

Hygrothermal Analysis of Laminated Composite Plates by Using Efficient Higher Order Shear Deformation Theory

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ABSTRACT

Hygrothermal analysis of laminated composite plates has been done by using an efficient higher order shear deformation theory. The stress field derived from hygrothermal fields must be consistent with total strain field in this type of analysis. In the present formulation, the plate model has been implemented with a computationally efficient C^0 finite element developed by using consistent strain field. Special steps are introduced to circumvent the requirement of C^1 continuity in the original plate formulation and C^0 continuity of the present element has been compensated in stiffness matrix calculations. The accuracy of the proposed C^0 element is established by comparing the results with those obtained by three dimensional elasticity solutions and other finite element analysis.

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Keywords: Finite element; Higher order; Static analysis; Laminated composites; Hygrothermal load.

1 INTRODUCTION

THE composite materials are widely used in civil and other engineering applications, due to their advantage of high stiffness and strength to weight ratio. Temperature and moisture variations often represent a significant factor, and sometimes the predominant causes of failure of composite structures. The deflection and stress analysis of laminated composite plates subjected to temperature and moisture has been the subject of research interest in recent years, but most of the researchers studied the effect of temperature. Whitney et al. [1] obtained three dimensional elasticity solutions using classical laminate theory to study the effect of environment on the stability, vibration, and bending behavior of plates. Wu et al. [2] used classical laminated plate theory to analyze symmetric and antisymmetric laminated plates subjected to temperature. However, this kind of approach is not sufficient for laminated plate by neglecting the effects of transverse shear stress in the laminates. The first order shear deformation theory developed by Rolfes et al. [3] which assumes a constant transverse shear strain across the thickness direction and a shear correction factor is generally required. Reddy [4] used higher-order shear deformation theory which accounts for parabolic distribution of the transverse shear strains throughout the thickness of the plate. Reddy et al. [5] used finite element analysis for composite plates subjected to thermal loading and compared the results with closed form solutions obtained by using YNS (Yang-Norris-Stavasky) shear deformation theory, which assumes constant shear deformation throughout the thickness and therefore it is inadequate to account for accurate shear distortion. Ram et al. [6] investigated the effect of hygrothermal environment on laminated composite plates using finite element method based on first order shear deformation theory. Kapania et al [7] used Discrete Kirchhoff Theory for analysis of plate but it shows good results for thin plates only. Chandrashekhara et al. [8] presented shear flexible finite element model for nonlinear static and dynamic analysis based on first order shear deformation theory. However, in order to avoid the use of shear correction factor various higher order theories [9-19] has been proposed

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by many researchers, but it requires C^1 continuity. Murakami [20] obtained an exact elasticity solution of the thermally induced cylindrical bending of layered composite plates. In case of thermal loading, a three dimensional thermal analysis has been carried out by Savoia et al. [21] in which some special boundary conditions are imposed at the edges which can cause significant mathematical problem for plates subjected to thermal loads. Three dimensional elasticity solutions carried out by Bhasker et al. [22] for thick laminates subjected to thermal loading gives non linear variation of inplane displacement throughout the thickness and thickness stretch/contraction effects in the transverse displacement. Kant et al. [23] developed a semi analytical model for thermo mechanical analysis where shear traction free conditions at the top and bottom of the plate has been assumed. Analytical and elasticity solutions are limited to simple boundary conditions and loadings. Shankara et al. [24] used the higher order shear deformation theory (HSDT) in finite element technique for the static analysis of anisotropic composite plates. A C^0 continuous nine noded Lagrangian element has been used for the analysis. Naganarayana et al. [25] has shown that unlike the first order formulations, the inconsistent strain field can also disturb the displacement recovery in the higher order shear deformable elements. Hence, possible errors due to such inconsistent terms are analyzed in analytical sense in this theory. Prathap et al. [26] has shown that stress resultant fields were of higher interpolation order than the strain field, hence higher degree terms did not participate in the stiffness matrix calculations. Therefore, stress field derived from temperature fields i.e. initial strains must be consistent with the total strain field used in the finite element formulations and also the C^0 continuity has not been compensated.

In the present study, a C^0 continuous type FE element model has been developed based on higher order shear deformation theory [4] for hygro-thermal analysis of laminated plates. The total number of unknowns in the present theory is independent of number of layers. In the present formulation, a nine noded Lagrangian element with seven degree of freedom per node has been used. Moreover, the conditions of zero transverse shear stresses on the top and bottom surfaces of the laminate have been enforced. Thus, the proposed plate model has been implemented with a computationally efficient C^0 finite element developed by using consistent strain field. Special steps were introduced (e.g. sampling at gauss points) to compensate this problem and C^0 element implementation has been compensated in computing stiffness matrix calculations.

2 MATHEMATICAL MODEL

A multilayered plate is a laminated structure obtained by stacking rectangular layers until the desired thickness and stiffness are reached. Laminae is considered homogeneous, perfectly bonded with each other. To analyze such a laminated structure, a Cartesian co-ordinate system x, y, z referred to the middle surface of the laminate is used. In plane, displacement is given by

$$u = u_0 + z\Psi_x + \alpha_1 z^2 + \beta_1 z^3 \quad (1)$$

$$v = v_0 + z\Psi_y + \alpha_2 z^2 + \beta_2 z^3 \quad (2)$$

where u_0 and v_0 are the displacements on mid-plane in x and y direction, respectively. Ψ_x and Ψ_y are total rotations about x and y axis, respectively. Transverse displacement is given by

$$w = w_0(x, y) \quad (3)$$

$$\gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w_0}{\partial x} = \Psi_x + 2\alpha_1 z + 3\beta_1 z^2 + \frac{\partial w_0}{\partial x} \quad (4)$$

$$\text{at } z = \pm h/2, \quad \gamma_{xz=0}, \quad \Psi_x + h\alpha_1 + 3\frac{h^2}{4}\beta_1 + \frac{\partial w_0}{\partial x} = 0, \quad \Psi_x - h\alpha_1 + 3\frac{h^2}{4}\beta_1 + \frac{\partial w_0}{\partial x} = 0$$

On solving

$$\beta_1 = -\frac{4}{3h^2} \left(\Psi_x + \frac{\partial w_0}{\partial x} \right) \& \alpha_1 = 0 \quad (5)$$

$$u = u_0 + z\Psi_x - \frac{4z^3}{3h^2}\Psi_x - \frac{4z^3}{3h^2}\frac{\partial w_0}{\partial x} \quad (6)$$

$$v = v_0 + z\Psi_y - \frac{4z^3}{3h^2}\Psi_y - \frac{4z^3}{3h^2}\frac{\partial w_0}{\partial y} \quad (7)$$

The displacement field expressed in Eq. (6) and (7) contains first order derivatives of w which requires C^1 continuity for finite element approximation. In order to avoid the usual difficulties associated with C^1 continuity requirement $\partial w/\partial x$ and $\partial w/\partial y$ are considered as independent field variables. For the sake of convenience to represent all variables as C^0 continuous, the derivative of w with respect to x and y is expressed as follows:

$$\theta_x = \frac{\partial w_0}{\partial x}, \quad \theta_y = \frac{\partial w_0}{\partial y}$$

with this, the in-plane displacement field may be rewritten as

$$u = u_0 + z\Psi_x - \frac{4z^3}{3h^2}\Psi_x - \frac{4z^3}{3h^2}\theta_x \quad (8)$$

$$v = v_0 + z\Psi_y - \frac{4z^3}{3h^2}\Psi_y - \frac{4z^3}{3h^2}\theta_y \quad (9)$$

The unknown nodal parameters for the present model

$$\{\delta\} = \{u_0 \quad v_0 \quad w_0 \quad \Psi_x \quad \Psi_y \quad \theta_x \quad \theta_y\} \quad (10)$$

The strain vectors corresponding to the displacement field

$$\{\varepsilon_b\} = \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{xy} \end{Bmatrix} = \{\varepsilon_1\} + z\{K_1\} + z^3\{K_2\} \& \{K_s\} = \begin{Bmatrix} \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} = \{\varepsilon_2\} + z^2\{K_3\} \quad (11)$$

where

$$\{\varepsilon_1\} = \begin{Bmatrix} u_{,0x} \\ v_{,0y} \\ u_{,0y} + v_{,0x} \end{Bmatrix}$$

$$\{\varepsilon_2\} = \begin{Bmatrix} \Psi_x \\ \Psi_y \end{Bmatrix} + \begin{Bmatrix} w_{,0x} \\ w_{,0y} \end{Bmatrix}$$

$$K_1 = \begin{Bmatrix} \Psi_{,x} \\ \Psi_{,y} \\ \Psi_{x,y} + \Psi_{y,x} \end{Bmatrix}$$

$$K_2 = C1 \begin{Bmatrix} \Psi_{,x} + \theta_{,x} \\ \Psi_{,y} + \theta_{,y} \\ \Psi_{x,y} + \Psi_{y,x} \end{Bmatrix} \quad \text{where} \quad C1 = -4/3h^2$$

$$K_3 = C2 \begin{Bmatrix} \Psi_x + \theta_x \\ \Psi_y + \theta_y \end{Bmatrix} \quad \text{where} \quad C2 = -4/h^2$$

where “,” are the derivative of variables with respect to x and y , respectively, for example $u_{0,x} = \partial u_0 / \partial x$. The stress strain relations for the k th orthotropic lamina of a laminate consisting of N layers having fibers oriented in any arbitrary orientation with respect to the reference axis (x) are

$$\{\bar{\sigma}\}_k = [\bar{Q}_{ij}] \{\bar{\varepsilon}\}_k \quad (12)$$

where

$$\{\bar{\varepsilon}\} = \{\varepsilon_b\} + \{K_S\}$$

where $\{\bar{\varepsilon}\}$ is the mechanical strain and $[\bar{Q}_{ij}]$ are the reduced stiffness coefficients of the k -th lamina. The temperature strain may also be included appropriately which is discussed in the next section.

3 FINITE ELEMENT FORMULATION

A nine noded isoparametric element shown in Fig. 2 with seven nodal unknowns (u_0 v_0 w_0 Ψ_x Ψ_y θ_x θ_y) per node has been used for the present analysis.

$$\{w\} = [N] \{\delta\} \quad (13)$$

i.e. $\{dw\} = [N] \{d\delta\}$. Now to find stress at any point

$$\{\bar{\sigma}\} = [\bar{Q}] \{\bar{\varepsilon}\} \quad (14)$$

where

$$\{\bar{\varepsilon}\} = \{\bar{\varepsilon} - \bar{\varepsilon}_k\} \quad (15)$$

Now $\{\bar{\varepsilon}\} = [H] \{\varepsilon\}$. where $[H]$ is a matrix of order 5×13 and comprises of term containing different order of z . The strain terms now need to be carefully analyzed $\{\varepsilon\}$ is the total strain and $\{\varepsilon_k\}$ is the initial strain or hygrothermal strain.

$$\{\varepsilon\} = [B] \{d\} \quad (16)$$

where $\{d\}$ is the vector of nodal displacements. In a finite element formulation, the displacement and temperatures/moistures are interpolated within the domain of the element using the same interpolations functions. The calculation of total strains involves differentiation of displacement fields and strain field functions will therefore be of lower order than shape functions. Hence, higher order terms of thermal stress components are not sensed by the total strain interpolations. Thus, a thermal load vector is created which corresponds to an initial strain (and stress) vector that is consistent with the total strain vector. Therefore, only the consistent part of the thermal stress should be computed when stress recovery is made from the nodal displacements. The inclusion of the inconsistent part in thermal stress recovery leads to thermal stress oscillations. Special steps were introduced there (e.g. sampling at reduced gauss points) to compensate for this problem. By applying virtual work method and equating work done by internal forces we get

$$[K]\{\delta\} = \{P\} \quad (17)$$

where $[K]$ is the element stiffness matrix and $\{P\}$ is nodal load vector and

$$[K] = \iint [B]^T [D][B] dx dy \quad (18)$$

and

$$\{P\} = \iint [N]^T q dx dy \quad (19)$$

Thermal loading is obtained by following Eq.

$$\{Pe^N\} = \iint [B]^T [H]^T \{F^N\} dx dy \quad (20)$$

where

$$\begin{aligned} \{F^N\}^T &= [N_x^N, N_y^N, N_{xy}^N, M_x^N, M_y^N, M_{xy}^N, 0, 0, \dots] \\ \{N_x^N, N_y^N, N_{xy}^N\}^T &= \sum_k^n \int_{z_{k-1}}^{z_k} \{\bar{Q}_{ij}\}_k \{\varepsilon\}_k dz \end{aligned} \quad (21)$$

$$\{M_x^N, M_y^N, M_{xy}^N\}^T = \sum_k^n \int_{z_{k-1}}^{z_k} \{\bar{Q}_{ij}\}_k \{\varepsilon\}_k \quad (22)$$

Here $i, j=1, 2, 6$. Also, $\{\varepsilon_k\}^T = [\varepsilon_x, \varepsilon_y, \varepsilon_{xy}]$. An arbitrary temperature distribution can be assumed without loss of generality of the form $T(x, y, z) = T_0 + \left(\frac{z}{h}\right)T_1(x, y)$ where T_0 is the initial constant temperature.

Case1. When temperature is uniform across the depth

$$\begin{aligned} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \end{Bmatrix} &= \begin{Bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{Bmatrix} \Delta T \\ \begin{Bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{Bmatrix} &= \begin{bmatrix} c^2 & s^2 & -2cs \\ s^2 & c^2 & 2cs \\ cs & -cs & c^2 - s^2 \end{bmatrix} [Q]_k \begin{Bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_{12} \end{Bmatrix}_k \end{aligned} \quad (23)$$

where $\alpha_1, \alpha_2, \alpha_{12}$ are the coefficient of thermal expansion referred to the principal material axes of the lamina and $\alpha_x, \alpha_y, \alpha_{xy}$ are the transformed coefficient of thermal expansion referred to the x - y coordinate system.

$$\{F\} = \iiint [B]^T [H]^T \begin{Bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{Bmatrix} T dv \quad (24)$$

Case2. When temperature is varying across the depth

$$\{F\} = \iiint [B]^T [H]^T \begin{Bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{Bmatrix} [1/2(T_U + T_L + z/h(T_U - T_L))] dv \tag{25}$$

where, T_U =Temperature at top surface and T_L =Temperature at bottom surface.

4 NUMERICAL RESULTS AND DISCUSSIONS

Numerical results are presented for symmetric and antisymmetric laminated rectangular plates as shown in Fig. 1 with different boundary conditions (BC).

BC I: $u = w = \Psi_x = \theta_x = 0$ at AB and CD
 $v = w = \Psi_y = \theta_y = 0$ at AD and BC

BC II: $u = v = w = \Psi_x = \theta_x = \Psi_y = \theta_y = 0$ at AB and CD
 $v = w = \Psi_y = \theta_y = 0$ at AD and BC

All the lamina are assumed to be of the same thickness and made of the same orthotropic materials. To illustrate the preceding thermal structural analysis, many problems are solved and comparisons are made with the results available in the literature.

4.1 Effect of different moisture concentration and different temperature on antisymmetric cross-ply

In this section, the effect of different moisture concentrations has been first investigated on the deflection of antisymmetric cross-ply ($0^0/90^0/0^0/90^0$) of a square laminate having side to thickness ratio of 100 using the material properties given in Table 1. The deflection w is plotted along x axis for the simply supported boundary conditions as shown in Fig. 3.

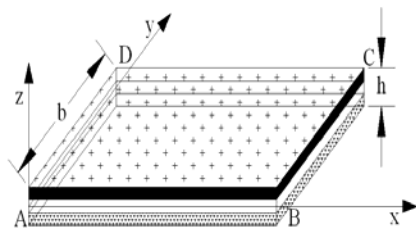


Fig. 1
Geometry of laminated composite plate.

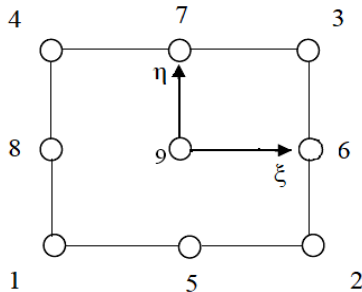


Fig. 2
Nine noded isoparametric elements with typical node numbering.

The Deflection is maximum near the centre of the supported edge and it vanishes along the diagonals. In case of clamped boundary conditions, the deflection is zero throughout the laminate boundary. Results closely match with Ram et al. [6]. The effect of different temperatures has also been analyzed in this section for the same laminate for simply supported boundary conditions using material properties given in Table 2. The deflection w is plotted along x axis for the simply supported boundary conditions as shown in Fig. 4. The observation made in case of moisture also applies to temperature. Results closely match with Ram et al. [6].

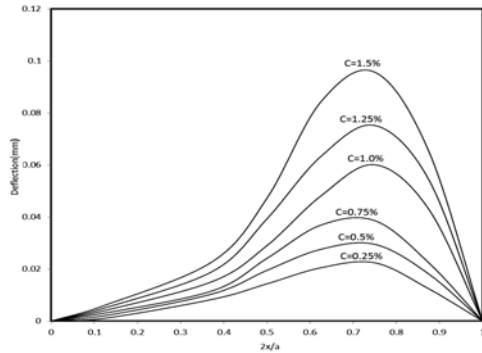


Fig. 3
Deflection along x axis at different moisture concentrations (C).

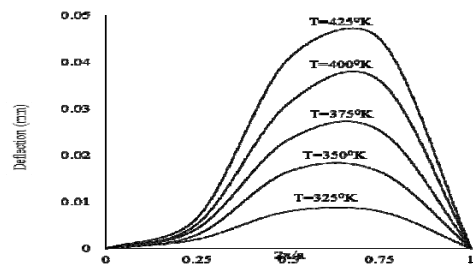


Fig. 4
Deflection along x axis at different at different temperatures (T).

Table 1

Material properties at different moisture concentrations $G_{13} = G_{12}$, $G_{23} = 0.5 G_{12}$, $\nu_{12} = 0.3$, $\beta_1 = 0$, and $\beta_2 = 0.44$ [6]

Elastic moduli (GPa)	Moisture Concentrations C (%)						
	0	0.25	0.5	0.75	1.0	1.25	1.5
E_1	130	130	130	130	130	130	130
E_2	9.5	9.25	9.0	8.75	8.5	8.5	8.5
G_{12}	6.0	6.0	6.0	6.0	6.0	6.0	6.0

Table 2

Material properties at different temperatures $G_{13} = G_{12}$, $G_{23} = 0.5 G_{12}$, $\nu_{12} = 0.3$, $\alpha_1 = -0.3 \times 10^{-6}$, and $\alpha_2 = 28.1 \times 10^{-6}$ [6]

Elastic moduli (GPa)	Temperatures T (K)					
	300	325	350	375	400	425
E_1	130	130	130	130	130	130
E_2	9.5	8.5	8.0	7.5	7.0	6.75
G_{12}	6.0	6.0	5.5	5.0	4.75	4.5

Table 3

Effect of aspect ratio and boundary conditions on the deflection \bar{w} for an isotropic rectangular plate subjected to uniform temperature in the xy plane and linearly varying through the thickness ($q=0, T_0=0, \nu=0.3$) with thickness ratio (t/a) =0.01

Reference	BCI			BCII		
	$a/b=1$	$a/b=1.5$	$a/b=2$	$a/b=1$	$a/b=1.5$	$a/b=2$
Present (4×4)	0.9586	0.5823	0.3697	0.2052	0.0338	0.0049
Present (6×6)	0.9592	0.5830	0.3705	0.2063	0.0355	0.0054
Present (8×8)	0.9593	0.5832	0.3706	0.2065	0.0357	0.0056
Present (10×10)	0.9594	0.5832	0.3707	0.2065	0.0357	0.0055
Present (12×12)	0.9594	0.5832	0.3707	0.2065	0.0358	0.0058
Present (16×16)	0.9594	0.5832	0.3707	0.2065	0.0358	0.0059
Prathap [26]	0.961	–	0.371	0.206	–	0.0048
Reddy [5]	0.9575	0.5822	0.3701	0.2063	0.036	0.00561
Timoshenko [28]	0.9578	0.5824	0.3702	–	–	–
Das and Rath [27]	0.957	0.582	0.370	0.206	0.036	0.0055

Entries inside the parenthesis indicate mesh division $\bar{w} = 10tw/aTa^2$.

4.2 Effect of aspect ratio and boundary conditions on isotropic plate

In this section, the effect of aspect ratio and boundary conditions on non-dimensional deflection \bar{w} for an isotropic rectangular plate subjected to temperature distribution that is uniform in the x - y plane and linearly varying through thickness is investigated using material properties given below [26]:

$$E_1 / E_2 = 25, \quad G_{12} / E_2 = 0.5, \quad G_{23} / E_2 = 0.2, \quad \nu_{12} = 0.25G_{12} = G_{13}, \quad \nu_{12} = \nu_{13}, \quad \alpha_2 = 3\alpha_1$$

The central deflection obtained is presented in Table 3 with those obtained by Prathap et al. [26] and other investigators [5, 26-28] (finite element analysis). The present result indicates accuracy and rate of convergence obtained by the proposed finite element model. The results obtained by proposed element are quite close to those obtained by Prathap et al. [26], Reddy et al. [5] and others [27, 28]. The effect of aspect ratio (a/b) and side to thickness ratio (t/a) on non-dimensional deflection \bar{w} for an isotropic rectangular plate subjected to temperature distribution that is uniform in the x - y plane and linearly varying through thickness is studied here. The material properties are the same as used in previous section. The central deflection has been presented in Table 4. The present results obtained are in good agreement with Prathap et al. [26], Reddy et al. [5] and others [27, 28].

4.3 Effect of aspect ratio, number of plies and their orientation on orthotropic plate

In this section, the effect of aspect ratio, number of plies and their orientation on non dimensional deflection \bar{w} for simply supported orthotropic plate subjected to temperature distribution that is sinusoidal in x - y plane and linearly varying through thickness has also been investigated. The material properties are the same as in the previous section. The central deflection has been presented in Table 5. The normalized deflection \bar{w} as obtained are compared with the results obtained by Prathap et al., the standard FE software MSC/NASTRAN and Reddy et al. [5]. The result obtained by proposed element are in good agreement with Prathap et al. [26]. It can be noticed that proposed element gives values between MSC/NASTRAN and Reddy et al. [5] respectively, because result obtained by Reddy et al. [5] were based on first order shear deformation theory and total strain field was not consistent with stress field.

4.4 Effect of thickness ratio on cross-ply ($0^\circ/90^\circ/0^\circ$) laminate

The effect of thickness ratio, on non dimensional deflection and stresses for simply supported cross ply ($0^\circ/90^\circ/0^\circ$) square laminate subjected to temperature distribution that is sinusoidal in x - y plane and linearly varying through thickness is presented. The material properties used are as given below [22]:

$$E_1 / E_2 = 25, \quad G_{12} / E_2 = 0.5, \quad G_{23} / E_2 = 0.2, \quad \nu_{12} = 0.25G_{12} = G_{13}, \quad \nu_{12} = \nu_{13}, \quad \alpha_2 = 1125\alpha_1$$

Table 4

Effect of aspect ratio and side to thickness ratio on deflection \bar{w} for an isotropic rectangular plate subjected to uniform temperature gradient

Thickness Ratio	Reference	BCI			BCII		
		$a/b=1$	$a/b=1.5$	$a/b=2$	$a/b=1$	$a/b=1.5$	$a/b=2$
0.01	Present (16x16)	0.9594	0.5832	0.3707	0.206	0.0358	0.0055
	Prathap [26]	0.961	-	0.371	0.206	-	0.0055
	Reddy [5]	0.9575	0.5822	0.3701	0.2063	0.0360	0.00561
	Timoshenko [28]	0.9578	0.5824	0.3701	-	-	-
	Das and Rath [27]	0.957	0.582	0.370	0.206	0.036	0.0055
0.05	Present (16x16)	0.9594	0.584	0.3707	0.213	0.0359	0.0066
	Reddy [5]	0.9576	0.5821	0.3700	0.2132	0.0390	0.0066
	Timoshenko [28]	0.9578	0.5824	0.3702	-	-	-
	Das and Rath [27]	0.960	0.584	0.371	0.213	0.039	0.0067
0.075	Present (16x16)	0.9594	0.5832	0.3707	0.2218	0.4319	0.0082
	Reddy [5]	0.9576	0.5821	0.3700	0.2219	0.0432	0.0085
	Timoshenko [28]	0.9578	0.5824	0.3702	-	-	-
	Das and Rath [27]	0.962	0.586	0.373	0.223	0.044	0.0085
0.1	Present (16x16)	0.9594	0.5838	0.3707	0.2328	0.0358	0.0106
	Reddy [5]	0.9576	0.5821	0.3700	0.2330	0.0492	0.0108
	Timoshenko [28]	0.9578	0.5824	0.3702	-	-	-
	Das and Rath [27]	0.967	0.589	0.375	0.235	0.050	0.0114

Entries inside the parenthesis indicate mesh division.

Table 5

Effect of aspect ratio on the deflection \bar{w} for simply supported rectangular laminated composite plates subjected to sinusoidal temperature gradient (thickness ratio $(t/a) = 0.01$)

Reference	$0^0/90^0/0^0$			0^0		$0^0/90^0$
	$a/b=1$	$a/b=1.5$	$a/b=2$	$a/b=1$	$a/b=1$	$a/b=1$
Present	1.0429	0.8802	0.6566	1.0332		1.1520
Prathap [26]	1.0249	0.8802	0.6566	1.0332		1.1434
NASTRAN	1.0028	0.8346	0.6108	1.0109		1.9374
Reddy [5]	1.0949	0.9847	0.7643	1.0313		1.6765

$$\bar{w} = 10tw/\alpha Ta^2.$$

Table 6

Deflections and stresses in $(0^0/90^0/0^0)$ square laminate

$S=a/h$		$\bar{u}(\mp h/2)$	$\bar{v}(\mp h/2)$	$\bar{w}(\mp h/2)$	$\bar{\sigma}_x(\pm h/2)$	$\bar{\sigma}_y(\mp h/2)$	$\bar{\tau}_{xy}(\mp h/2)$
100	Present	± 16.17	± 16.15	10.30	± 971.6	± 1075	± 50.76
	Elasticity [22]	± 16.00	± 16.17	10.26	± 965.4	± 1065	± 50.53
50	Present	± 16.42	± 16.42	10.45	± 974.6	± 1074	± 51.58
	Elasticity [22]	± 16.02	± 16.71	10.50	± 967.5	± 1063	± 51.41
20	Present	± 17.75	± 17.75	11.30	± 965.1	± 1059	55.45
	Elasticity [22]	± 16.17	± 20.34	12.12	± 982.0	± 1051	± 57.35
10	Present	± 22.20	± 22.14	14.13	± 945.6	± 1029	± 68.43
	Elasticity [22]	± 16.61	± 31.95	17.39	± 1026	± 1014	± 76.29

z values are given in parentheses: (x,y) values are $(a/2,a/2)$ for $\bar{w}, \bar{\sigma}_x, \bar{\sigma}_y$ and $(0,0)$ for $\bar{\tau}_{xy}$.

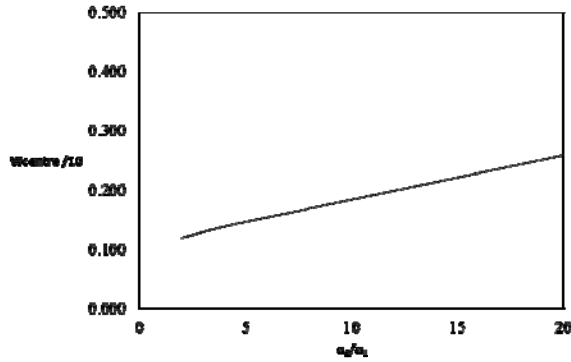


Fig. 5
Variation of central deflection with the ratio, α_2/α_1 .

The deflections and stresses obtained are presented in Table 6. The results obtained by the proposed element are in good agreement with elasticity solution given by Bhaskar et al. [22] for thin plates to moderately thick plates.

4.5 Effect of coefficient of thermal expansion ratio (α_2/α_1) on angle-ply

In this section, the variation of non dimensional transverse centre deflection for a four layer ($45^\circ/-45^\circ/-45^\circ/45^\circ$) angle ply square laminate under sinusoidal temperature in x - y plane and linearly varying through thickness are shown in Fig 5. The deflection increases with the increase in ratio, α_2/α_1 . The material properties used are the same as in section 4.2. The central deflections are plotted for thickness ratio ($a/h=20$) in Fig 5.

5 CONCLUSIONS

- The proposed efficient higher order shear deformation theory calculates deflections and stresses in an efficient manner for isotropic, multi-layered plate of orthotropic cross ply and angle ply laminate subjected to hygro-thermal load of uniform and sinusoidal distribution.
- The results provided by proposed model for isotropic plates for different aspect ratio, and boundary conditions converge very rapidly. The deflections obtained for laminated composite plates for different aspect ratio, thickness ratio and boundary conditions exactly matches with Prathap et al. [26], Reddy et al. [5] and others.
- The results obtained by proposed model for different aspect ratio for orthotropic plates under sinusoidal temperature load closely matches with Prathap et al. [26] in which the inconsistent part of the thermal stress was compensated by introducing special steps at reduced gauss points etc, but the present results based on consistent strain field are not matching closely with those obtained by Reddy et al. [5] where the thermal stress was not consistent with strain field.
- The results obtained for deflection and stresses in case of cross ply ($0^\circ/90^\circ/0^\circ$) square laminate by the proposed model closely match with elasticity solution given by Bhaskar et al. [22] for moderately thin to thick plates. However, it is observed that in-plane displacement and transverse deflection values vary considerably for $a/h \geq 20$. Based on the above observation it may be concluded that the proposed finite element model is quite efficient in calculating deflections and stresses very accurately for laminated composite plates subjected to hygro-thermal loadings in the range of very thin to moderately thick plates.

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