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Optimal Nonlinear Energy Sinks in Vibration Mitigation of the Beams Traversed by Successive Moving Loads

D. Younesian[1,*], A. Nankali[1,2], E. Motieyan[1]

[1]*Center of Excellence in Railway Transportation, School of Railway Engineering, Iran University of Science and Technology, 16846-13114, Tehran , Iran*
[2]*Department of Mechanical and Aerospace Engineering, New-Mexico State University, Las Cruces, USA*

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ABSTRACT

Optimal Nonlinear Energy Sink (NES) is employed in vibration suppression of the beams subjected to successive moving loads in this paper. As a real application, a typical railway bridge is dynamically modeled by a single-span beam and a traveling high-speed train is simulated by a series of successive moving loads. Genetic algorithm is employed as the optimization technique and optimal parameters of the NES system are accordingly obtained. It is found that the NES can remarkably suppress the vibration level particularly in vicinity of the critical speeds. A sensitivity analysis is then carried out and robustness of the optimal NES is investigated. A parametric study is performed and performance of the optimal NES is evaluated for different values of the load speeds, load magnitudes, load intervals and mass ratios.

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Keywords: Nonlinear Energy Sink (NES); Vibration suppression; Beam; Successive moving load; Genetic algorithm

1 INTRODUCTION

NONLINEAR vibration absorbers have recently received much attention due to their flexible capabilities in supporting the goal of either maximizing the suppression bandwidth or minimizing the maximum displacement. Optimization of the nonlinear passive control systems were firstly investigated in 1950's [1]. A practical method applying in design of a nonlinear vibration absorber was developed by Rice and McCraith in [2]. Application of a nonlinear active vibration absorber employing in the flexible structures, theoretically and experimentally was examined by Oueini et al. [3-4] based on the saturation phenomenon. Concept of the Nonlinear Energy Sink (NES) and Energy Pumping methods was significantly extended by Vakakis et al. [5-6]. The NES absorbs vibration energy in a one-way pattern from the main system toward the absorber and locally dissipates this energy, without spreading it back to the linear system.

Many theoretical and experimental research projects have been conducted in order to examine the applications of NES in vibration mitigation of practical cases. Suppression of aero-elastic instability by using the NES has been studied in [7-8]. Nucera et al. used the numerical and experimental procedures to show that the nonlinear energy sink is a feasible and robust strategy for seismic mitigation [9]. In 2007, Georgiades and Vakakis studied the vibration of a linear flexible beam under a shock excitation and showed that the NES concept can be used properly in flexible systems [10]. Viguie et al. addressed the problem of stabilizing the dynamics of the drill-string system by means of a nonlinear energy sink [11]. Passive control systems and particularly the Tuned Mass Dampers (TMDs)

* Corresponding author. Tel.: +98 21 77490 1029; Fax: +98 21 7745 1568.
E-mail address: younesian@iust.ac.ir (D. Younesian).

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have been widely employed in vibration suppression of the bridges excited by moving loads [12-16]. Younesian and Esmailzadeh [17] presented a nonlinear absorption concept which can be used in order to suppress the vibration of rotating beams. Spectral analysis of the beam's vibration with uncertain natural frequencies under a random train of moving forces was studied by Gładysz and Śniady [18]. Bryja [19] provided a general outline of the stochastic response analysis of suspension bridge subjected to randomly fluctuated wind with time-dependent mean velocity. More recently, Samani and Pellicano [20] studied a NES system applying in vibration suppression of a beam excited by a single moving load. They showed that the NES system can effectively reduce the vibration level of the system. Reviewing the literature reveals that there are two significant features which can be considered in investigations of the NES systems. These features are listed as follows:
1-Because of the strongly nonlinear behavior of the NES, optimal design of a NES has been always a challenging problem.
2-For the most cases, including the non-transient excitations, passive vibration control systems (TMDs and NES) are suitable suppression devices.

The main examinations of the present paper are directed to cover the lack of knowledge which was already presented as two features. In other words, for the first time in this paper, based on a classical optimization procedure, optimal values of the NES parameters are obtained for a real railway bridge traversed by series of moving loads. Successive moving loads as one of the non-transient excitations are employed to simulate the moving wheels of train. The distance between the moving loads can then significantly affect the performance and optimal parameters of the NES system. Galerkin method is employed as the solution technique. Root Mean Square (RMS) of the bridge response is adopted as the objective function and optimal parameters of the NES system are consequently obtained. Effects of different parameters including the load speed, load distance and magnitude, mass ratio and the bridge length on optimal values of the NES parameters are studied. Series of numerical simulations are then carried out and performance of the NES system is evaluated.

2 MATHEMATICAL MODELLING

As shown in Fig.1 a simply-supported beam with span length L, subjected to a series of loads P_s with uniform distance d, are considered as a bridge traveled by a train. Train wheel loads are modeled by a successive concentrated loads moving on the beam. As it is shown in Fig. 1, the NES system consists of a mass attached to the bridge by means of a nonlinear spring and a linear viscous damper.

Moving wheel loads of the train can be mathematically modeled as the following function [21-22]:

$$F(x,t)=\sum_{j=1}^{N} p.u_j(x,t,V,L) \tag{1}$$

where

$$u_j(x,t,V,L)=\delta\left[x-V(t-t_j)\right]\left[H(t-t_j)-H\left(t-t_j-\frac{L}{V}\right)\right] \tag{2}$$

In which, δ denotes the Dirac delta function, x is the longitudinal coordinate, H is a unit step function, t_j denotes arriving time of the jth load so that $t_j=(j-1)\ d/V$, and N is total number of moving loads (number of carriages). Actually, the action of the jth moving load is turned on by the term $H(t-t_j)$ when it enters the beam, and turned off by the term $H(t-t_j-L/V)$ when it leaves the beam.

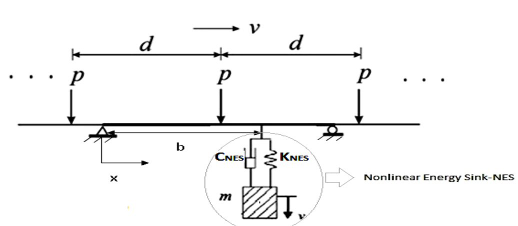

Fig. 1
Simple model of a railway bridge and vehicle series and an attached NES system.

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3 EQUATIONS

The bridge is modeled as an Euler–Bernoulli beam and consequently, the coupled nonlinear equations of the beam-NES motion can be derived as:

$$EIy_{xxxx} + C\dot{y} + M\ddot{y} + \left\{ K_{NES}\left[y(b,t) - v(t)\right]^3 + C_{NES}\left[\dot{y}(b,t) - \dot{v}(t)\right]\right\} \times \delta(x-b) = F(x,t) \tag{3}$$

where

$$C = 2\xi M \omega_n \tag{4}$$

In which ξ denotes the beam-damping coefficient and ω_n is nth natural frequency of the bridge. Also, the equation of motion of the attached NES can be obtained as following equation:

$$m\ddot{v}(t) + C_{NES}(\dot{v}(t) - \dot{y}(b,t)) + K_{NES}(v(t) - y(b,t))^3 = 0 \tag{5}$$

where *y* denotes deflection of the beam which is the function of position *x* and time *t*. *M* is the mass per unit length, *E* is the modulus of elasticity and *I* denotes moment of inertia of the bridge. K_{NES} ,C_{NES} and *m* are the nonlinear spring stiffness, damping and mass of the NES system respectively and displacement of the NES mass is represented by $v(t)$. The mass ratio (β) is defined as the NES mass divided by the bridge mass as follows:

$$\beta = \frac{m}{ML} \tag{6}$$

Galerkin method is employed as the strong technique to solve the Eqs. (3) and (5). The first five vibration modes of the system are taken into account in numerical simulations. Let φ_n denotes the n*th* vibration mode of the beam which satisfies the boundary conditions. Therefore, the deflection of the simply supported beam *y*(*x, t*) is assumed to be:

$$y(x,t) = \sum_{n=1}^{\infty} \phi_n(x) q_n(t) \tag{7}$$

where $q_n(t)$ is the generalized coordinate corresponding to the nth mode. Substituting Eq. (7) into Eq. (3), multiplying both sides of the equation by φ_n, and integrating with respect to *x* over the length of the beam (*L*), yields to the following equation:

$$\left(\frac{ML}{2}\right)\ddot{q}_n + \dot{q}_n\left(\frac{CL}{2}\right) + q_n\left(\frac{EI(n\pi)^4}{2L^3}\right) + \sin\left(\frac{n\pi b}{L}\right)\left\{K_{NES}\left[\sum_{n=1}^{\infty} q_n(t)\sin\left(\frac{n\pi b}{L}\right) - v(t)\right]^3 + C_{NES}\left[\sum_{n=1}^{\infty}\dot{q}_n(t)\sin\left(\frac{n\pi b}{L}\right) - \dot{v}(t)\right]\right\} = p\sum_{n=1}^{\infty} f_n(t,V,L) \tag{8}$$

in which

$$f_n(t,V,L) = \sin\frac{n\pi v(t-t_j)}{L} H(t-t_j) + (-1)^{n+1}\sin\frac{n\pi v\left(t - t_j - \frac{L}{V}\right)}{L} H\left(t - t_j - \frac{L}{V}\right) \tag{9}$$

Moreover, the equation of motion of the attached NES can be written as:

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$$m\ddot{v}+C_{NES}\left[\dot{v}-\sum_{n=1}^{\infty}\dot{q}_n(t)\sin\left(\frac{n\pi b}{L}\right)\right]+K_{NES}\left[v-\sum_{n=1}^{\infty}q_n(t)\sin\left(\frac{n\pi b}{L}\right)\right]^3=0 \tag{10}$$

Nonlinear set of coupled equations of motion including Eqs. (8) and (10) can be solved by numerical integration methods and the time responses consequently can be obtained. The critical speed of the train traveling over the beam can be found as [16]

$$v_c=\frac{\omega_1 d}{2\pi} \tag{11}$$

4 NUMERICAL RESULTS

For a railway bridge having properties listed in Table 1, a numerical simulation is carried out and deflection of the bridge is obtained for a point placed at the mid-span. The bridge is assumed to be excited by the train having the properties listed in Table 2. The deflection of a bridge, having length of 40 meters, is illustrated in Fig. 2. A case with the vibration mitigation system (NES) is compared with a case which has no absorber. As it is seen, there are two phases predictable in time responses. At the first phase related to the forced vibration, the deflection increases for the beam which has no NES. The NES keeps the deflection level constant during the passage of the successive moving loads. At the second phase where the successive moving loads completely passed on the bridge, suppression rate is much faster in presences of the NES. Comparing the RMS value of the bridge response, before and after installation of the NES system, indicates that the RMS reduces up to 42% for the bridge in the case of critical speed.

Table 1
Bridge properties [16]

Parameter	Symbol	Value	Unit
Young module	E	29.43	Gpa
Moment of Inertia	I	7.52	m^4
Mass per unit length of bridge	M	34056	Kg/m
Bridge span length	L	20 and 40	m
Bridge Damping Ratio	ζ	.01	-

Table 2
Train properties [16]

Parameter	Symbol	Value	Unit
Load	P	500	KN
Wagons Number	N	20	-
Load Interval distance	d	20	m

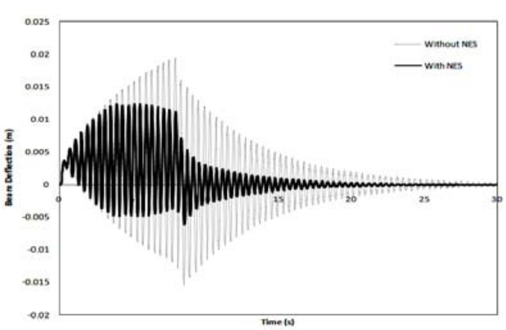

Fig. 2
Bridge Deflection on its center L=40 m, v= 50 m/s.

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5 OPTIMAL DESIGN OF THE NES

Genetic Algorithm (GA) method is adopted here as the optimization technique. The RMS value of the bridge response is considered as following equation:

$$\text{Objective function } = \sqrt{\frac{1}{T}\int_0^T y(t)^2 dt} \tag{12}$$

A computer program has been developed using MATLAB-R2008 software based on the flow-chart of the Genetic Algorithm procedure, illustrated in Fig. 3. Sensitivity analysis of the NES system performed and effects of the nonlinear stiffness and damping on the system are shown in Fig. 4. As it is seen, for the smaller values of damping, nonlinear stiffness has a more significant effect on the NES system. Sensitivity of the optimal NES system to the optimal values is analyzed in Fig. 5. This diagram shows how the off-tuning of the damper and nonlinear spring can affect the performance of the NES system. Vertical axis represents percentage of the response deviation with respect to the optimal system. Characters α_c and α_K denote deviation with respect to optimal values of the NES damping and nonlinear stiffness.

$$\alpha_c = \frac{C_{NES}}{\text{Optimal value of } C_{NES}} \tag{13}$$

$$\alpha_K = \frac{K_{NES}}{\text{Optimal value of } K_{NES}} \tag{14}$$

It is seen that the optimal NES applied for a longer bridge is more sensitive to the damping while the stiffness is significant parameter which has a great effect on the optimal NES in a shorter bridge.

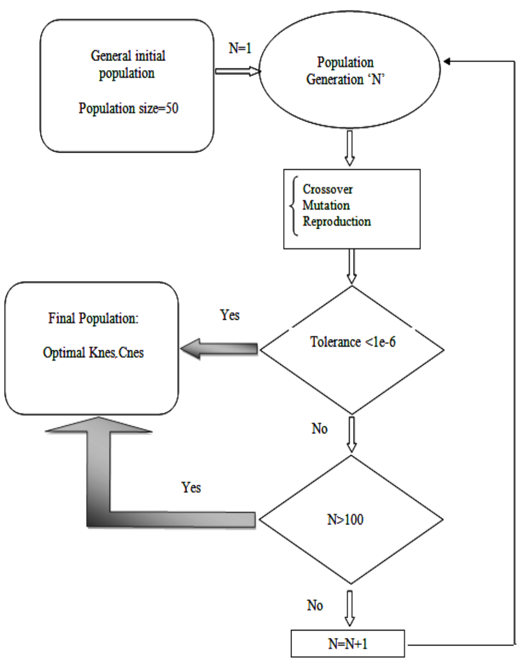

Fig. 3
Flow-chart of the implemented Genetic Algorithm.

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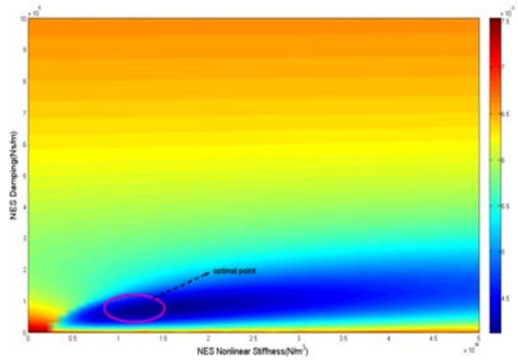

Fig. 4
Detection of an optimal point in damping-stiffness plane.

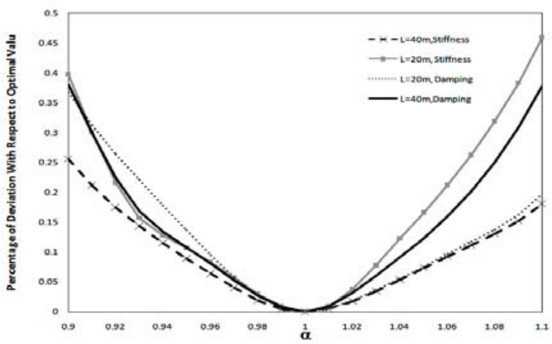

Fig. 5
Robustness of the optimal system.

6 PARAMETRIC STUDY

A parametric study is carried out in this section and effects of different parameters including the load magnitude, train speed and load interval distance on the performance of the optimal NES are studied.

6.1 Mass ratio

Effects of the mass ratio on the optimal value of the nonlinear stiffness and also performance of the NES system are studied in this section. Fig. 6 shows the optimal nonlinear stiffness which is plotted against the mass ratio β. As it is seen, optimal nonlinear stiffness, K_{NES}, is an increasing function of the mass ratio. The NES mass has a significant effect on the NES performance. Fig. 7 illustrates the effect of NES mass on reduction factor of the suppression system. It is clear that the reduction factor is abruptly reduced for the smaller values of mass ratio ($\beta < 0.03$). The saturation phenomenon occurs after this specific value of mass ratio and the slope of the curve asymptotically approaches to zero. In other words, any excessive mass cannot significantly reduce the vibration level.

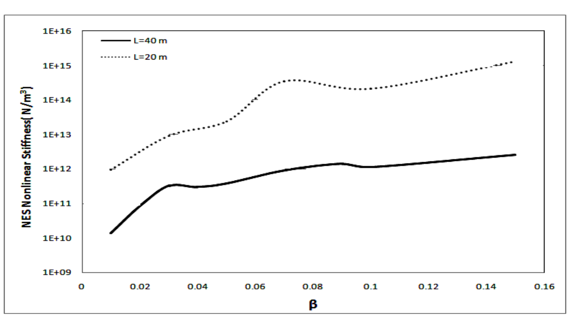

Fig.6
Optimal nonlinear stiffness versus mass ratio.

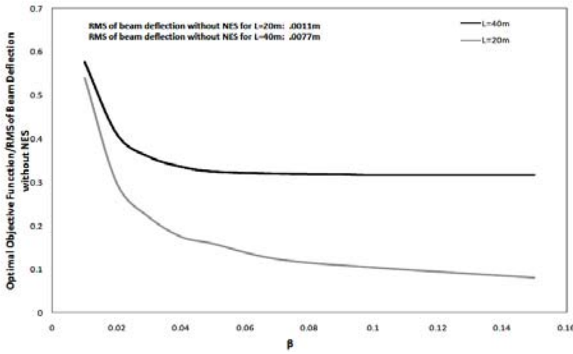

Fig. 7
Optimal objective function versus mass ratio.

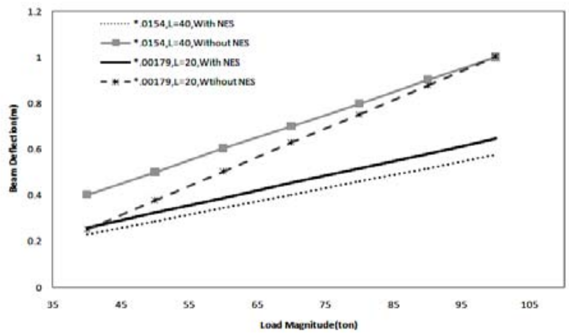

Fig. 8
Beam deflection versus load magnitude obtained for critical speed.

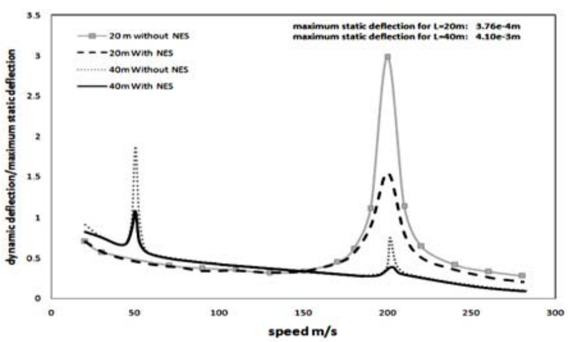

Fig. 9
Beam deflection versus load speed.

6.2 Load magnitude

Since variety of trains with different axle loads may travel on the bridge, a range of common loads should be taken into account in NES optimal design. Fig. 8 shows deflection (RMS value) of the 20 and 40 m bridges against the load magnitude, in case of critical speed. Unlike the linear passive control systems, it is observed that the NES performance is remarkably dependent on the moving load magnitude. As it is seen, the divergence between the responses is enhanced for the larger values of the moving loads. NES, it is implied , more efficiently mitigates the vibration of the bridge subjected to the larger moving loads.

6.3 Train speed

Effect of the train speed on the optimal NES performance is studied in this section. Fig. 9 shows the effect of the train speed on the bridge response in the cases of with and without NES. It is clearly seen that the optimal NES has its maximum suppression effects at the vicinity of the critical speeds.

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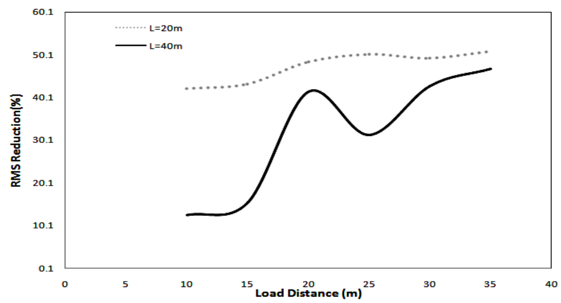

Fig. 10
Percentage of RMS reducation for *L*=20 and 40 m.

Table 3
Comparison between various optimization results

Case	Condition	Bridge Length (m)	NES stiffness (N/m^3)	NES damping (NS/m)	Mass Ratio β	RMS (m)	Reduction (%)
1	R	20	without NES system			1.10e-3	0
2	R	20	9.56e11	9.92e4	.01	5.94e-4	47
3	R	20	2.16e14	4.38e5	0.1	1.15e-4	89.5
4	NR	20	2.59e14	4.75e4	.01	1.70e-4	23
5	R	40	without NES system			77e-4	0
6	R	40	1.39e10	7.46e4	.01	44.3e-4	42.8
7	NR	40	1.54e11	3.00e4	.01	1.7e-3	5
8	R	40	1.14e12	9.13e4	0.1	2.43e-3	68.3

R: Resonance
NR: Non-resonance

6.4 Load interval distance effect

Performance of the optimal NES in different values of the load interval distances is investigated in this section as is shown in Fig. 10. One can generally conclude that the NES has the better performance in shorter span bridges. It is also found that the RMS reduction percentage is generally enhanced by increasing the load span. Table 3 summarizes the results of optimal NES application for different conditions in both the 20 and 40-m length of bridges. Two mass ratios of β=0.01 and β=0.1 are taken into account and different cases including uncontrolled system, resonant system under critical speed and non-resonant case are studied. It is seen that the optimal NES with mass ratio of 10% can remarkably suppress the vibration level up to 89.5 % in a resonant case.

7 CONCLUSIONS

Application of an optimal nonlinear energy sink system in vibration mitigation of railway bridges was studied in this paper. Train was modeled as a series of moving loads traveling over the beam. Root Mean Square (RMS) value of the bridge response was adopted as the objective function. Genetic Algorithm was employed and optimal parameters of the NES system was obtained as a function of the bridge properties and load conditions. For the first time in this paper, properties of the moving load including its magnitude, speed and sequence interval was taken into account in optimization procedure. Unlike the conventional linear passive control systems, it was found that the optimal NES is completely dependent on the moving load parameters. Application of the NES system for different train speed conditions demonstrated that the mitigation system is more efficient in the vicinity of critical speeds. It was also found that the optimal NES systems applying to the longer and shorter bridge are more sensitive to damping and off-tuned stiffness, respectively. Moreover, it was shown that NES performance is monotonically enhanced for the larger values of the moving loads. Finally, it was found that the NES efficiency is amplified in the case of shorter span bridges and the RMS reduction percentage is generally enhanced for longer load spans.

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