3(x_1, x_3, t) = T(x_1, x_3, t) = C(x_1, x_3, t) = P(x_1, x_3, t) = 0
\end{aligned}$$

for $t > 0$ when $x_3 \to \infty$. To facilitate the solution, following dimensionless quantities are introduced:

$$\begin{aligned}
&x_1' = \frac{\omega_1^*}{c_1} x_1, \quad x_3' = \frac{\omega_1^*}{c_1} x_3, \quad u_1' = \frac{\omega_1^*}{c_1} u_1, \quad u_3' = \frac{\omega_1^*}{c_1} u_3, \quad t' = \omega_1^* t,\\
&t_{31}' = \frac{t_{31}}{\beta_1 T_0}, \quad t_{33}' = \frac{t_{33}}{\beta_1 T_0}, \quad T' = \frac{\beta_1}{\rho c_1^2} T, \quad C' = \frac{\beta_2 C}{\rho c_1^2},\\
&P' = \frac{P}{\beta_2}, \quad \gamma' = \omega_1 \gamma, \quad \tau_0' = \omega_1^* \tau_0, \quad \tau^{0'} = \omega_1^* \tau^0
\end{aligned} \tag{7}$$

where

$$c_1^2 = \frac{\lambda + 2\mu}{\rho} \quad \text{and} \quad \omega_1^* = \frac{\rho C_E c_1^2}{K}$$

The displacement components, $u_1(x_1, x_3, t)$ and $u_3(x_1, x_3, t)$, may be written in terms of the potential functions $\phi(x_1, x_3, t)$ and $\psi(x_1, x_3, t)$ in dimensionless form are given by

$$u_1 = \frac{\partial \phi}{\partial x_1} - \frac{\partial \psi}{\partial x_3}, \quad u_3 = \frac{\partial \phi}{\partial x_3} + \frac{\partial \psi}{\partial x_1} \tag{8}$$

Laplace and Fourier transformsare defined as

$$\hat{f}(x_1, x_3, s) = \int_o^\infty e^{-st} f(x_1, x_3, t) dt$$
$$\tilde{f}(\xi, x_3, s) = \int_{-\infty}^\infty e^{i\xi x_1} \hat{f}(x_1, x_3, s) dx_1 \tag{9}$$

Applying Laplace and Fourier transform defined by Eq. (9) on Eqs (3)-(5), after using Eqs. (6)-(8)(suppressing the primes for convenience) and eliminating $\tilde{\phi}$, $\tilde{T}$, $\tilde{C}$ and $\tilde{\psi}$ from the resulting expressions, we obtain

$$(\frac{d^6}{dx_3^6} + R\frac{d^4}{dx_3^4} + Q\frac{d^2}{dx_3^2} + S)(\tilde{\phi}, \tilde{T}, \tilde{C}) = 0 \tag{10}$$

$$(\frac{d^2}{dz^2} - \lambda_4^2)\tilde{\psi} = 0 \tag{11}$$

where

$$R = \frac{F - 3\xi^2 E}{E}, \quad Q = \frac{G - 2F\xi^2 + 3\xi^4 E}{E}, \quad S = \frac{F\xi^4 - G\xi^2 + H - E\xi^6}{E},$$
$$\lambda_4^2 = \xi^2 + \frac{s^2}{\delta}, \quad E = 1 - \varepsilon_2, \quad F = \varepsilon_2(f_1 + a_2 f_2 + s^2) + \varepsilon_1 f_1(\varepsilon_2 + 2a_1 + a_1^2) - f_1,$$
$$G = -[\varepsilon_2 a_2 f_2 (f_1 + s^2 + \varepsilon_1 f_1) + s^2 f_1(\varepsilon_2 + \varepsilon_1 a_1^2)], \quad H = \varepsilon_2 a_2 s^2 f_1 f_2,$$
$$a_1 = \frac{a(\lambda + 2\mu)}{\beta_1 \beta_2}, \quad a_2 = \frac{K}{bD\rho C_E}, \quad \delta = \frac{\mu}{\lambda + 2\mu}, \quad \varepsilon_1 = \frac{\beta_1^2 T_0}{\rho C_E(\lambda + 2\mu)},$$
$$\varepsilon_2 = \frac{b(\lambda + 2\mu)}{\beta_2^2}, \quad f_1 = s(1 + \tau_0 s), \quad f_2 = s(1 + \tau^0 s)$$

The roots of Eq.(10) are $\pm\lambda_i (i = 1, 2, 3)$ and the root of Eq. (11) is $\pm\lambda_4$. Making use of the radiation conditions that $\tilde{\phi}, \tilde{T}, \tilde{C}, \tilde{\psi} \to 0$ as $x_3 \to \infty$, the solutions of Eqs .(10) and (11) can be written as

$$\tilde{\phi} = A_1 e^{-\lambda_1 x_3} + A_2 e^{-\lambda_2 x_3} + A_3 e^{-\lambda_3 x_3} \tag{12}$$
$$\tilde{T} = d_1 A_1 e^{-\lambda_1 x_3} + d_2 A_2 e^{-\lambda_2 x_3} + d_3 A_3 e^{-\lambda_3 x_3} \tag{13}$$
$$\tilde{C} = e_1 A_1 e^{-\lambda_1 x_3} + e_2 A_2 e^{-\lambda_2 x_3} + e_3 A_3 e^{-\lambda_3 x_3} \tag{14}$$
$$\tilde{\psi} = A_4 e^{-\lambda_4 x_3} \tag{15}$$

where

$$d_i = \frac{\lambda_i^4 P_1 - \lambda_i^2 P_2 + P_3}{P_{12}}, \quad e_i = \frac{\lambda_i^6 + \lambda_i^4 P_4 + P_5}{P_{12}}, \quad P_{12} = \varepsilon_2 \lambda_i^4 - \lambda_i^2 P_{10} + P_{11}, \quad \text{(i=1,2,3)}$$
$$P_1 = \varepsilon_1(a_1 + \varepsilon_2 f_1), \quad P_2 = 2P_1\xi^2 + P_6, \quad P_3 = P_1\xi^4 + P_6\xi^2, \quad P_4 = P_5 - 3\xi^2,$$
$$P_5 = 3\xi^4 - 2P_9\xi^2 + P_9\xi^4, \quad P_6 = \varepsilon_1 \varepsilon_2 a_2 f_1 f_2, \quad P_7 = \varepsilon_2(f_1 + a_2 f_2) + \varepsilon_1 a_1^2 f_1,$$
$$P_8 = \varepsilon_2 a_2 f_1 f_2, \quad P_9 = f_1(\varepsilon_1 a_1 - 1), \quad P_{10} = P_7 + 2\varepsilon_2 f_1 \xi^2, \quad P_{11} = \varepsilon_2 \xi^4 + P_7 \xi^2 + P_8$$

4 BOUNDARY CONDITIONS

Consider a normal line load F_1 per unit length, acting in the positive x_3-axis on the plane boundary $x_3 = 0$ along the x_2-axis and a tangential line load F_2 per unit length, acting at the origin in the positive x_1- axis, then boundary conditions are

$$(i)\ t_{33} = -F_1\psi_1(x_1)H(t) \tag{16}$$

$$(ii)\ t_{31} = -F_2\psi_2(x_1)H(t) \tag{17}$$

$$(iii)\ \frac{\partial T}{\partial x_3} = 0 \tag{18}$$

$$(iv)\ \frac{\partial C}{\partial x_3} = 0 \tag{19}$$

where $H(t) = 1$ for $t \geq 0$, $H(t) = 0$ for $t < 0$, F_1 and F_2 are the magnitude of forces, $\psi_1(x)$ and $\psi_2(x)$ specify the vertical and horizontal load distributions respectively as shown in appendix I, Fig.2. $H(t)$ is the Heavy step unit function. Making use of Eqs.(7) and (8) in the boundary conditions Eqs. (16)-(19) and applying the Laplace and Fourier transforms defined by Eq. (9), then substituting values of $\tilde{\phi}, \tilde{T}, \tilde{\psi}, \tilde{C}$ from the Eqs. (12)-(15), we obtain the expressions of displacement stresses, temperature distribution and concentration as

$$\tilde{u}_1 = -i\xi(A_{11}e^{-\lambda_1 x_3} + A_{12}e^{-\lambda_2 x_3} + A_{13}e^{-\lambda_3 x_3}) + \lambda_4 A_{14}e^{-\lambda_4 x_3} \tag{20}$$

$$\tilde{u}_3 = -(\lambda_1 A_{11}e^{-\lambda_1 x_3} + \lambda_2 A_{12}e^{-\lambda_2 x_3} + \lambda_3 A_{13}e^{-\lambda_3 x_3} + i\xi A_{14}e^{-\lambda_4 x_3}) \tag{21}$$

$$\tilde{t}_{33} = S_1 A_{11}e^{-\lambda_1 x_3} + S_2 A_{12}e^{-\lambda_2 x_3} + S_3 A_{13}e^{-\lambda_3 x_3} + S_4 A_{14}e^{-\lambda_4 x_3} \tag{22}$$

$$\tilde{t}_{31} = R_1 A_{11}e^{-\lambda_1 x_3} + R_2 A_{12}e^{-\lambda_2 x_3} + R_3 A_{13}e^{-\lambda_3 x_3} + R_4 A_{14}e^{-\lambda_4 x_3} \tag{23}$$

$$\tilde{T} = d_1 A_{11}e^{-\lambda_1 x_3} + d_2 A_{12}e^{-\lambda_2 x_3} + d_3 A_{13}e^{-\lambda_3 x_3} \tag{24}$$

$$\tilde{C} = e_1 A_{11}e^{-\lambda_1 x_3} + e_2 A_{12}e^{-\lambda_2 x_3} + e_3 A_{13}e^{-\lambda_3 x_3} \tag{25}$$

where

$$A_{11} = \frac{W_1(-R_4F_1\tilde{\psi}_1(\xi) + S_4F_2\tilde{\psi}_2(\xi))}{\Delta}, \quad A_{12} = \frac{W_2(R_4F_1\tilde{\psi}_1(\xi) - S_4F_2\tilde{\psi}_2(\xi))}{\Delta},$$

$$A_{13} = \frac{W_3(-R_4F_1\tilde{\psi}_1(\xi) + S_4F_2\tilde{\psi}_2(\xi))}{\Delta}, \quad g_1 = \frac{\lambda}{\beta_1 T_0}, \quad g_2 = \frac{\rho c_1^2}{\beta_1 T_0}, \quad g_3 = \frac{\mu}{\beta_1 T_0},$$

$$A_{14} = \frac{F_1\tilde{\psi}_1(\xi)(W_1R_1 - W_2R_2 + W_3R_3) - F_2\tilde{\psi}_2(\xi)(W_1S_1 - W_2S_2 + W_3S_3)}{\Delta}$$

$$Si = -\xi^2 g_1 + \lambda_i^2 g_2 - a_i - b_i, S_4 = \imath\xi\lambda_4(g_2 - g_1), R_i = 2\imath\xi\lambda_i g_3, R_4 = -g_3(\lambda_4^2 + \xi^2)$$

$$W_1 = \frac{\lambda_2\lambda_3(d_2e_3 - d_3e_2)}{s}, \quad W_2 = \frac{\lambda_1\lambda_3(d_1e_3 - d_3e_1)}{s}, \quad W_3 = \frac{\lambda_1\lambda_2(d_1e_2 - d_2e_1)}{s}$$

and

$$\Delta = \begin{vmatrix} S_1 & S_2 & S_3 & S_4 \\ R_1 & R_2 & R_3 & R_4 \\ \lambda_1 d_1 & \lambda_2 d_2 & \lambda_3 d_3 & 0 \\ \lambda_1 e_1 & \lambda_2 e_2 & \lambda_3 e_3 & 0 \end{vmatrix}$$

5 PARTICULAR CASE

Neglecting diffusion effect $(\beta_2 = b = a = 0)$: in Eqs. (20)-(25), we obtain the corresponding expression for thermoelastic medium.

$$\tilde{u}_1 = -i\xi(B_{11}e^{-\lambda_1 x_3} + B_{12}e^{-\lambda_2 x_{13}}) + \lambda_4 B_{13}e^{-\lambda_4 x_3} \tag{26}$$

$$\tilde{u}_3 = -(\lambda_1(B_{11}e^{-\lambda_1 x_3} + \lambda_2 B_{12}e^{-\lambda_2 x_3} + i\xi B_{13}e^{-\lambda_4 x_3}) \tag{27}$$

$$\tilde{t}_{33} = S_1 B_{11}e^{-\lambda_1 x_3} + S_2 B_{12}e^{-\lambda_2 x_3} + S_4 B_{13}e^{-\lambda_4 x_3} \tag{28}$$

$$\tilde{t}_{31} = R_1 B_{11}e^{-\lambda_1 x_3} + R_2 B_{12}e^{-\lambda_2 x_3} + R_4 B_{13}e^{-\lambda_4 x_3} \tag{29}$$

$$\tilde{T} = d_1 B_{11}e^{-\lambda_1 x_3} + d_2 B_{12}e^{-\lambda_2 x_3} \tag{30}$$

where

$$B_{11} = \frac{X_1[F_1 R_4 \tilde{\psi}_1(\xi) - F_2 S_4 \tilde{\psi}_2(\xi)]}{\Delta_0}, \quad B_{12} = -\frac{X_2[-F_1 R_4 \tilde{\psi}_1(\xi) + F_2 S_4 \tilde{\psi}_2(\xi)]}{\Delta_0}$$

$$B_{13} = \frac{F_1\tilde{\psi}_1(\xi)[R_{12}X_2 - R_{11}X_1] + F_2\tilde{\psi}_2(\xi)[S_{11}X_1 - S_{12}X_2]}{\Delta_0},$$

$$X_1 = \frac{\lambda_2 d_2}{s}, \quad X_2 = \frac{\lambda_1 d_1}{s}, \quad S_i = -\xi^2 g_1 + \lambda_i^2 g_2 - a_i, \quad S_4 = \iota\xi\lambda_4(g_2 - g_1),$$

$$R_i = 2\iota\xi\lambda_i g_3, \quad R_4 = -g_3(\lambda_4^2 + \xi^2)$$

for i= 1,2

$$\Delta_0 = \begin{vmatrix} S_{11} & S_{12} & S_4 \\ R_{11} & R_{12} & R_4 \\ \lambda_1 d_1 & \lambda_2 d_2 & 0 \end{vmatrix}$$

Case 1. Uniformly distributed force:

The solution due to uniformly distributed force applied on the half-space is obtained by setting

$$[\psi_1(x_1), \psi_2(x_1)] = H(x_1 + a) - H(x_1 - a) \tag{31}$$

Applying Laplace and Fourier transforms defined by (9) on Eq. (31) yield

$$[\tilde{\psi}_1(\xi), \tilde{\psi}_2(\xi)] = 2\frac{\sin(\xi a)}{\xi} \tag{32}$$

Case 2. Linearly distributed force:

The solution due to linearly distributed force over a strip of non-dimensional width 2*d*, applied on the half-space is obtained by setting

$$[\psi_1(x_1), \psi_2(x_2)] = \begin{cases} 1 - \dfrac{|x_1|}{d} & \text{if} \quad |\mathrm{x}_1| \le \mathrm{d} \\ 0 & \text{if} \quad |\mathrm{x}_1| > \mathrm{d} \end{cases} \tag{33}$$

In Eqs. (16)-(17), applying Laplace and Fourier transforms defined by (9) on Eq. (33) gives

$$[\tilde{\psi}_1(\xi),\tilde{\psi}_2(\xi)] = \frac{2(1-\cos(\xi d))}{\xi^2 d}, \quad \xi \neq 0 \tag{34}$$

Case 3. Moving force:
The solution due to an impulsive force, moving along the x_1-axis with uniform speed V at $x_3 = 0$ is obtained by setting

$$[\psi_1(x_1),\psi_2(x_1)]H(t) = \psi(x_1,t) = \delta(x_1 - Vt) \tag{35}$$

In Eqs. (16)-(17). Applying Laplace and Fourier transform's defined by Eq .(9) on the Eq. (35) yield

$$[\tilde{\psi}_1(\xi),\tilde{\psi}_2(\xi)]/s = \psi(\xi,s) = \frac{1}{s - i\xi V} \tag{36}$$

Substituting the values of $\tilde{\psi}_1(\xi),\tilde{\psi}_2(\xi)$ from Eqs. (32), (34) and (36) in Eqs. (20)-(25), the corresponding expressions for uniformly distributed force, linearly distributed force and moving force, respectively are obtained.

6 APPLICATIONS

Inclined line load: for an inclined line load F_0, per unit length, we have

$$F_1 = F_0 \cos\delta, \quad F_2 = F_0 \sin\delta \tag{37}$$

Using Eq. (37) in Eqs. (20)-(25) and with the aid of Eqs . (32),(34) and (36) we obtain the expressions for uniformly distributed force, linearly distributed force and moving force respectively.

7 SPECIAL CASE

For the case of coupled thermoelasticity, the thermal relaxation times vanish, i.e $\tau_0 = \tau^0 = 0$ and consequently, we obtain the corresponding expressions of thermoelastic diffusion and thermoelasticity by putting these values in Eqs. (20)-(25).

8 INVERSION OF THE TRANSFORM

The transformed stresses and temperature distribution are functions of x_2, the parameters of Laplace and Fourier transforms s and ξ, respectively, and hence are of the form $\tilde{f}(\xi, x_2, s)$. To obtain the solution of the problem in the physical domain, we must invert the fourier transform in Eqs. (20)-(25) using

$$\begin{aligned} \overline{f}(x_1,x_2,s) &= \frac{1}{2\pi}\int_{-\infty}^{\infty} \overline{f}(\xi,x_2,s)\, e^{-i\xi x_1}\,\mathrm{d}\xi \\ \overline{f}(x_1,x_2,s) &= \frac{1}{2\pi}\int_{-\infty}^{\infty} [\overline{f}_e \cos(\xi,x_2) - i\overline{f}_0 \sin(\xi,x_2)]\,\mathrm{d}\xi \end{aligned} \tag{38}$$

where f_e and f_o are respectively even and odd parts of the function $\overline{f}(\xi, x_2, p)$. Thus, Eq. (38) give us the Laplace transform $\overline{f}(\xi, x_2, s)$ of the function $f(x_1, x_2, t)$. Now, for the fixed values of ξ, x_1 and x_2, the function $\overline{f}(x_1, x_2, s)$ in the Eq. (38) can be considered as the Laplace transformed function $\overline{g}(p)$ of some function $g(t)$. Following Honig and Hirdes [27], the Laplace transformed function $\overline{g}(p)$ can be converted as given below. The function $g(t)$ can be obtained by using

$$g(t) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} e^{st}\overline{g}(s)\,\mathrm{d}s \tag{39}$$

where c is an arbitrary real number greater than all the real parts of the singularities of $\overline{g}(p)$. Taking $s = c + ix_2$ we get

$$g(t) = \frac{e^{ct}}{2\pi i}\int_{-\infty}^{\infty} e^{itx_2}\overline{g}(c + ix_2)\,\mathrm{d}x_2 \tag{40}$$

Now, taking $e^{-ct}g(t)$ as $h(t)$ and expanding it as a Fourier series in [0, 2L], we obtain approximately the formula

$$g(t) = g_\infty(t) + E'_D$$

where

$$g_\infty(t) = \frac{c_0}{2} + \sum_{k=1}^{\infty} c_k, \quad 0 \le t \le 2L, \quad c_k = \frac{e^{ct}}{L}\Re[e^{\frac{ik\pi t}{L}}\overline{g}(c + \frac{ik\pi}{L})] \tag{41}$$

E_D is the discretization error and can be made arbitrarily small by choosing c large enough .The values of c and L are chosen according to the criteria outlined by Honig and Hirdes [27]. Since the infinite series in Eq. (41) can be summed up only to a finite number of N terms, the approximate value of $g(t)$ becomes

$$g_N(t) = \frac{c_0}{2} + \sum_{k=1}^{N} c_k, \quad 0 \le t \le 2L \tag{42}$$

Now, we introduce a truncation error E_T that must be added to the discretization error to produce the total approximation error in evaluating $g(t)$ using the above formula. Two methods are used to reduce total error. The discretization error is reduced by using the 'Korrecktur' method and 'ε-algorithm' described in Honig and Hirdes [27].

The 'Korrecktur' method formula, to evaluate the function $g(t)$ is

$$g(t) = g_\infty(t) - e^{-2cL}g_\infty(2L + t) + E_{D'}$$

where $|E'_D| << |E_D|$. Thus the approximate value of $g(t)$ becomes

$$g_{N_k}(t) = g_N(t) - e^{-2cL}g_{N'}(2L + t) \tag{43}$$

where N' is an integer such that $N' < N$.

We shall now describe the ε-algorithm which is used to accelerate the convergence of the series in Eq. (42). Let N be a natural number and $S_m = \sum_{k=1}^{m} c_k$ be the sequence of partial sums of Eq. (42). We define the ε-sequence by

$$\varepsilon_{0,m} = 0, \varepsilon_{1,m} = S_m, \quad \varepsilon_{n+1,m} = \varepsilon_{n-1,m+1} + \frac{1}{\varepsilon_{n,m+1} - \varepsilon_{n,m}}, \quad n,m=1,2,3,...$$

It can be shown by Honig and Hirdes [27] that the sequence $\varepsilon_{1,1}, \varepsilon_{3,1}, ..., \varepsilon_{N,1}$ converges to $g(t) + E_D - c_0 / 2$ faster than the sequence of partial S_m, $m = 1,2,3,...$. The actual procedure to invert the Laplace Transform reduces to the study of Eq. (43) together with the ε-algorithm. The last step is to calculate the integral in Eq. (38). The method for evaluating this integral is described in Press et al. [28], which involves the use of Romberg's integration with an adaptive step size. This also uses the results from successive refinements of the extended trapezoidal rule followed by extrapolation of the results to the limit when the step size tends to zero.

9 NUMERICAL RESULT AND DISCUSSION

With the view of illustrating theoretical results obtained in preceding section, we now present some numerical results. The material parameter chosen for this purpose are Eringen [29] and Thomas [30]

$$\lambda = 9.4\times10^{10} \text{dyn cm}^{-2}, \quad \mu = 4.0\times10^{10} \text{dyn cm}^{-2}, \quad \rho = 1.74\times10^{3} \text{ g cm}^{-3},$$

$$C_E = 1.0 \text{ Cal gm}^{-1}\ {}^{o}\text{c}^{-1}, \quad K = 0.435\times10^{2} \text{ Cal cm}^{-1}\text{sec}^{-10}\ {}^{o}\text{c}^{-1}, \quad T_0 = 20^{o}\text{c},$$

$$\alpha_t = 1.78\times10^{-5} \text{ k}^{-1}, \quad \alpha_c = 1.98\times10^{-4} \text{ m}^{3}\text{kg}^{-1}, \quad a = 1.2\times10^{4} \text{ m}^{2}\text{s}^{-2}\text{k}^{-1},$$

$$b = 0.9\times10^{6} \text{ m}^{5}\text{kg}^{-1}\text{s}^{-2}, \quad D = 0.85\times10^{-8} \text{ kg s m}^{-3}, \quad \tau_0 = 0.02 \text{ s}, \quad \tau^0 = 0.01 \text{ s}$$

The values of normal stress (t_{33}), tangential stress (t_{31}), temperature distribution T and mass Concentration C are presented graphically for L-S theory with thermoelastic diffusion and without diffusion effects with distance x in the range $0 \le x \le 10$. The solid line, dashed line and small dashed line corresponds for thermoelastic diffusion (LSWD) and solid line, dash line and small dashed line with centre symbols corresponds for thermoelastic theory (LSD) due to various sources for $\delta = 0$ (Initial angle), $\delta = 45$ (Intermediate angle), $\delta = 90$ (Extreme angle)

9.1 Uniformly distributed force

Fig. 3 depicts the variations of t_{33} with distance x. It is noticed that values of LSWD at $\delta = 0,45,90$ increase in range $0 \le x \le 2$, magnitude of LSWD at $\delta = 0,90$ is greater as compared at $\delta = 45$, whereas values for LSD show opposite behavior as compared to LSWD at $\delta = 45,90.$ Also value of LSWD at $\delta = 0$ increases near the point of application of source, then exhibits small variations about zero. The variations for t_{31} with x is shown in Fig. 4. It is evident that the values for LSWD and LSD at $\delta = 0,45,90$ increases initially in the range $0 \le x \le 2$, magnitude for LSWD is greater as compared to those for LSD and in remaining range values show variation about origin except for LSWD at $\delta = 0$, which shows oscillatory behavior with decreasing magnitude. It is evident from Fig. 5 that the values of temperature distribution for LSWD and LSD at $\delta = 0,45$ show opposite behavior near the point of application of source, whereas value of LSWD and LSD for $\delta = 90$ shows similar behavior i.e. their values decreases in range $0 \le x \le 2$, with further increase in x, values for LSWD and LSD converges toward origin. As clear by Fig. 6 which is a plot of variation for mass concentration C with distance x, which indicate that values of LSWD for $\delta = 0,45$ shows similar behavior in entire range with significant difference in their magnitude, while steady state is observed in case of $\delta = 90$ for LSWD.

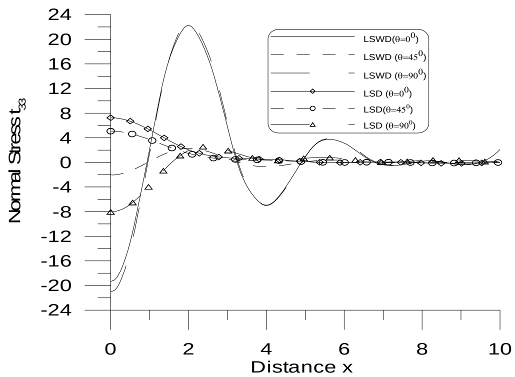

Fig. 3
Variation of normal stress t_{33} with distance x (uniformly distributed normal force).

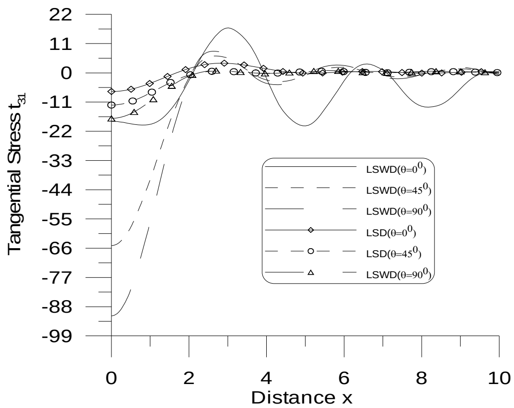

Fig. 4
Variation of tangential stress t_{31} with distance x (Uniformly distributed force).

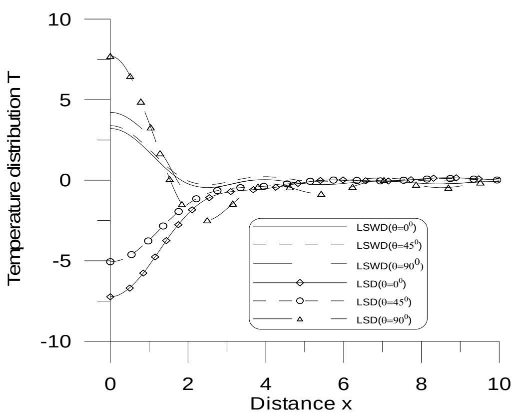

Fig. 5
Variation of temperature distribution T with distance x (uniformly distributed normal force).

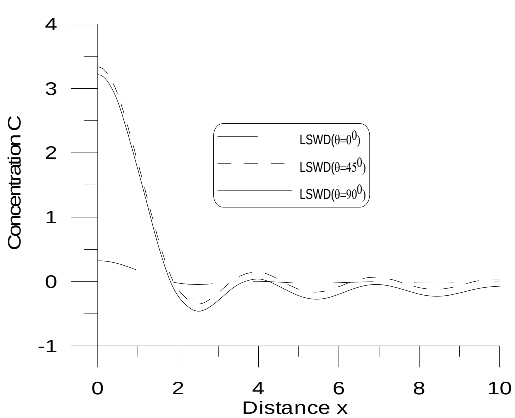

Fig. 6
Variation of mass concentration C with distance x (uniformly distributed normal force).

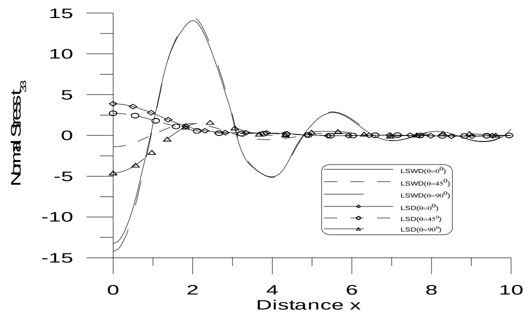

Fig. 7
Variation of normal stress with distance *x* (Linearly distributed normal force).

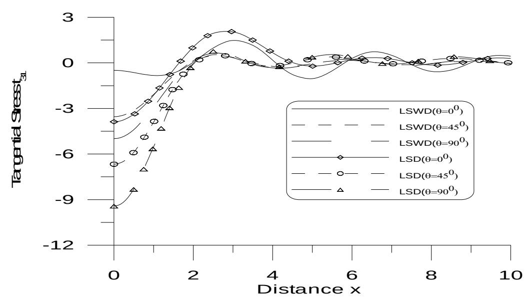

Fig. 8
Variation of tangential stress t_{31} with distance *x* (linearly distributed force).

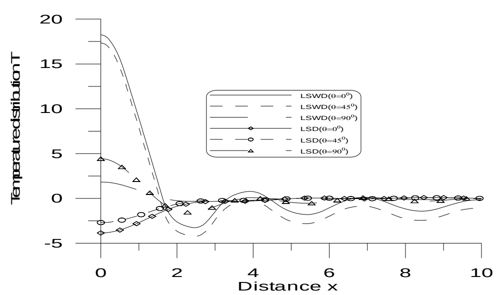

Fig. 9
Variation of temperature distribution *T* with distance *x* (linearly distributed normal force).

9.2 Linearly distributed force

It is noticed from Fig. 7 variations of t_{33} with distance x that values of t_{33} for LSWD at $\delta = 0,45$ show opposite behavior as observed for LSD i.e. values for LSWD increase abruptly whereas values for LSD decrease in range $0 \le x \le 2$. Also trend of variations of t_{33} for LSWD and LSD are similar i.e. their value increases initially. Magnitude of LSWD being greater than LSD with further increase in distance *x*, values for both LSWD and LSD at $\delta = 0,45$ exhibit variations about zero value. The trend of variations of t_{31} is shown by Fig. 8. It is noticed that values of t_{33} for LSD increase with greater magnitude in range $0 \le x \le 2$ as compared to LSWD, then with increase in distance *x* values converges towards zero value.

Fig. 9 shows the variations of *T* with distance *x*. It is noticed that values of *T* for LSWD at $\delta = 0,45,90$ and for LSD at $\delta = 90$ decreases in range $0 \le x \le 2.5$, whereas opposite behavior is noticed for LSD at $\delta = 0,45$, which reveals the impact of diffusion. With increase in *x*, Values of LSWD for $\delta = 0,45,90$ oscillates with decreasing magnitude, while rest of them shows steady state about zero value. As clear by Fig.10 which is a plot for mass concetration *C* with distance *x*. It is noticed that values of *C* for LSWD at $\delta = 0,45$ shows similar behavior magnitude being greater at whereas steady state is noticed at $\delta = 90$.

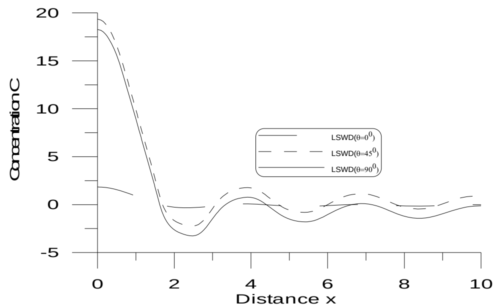

Fig. 10
Variation of mass concentration C with distance x (linearly distributed force).

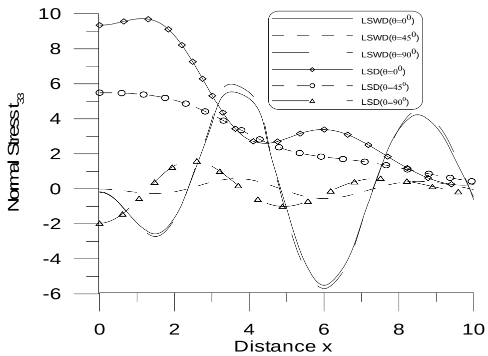

Fig. 11
Variation of normal stress t_{33} with distance x (moving force)

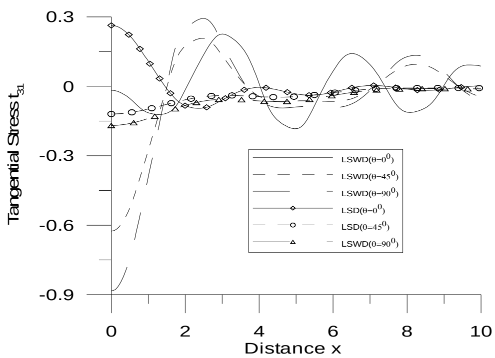

Fig. 12
Variation of tangential stress t_{31} with distance x (moving force).

9.3 Moving force

It seems from Fig. 11 which is plot for t_{33} with distance *x*, values of t_{33} for LSWD at $\delta = 0, 90$ oscillate with greater magnitude as compared to values of LSWD at $\delta = 45$ and for LSD at $\delta = 90$, while values for LSD at $\delta = 0, 45$ decrease in entire range. Fig. 12 depicts the variations of t_{31} with distance *x*. It is observed that values of t_{31} for LSWD at all δ, show opposite behaviour except at $\delta = 0$ in the interval $0 \le x \le 1.5$ and in second half of range. Also values for LSD at $\delta = 0$ decreases, while for $\delta = 45, 90$, values increases in the interval $0 \le x \le 2$, in remaining range values converges towards small variations.

It is evident by the Fig. 13 which is plot for *T* with *x* that the value of T for LSWD for all δ and for LSD at $\delta = 90$ show oscillatory behaviour, magnitude being greater for LSD, which reveals the impact of diffusion. Also values of T for LSD at $\theta = 0, 45$ increases abruptly in entire range and converges towards origin. Fig. 14 shows the variations of *C* with *x*. It is noticed that values of *C* for LSWD at all δ decreases sharply in range $0 \le x \le 2$, with increase in *x*, values for LSWD increases and decreases alternately.

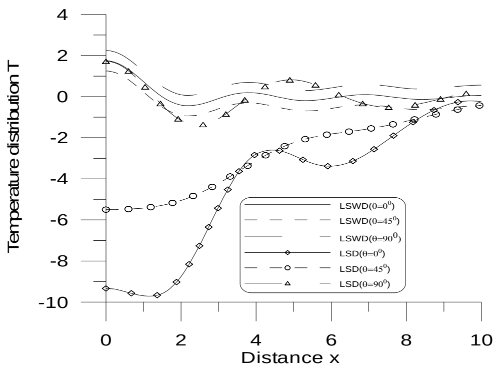

Fig. 13
Variation of temperature distribution T with distance x (moving force).

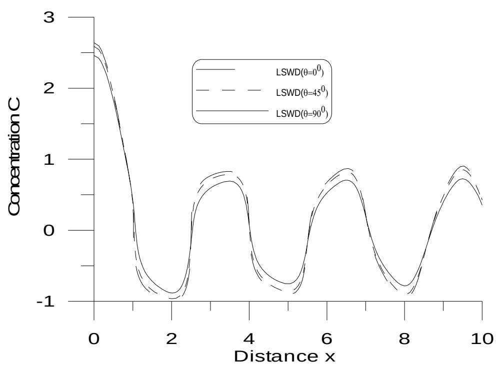

Fig. 14
Variation of mass concentration C with distance x (moving force).

10 CONCLUSIONS

It is observed from figures that the trends of variations of t_{33}, t_{31}, T and C on the application of distributed sources are similar in nature with significant difference in degree of sharpness. The present theoretical results provide useful information for experimental researches/ seismologist working in the field of mining tremors and drilling into crust of the earth. Also phenomenon of diffusion is studied. Therefore it can be used for improving the conditions of oil extraction. Therefore, this theory has practical utilities in investigating various types of geophysical and industrial applications.

APPENDIX A

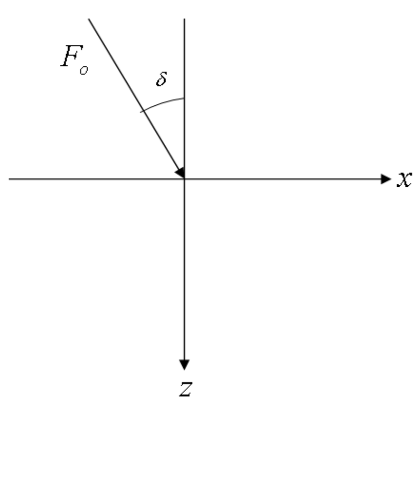

Fig. 1
Inclined load over thermodiffusive elastic half-space.

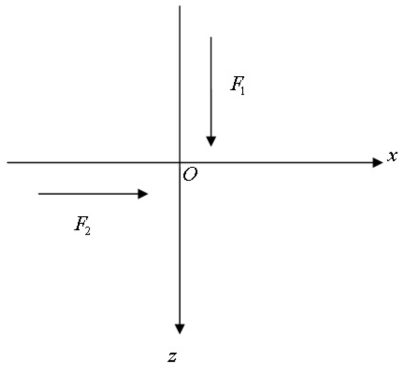

Fig. 2
Normal and tangential loadings.

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