Free Vibrations of Continuous Grading Fiber Orientation Beams on Variable Elastic Foundations

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ABSTRACT
Free vibration characteristics of continuous grading fiber orientation (CGFO) beams resting on variable Winkler and two-parameter elastic foundations have been studied. The beam is under different boundary conditions and assumed to have arbitrary variations of fiber orientation in the thickness direction. The governing differential equations for beam vibration are being solved using Generalized Differential Quadrature (GDQ) method. Numerical results are presented for a beam with arbitrary variation of fiber orientation in the beam thickness and compared with similar discrete laminate beam. The main contribution of this work is to present useful results for continuous grading of fiber orientation through thickness of a beam on variable elastic foundation and its comparison with similar discrete laminate composite beam. The results show the type of elastic foundation plays very important role on the natural frequency parameter of a CGFO beam. According to the numerical results, frequency characteristics of the CGFO beam resting on a constant Winkler elastic foundation is almost the same as of a composite beam with different fiber orientations for large values of Winkler elastic modulus, and fiber orientations has less effect on the natural frequency parameter. The interesting results show that normalized natural frequency of the CGFO beam is smaller than that of a similar discrete laminate beam and tends to the discrete laminated beam with increasing layers. It is believed that new results are presented for vibrational behavior of CGFO beams are of interest to the scientific and engineering community in the area of engineering design.

Keywords: continuous grading fiber orientation; free vibrations; beam; elastic foundation; GDQ method

1 INTRODUCTION
FUNCTIONALLY graded materials (FGMs) are heterogeneous materials in which the elastic and thermal properties change from one surface to the other, gradually and continuously. These structures support applied external forces efficiently by virtue of their geometrical shapes. Suresh and Mortensen [1] studied on functionally graded materials and describe the fundamentals of FGMs. Pradhan et al. [2] assumed that material properties follow a through-thickness variation according to a power-law distribution in terms of the volume fractions of constituents. FGM is a class of composite that has a smooth and continuous variation of material properties from one surface to another and thus can alleviate the stress concentrations found in laminated composites.

Beams and columns supported along their length are very common in structural configurations. Beam structures are often found to be resting on earth in various engineering applications. These include railway lines, geotechnical areas, highway pavement, building structures, offshore structures, transmission towers and transversely supported

The differential quadrature method (DQM) is found to be a simple and efficient numerical technique for solving partial differential equations as reported by Bellman et al. [8]. The mathematical fundamental and recent developments of GDQ method as well as its major applications in engineering were discussed in detail in book [9]. Combination of the state-space method and the technique of DQ were used for free vibration of generally laminated beams by Chen et al. [10]. They did this by discretizing the state space formulations along the axial direction using the technique of DQ, new state equations at discrete points were established. Chen [11] used DQM to determine vibration characteristics of cross-ply laminated plates subjected to cylindrical bending. Khalili [12] used a mixed Ritz-DQ method to study the dynamic behavior of functionally graded beams subjected to moving loads and considered the material properties of the FG beam vary through the thickness according to exponential and power-law functions. Pradhan [13] studied thermo-mechanical vibration of FGM sandwich beam under variable elastic foundations by using GDQ method.

In this paper, we study the dynamic behavior of a new type of materials called "functionally graded fiber volume or fiber orientation materials". These kinds of materials have some advantages over discrete laminated ones. For these materials significant improvements are found in their applications due to the reduction in spatial mismatch of mechanical material properties. This research is in the continuation of our previous work [14] to consider vibrational behavior of CGFO beams on variable elastic foundation and its comparison with discrete laminated beam. The present work provides an enhanced insight into the mechanical behavior of this type of materials. For this purpose, a semi-analytic solution procedure for the free vibrations analysis of continuous grading fiber orientation beam on variable elastic foundation is presented. A detailed parametric study is carried out to highlight the influences of fiber orientation in the beam's thickness, material property graded indexes, coefficients of elastic foundations with constant, linear and parabolic modulus on the vibration frequencies of beams and finally comparison is made with similar discrete laminate composite beams.

### 2 PROBLEM DESCRIPTION

Consider a functionally graded fiber orientation beam with its coordinate system \((x, z)\) as shown in Fig. 1. Three different Winkler elastic foundations including:

a: Winkler elastic foundation with constant modulus \(k(x) = k_0\)
b: Winkler elastic foundation with linear variation type \(k(x) = k_0(1 - \alpha x)\)
c: Winkler elastic foundation with nonlinear variation type \(k(x) = k_0(1 - \beta x^2)\)

are considered in this study. The beam is divided into \(N\) fictitious thin layers in the thickness direction, thus each layer can be considered as a plane stress state. The on-axis and off-axis coordinate systems coincide with the "1-2" and "x-y" directions respectively. The mechanical constitutive relations, which relates the stresses to the strains for the \(K\)th layer is expressed as [15]:

\[
\begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\sigma_3 \\
\sigma_4 \\
\sigma_5 \\
\sigma_6
\end{bmatrix} =
\begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\sigma_3 \\
\sigma_4 \\
\sigma_5 \\
\sigma_6
\end{bmatrix}
\begin{bmatrix}
\varepsilon_1 \\
\varepsilon_2 \\
\varepsilon_3 \\
\varepsilon_4 \\
\varepsilon_5 \\
\varepsilon_6
\end{bmatrix}
\]

where \(Q_i\) is the stiffness of the beam at the \(K\)th layer.

The solution presented here is applicable for arbitrary variation of material composition through the thickness of the beam. For the beam, we assume the following specific power-law variation of the fiber orientation [14]:

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\[
\phi = \phi_0 + (\phi_o - \phi_0) \left( \frac{1}{2} + \frac{z}{h} \right)^p
\]

(2)

where \( \phi_0 \) and \( \phi_o \) denote the fiber orientation on the \( z = -\frac{h}{2} \) and \( z = \frac{h}{2} \) respectively and may typically range from 0° to 90°. The power \( p \) denotes the manner in which the orientation of the fibers varies through the thickness. Fig. 2 shows the variations of the fiber orientations through the thickness \( (\eta = \frac{z}{h}) \). In this figure the fiber orientations are assumed as \( \phi_0 = 0^o \) and \( \phi_o = 90^o \) on the lower and upper surfaces respectively.

For continuous grading fiber orientation beam resting on variable two-parameter elastic foundation, the linear governing equation can be expressed as [7]:

\[
-\frac{D_w}{\rho} \frac{\partial^4 W}{\partial x^4} + \frac{\partial}{\partial x} \left( k(x) \frac{\partial W}{\partial x} \right) - k(x)w - \rho A \frac{\partial^2 W}{\partial t^2} = 0 \quad 0 \leq x \leq L
\]

(3)

where \( k(x) \) and \( k_o(x) \) are Winkler and shearing layers elastic coefficients of the foundation. To obtain the natural frequency, the above equation is formulated as an eigen value problem by using the following periodic function:

\[
w(x,t) = W(x)e^{i\omega t}
\]

(5)

where \( W(x) \) is the mode shape of the transverse motion of the beam, therefore:

\[
-\frac{D_w}{\rho} \frac{\partial^4 W}{\partial x^4} + \frac{\partial}{\partial x} \left( k(x) \frac{\partial W}{\partial x} \right) + k(x)W + \rho A \omega^2 W = 0 \quad 0 \leq x \leq L
\]

(6)

Eq. (6) is a fourth-order ordinary differential equation. Thus, it requires four boundary conditions. The following two types of boundary conditions are considered.

Simply supported edge

\[
W = 0, \frac{\partial^2 W}{\partial x^2} = 0 \quad \text{at} \quad x = 0 \text{ or } x = L
\]

(7)

Clamped edge

\[
W = 0, \frac{\partial W}{\partial x} = 0 \quad \text{at} \quad x = 0 \text{ or } x = L
\]

(8)

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Fig. 1
Various Winkler elastic foundations along the axial direction: (a) linear type, (b) parabolic type.
3 GDQ SOLUTION OF GOVERNING EQUATION

The generalized differential quadrature (GDQ) approach is used to solve the governing equation of the beam. The GDQ approach was developed by Shu and coworkers [9, 16] based on the (DQ) technique [8]. It approximates the spatial derivative of a function of given grid point as a weighted linear sum of all the functional value at all grid point in the whole domain. The computation of weighting coefficient by GDQ is based on an analysis of a high order polynomial approximation and the analysis of a linear vector space. The weighting coefficients of the first-order derivative are calculated by a simple algebraic formulation, and the weighting coefficient of the second-and higher-order derivatives are given by a recurrence relationship. The details of the GDQ method can be found in [9, 16]. In the GDQ method, the \( n \)th order of a continuous function \( f(x, z) \) with respect to \( x \) at a given point \( x_i \) can be approximated as a linear sum of weighting values at all of the discrete point in the domain of \( x \), i.e. [9]:

\[
\frac{\partial^nf(x, z)}{\partial x^n} = \sum_{k=1}^{N} c_{ik} f(x_k, z), \quad (i = 1, 2, \ldots, N, \quad n = 1, 2, \ldots, n-1)
\]  

(9)

where \( N \) is the number of sampling points, and \( c_{ik}^n \) is the \( x_i \) dependent weight coefficients.

In order to determine the weighting coefficients \( c_{ij}^n \), the Lagrange interpolation basic functions are used as test functions, and explicit formulation for computing these weighting coefficients can be obtained [9,16]:

\[
c_{ij}^{(n)} = \frac{M^{(i)}(x_j)}{(x_i - x_j)M^{(i)}(x_j)} , \quad i,j = 1, 2, \ldots, N, i \neq j
\]  

(10)

where

\[
M^{(i)}(x_j) = \prod_{j \neq i}^{N} (x_i - x_j)
\]  

(11)

For the first-order derivative; i.e. \( n=1 \) and for higher-order derivative, one can use the following relations iteratively:

\[
c_{ij}^{(n)} = \left( c_{ij}^{(n-1)} + c_{ij}^{(n-1)} - c_{ij}^{(n-1)} \right), \quad i,j = 1, 2, \ldots, N, \quad i \neq j, \quad n = 2, 3, n-1
\]  

(12)

\[
c_{ij}^{(n)} = - \sum_{j \neq i}^{N} c_{ij}^{(n)}, \quad i = 1, 2, \ldots, N, \quad n = 1, 2, \ldots, N-1
\]  

(13)
A simple and natural choice of the grid distribution is the uniform grid spacing rule. However, it was found that non-uniform grid spacing yields results with better accuracy [17]. Hence, in this work, the Chebyshev-Gauss-Labatto quadrature points are used, that is [9]

\[ x_i = \frac{1}{2} \left[ 1 - \cos \left( \frac{i-1}{N-1} \pi \right) \right] \quad i = 1, 2, \ldots, N \]  

(14)

4 RESULTS AND DISCUSSION

First, verification study of the results is considered for an isotropic beam resting on Winkler elastic foundation in Table 1. As observed there is good agreement between the present results with similar ones obtained by Zhou Ding [3].

In this section, we characterize the response of the CGFO beam resting on different types of two-parameter elastic foundation. It is assumed the beam has the following mechanical properties [15]:

\[ \frac{E_2}{E_T} = 25, \quad \frac{G_{12}}{E_T} = 0.2, \quad \frac{G_{13}}{E_T} = 0.5, \quad v = 0.25 \]

First the convergence of the method is investigated in evaluating the natural frequency parameter \( \Omega = \omega h^2 \sqrt{\frac{\rho A}{E_T}} \).

The results are prepared for a graded beam with a linear variation of fiber orientation \((p=1)\) from \( \phi_1 = 0^\circ \) at \( z = -\frac{h}{2} \) to \( \phi_0 = 90^\circ \) at \( z = \frac{h}{2} \) and is shown in Fig. 3. Fast rate of convergence of the method is evident at different boundary conditions and it is found that only ten DQ grid in the thickness direction can yield accurate results. It is also observed for the considered system the formulation is stable while increasing the number of points and that the use of 50 points guarantees convergence of the procedure.

Now we compare a continuous grading fiber orientation beam with a linear variations of fiber orientation \((p = 1)\), from \( \phi_1 = 0^\circ \) at \( z = -\frac{h}{2} \) to \( \phi_0 = 90^\circ \) at \( z = \frac{h}{2} \) with discretely laminated 2-layer \([0^\circ/90^\circ]\), 3-layer \([0^\circ/45^\circ/90^\circ]\), 4-layer \([0^\circ/30^\circ/60^\circ/90^\circ]\) and 7-layer \([0^\circ/15^\circ/30^\circ/\ldots/90^\circ]\) respectively. This comparison is shown in Table 2, for various values of the wave numbers. It results the natural frequency of the CGFO beam is smaller that of a discrete laminate composite one. Also it is found that by increasing the layers of a discrete laminate composite beam, its natural frequency decreases and tends to a similar functionally graded fiber orientation one. Here we consider the effect of various Winkler elastic foundations on the CGFO beam with simply supported ends. Fig. 4 shows variations of the natural frequency parameter of a functionally graded fiber orientation as well as composite beams with different fiber orientations versus various constant Winkler elastic foundations. In this figure the shearing layers elastic coefficient \(k_1\) is assumed to be unity while Winkler elastic modulus \(k\) is considered to vary from 10 to 100,000. From this figure one could observe that for \( k > 10000 \), the natural frequency parameter of the CGFO beam as well as composite one with different fiber orientation are the same. In other words for the large values of Winkler elastic modulus \(k\), fiber orientations has less effect on the natural frequency parameter. Fig. 5 shows variations of the natural frequency parameter of continuous grading fiber orientation beam with a linear variation of fiber orientation \((p=1)\) from \( \phi_1 = 0^\circ \) at \( z = -\frac{h}{2} \) to \( \phi_0 = 90^\circ \) at \( z = \frac{h}{2} \) resting on different types of Winkler elastic foundation. As observed, different types of Winkler elastic foundation doesn’t affect on the natural frequency parameter of CGFO beam for Winkler elastic constant \(k_0\), ranges 10< \( k_0 < 1000 \) and then for \( k_0 > 10000 \), the natural frequency parameter of a CGFO beam resting on a variable Winkler elastic foundation decreases from constant type to parabolic and then linear types. In this figure the ends of the CGFO beam is simply supported. The effect of Winkler elastic foundation coefficients with constant modulus on the natural frequency parameter of a CGFO beam with simply supported ends is illustrated in Fig. 6 for different shearing layer coefficient. As it could be observed the natural frequency parameter converges with increasing the shearing layer elastic coefficient. For further study, the first three natural frequency parameters of the CGFO beam on Winkler elastic foundation with various linear modulus \((\alpha)\) as
well as parabolic modulus ($\beta$) is shown for different boundary conditions in Tables 3. and 4. As noticed the natural frequencies parameter decrease with the increase of linear and parabolic modulus.

### Table 1
Comparison of the frequency parameters of an isotropic beam resting on parabolic type of Winkler elastic foundation ($k = k_0(l - \beta x^2), k_0 = 0, \lambda_i = \frac{\rho M}{EI} \Omega_i^2$)

<table>
<thead>
<tr>
<th>$k_0$</th>
<th>$\beta$</th>
<th>$\lambda_1$</th>
<th>$\lambda_2$</th>
<th>$\lambda_3$</th>
<th>$\lambda_4$</th>
<th>$\lambda_5$</th>
<th>$\lambda_6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000</td>
<td>0.4</td>
<td>5.597</td>
<td>7.022</td>
<td>9.675</td>
<td>12.675</td>
<td>15.763</td>
<td>18.882</td>
</tr>
<tr>
<td></td>
<td>0.8</td>
<td>5.5961</td>
<td>7.0231</td>
<td>9.6744</td>
<td>12.6743</td>
<td>15.7636</td>
<td>18.8818</td>
</tr>
<tr>
<td>1500</td>
<td>0.4</td>
<td>6.138</td>
<td>7.321</td>
<td>9.792</td>
<td>12.728</td>
<td>15.792</td>
<td>18.8905</td>
</tr>
<tr>
<td></td>
<td>0.8</td>
<td>6.1382</td>
<td>7.3207</td>
<td>9.7923</td>
<td>12.7273</td>
<td>15.7912</td>
<td>18.8979</td>
</tr>
<tr>
<td>2000</td>
<td>0.4</td>
<td>6.564</td>
<td>7.587</td>
<td>9.905</td>
<td>12.780</td>
<td>15.819</td>
<td>18.913</td>
</tr>
<tr>
<td></td>
<td>0.8</td>
<td>6.5642</td>
<td>7.5865</td>
<td>9.9063</td>
<td>12.7797</td>
<td>15.8187</td>
<td>18.9140</td>
</tr>
</tbody>
</table>

### Table 2
Comparison of natural frequency parameter of CGFO beam with discretely laminated beam ($k_0=1, k = 2000$)

<table>
<thead>
<tr>
<th>$M$</th>
<th>$[0^\circ/90^\circ]$</th>
<th>$[0^\circ/45^\circ/90^\circ]$</th>
<th>$[0^\circ/30^\circ/60^\circ/90^\circ]$</th>
<th>$[0^\circ/15^\circ/30^\circ/../90^\circ]$</th>
<th>CGFO($0^\circ/90^\circ$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>57.9724</td>
<td>57.7744</td>
<td>57.4724</td>
<td>57.2721</td>
<td>56.9237</td>
</tr>
<tr>
<td>2</td>
<td>152.800</td>
<td>152.603</td>
<td>152.401</td>
<td>152.201</td>
<td>147.939</td>
</tr>
<tr>
<td>3</td>
<td>333.937</td>
<td>331.146</td>
<td>328.350</td>
<td>325.550</td>
<td>318.989</td>
</tr>
<tr>
<td>4</td>
<td>589.911</td>
<td>584.917</td>
<td>580.917</td>
<td>577.917</td>
<td>563.160</td>
</tr>
<tr>
<td>5</td>
<td>920.096</td>
<td>912.278</td>
<td>900.278</td>
<td>889.278</td>
<td>878.219</td>
</tr>
<tr>
<td>6</td>
<td>1324.07</td>
<td>1312.80</td>
<td>1295.51</td>
<td>1278.51</td>
<td>1263.72</td>
</tr>
<tr>
<td>7</td>
<td>1801.68</td>
<td>1786.35</td>
<td>1762.80</td>
<td>1740.80</td>
<td>1719.52</td>
</tr>
<tr>
<td>8</td>
<td>2352.88</td>
<td>2332.84</td>
<td>2302.08</td>
<td>2281.55</td>
<td>2245.55</td>
</tr>
<tr>
<td>9</td>
<td>2977.62</td>
<td>2952.26</td>
<td>2913.33</td>
<td>2887.34</td>
<td>2841.78</td>
</tr>
<tr>
<td>10</td>
<td>3675.90</td>
<td>3644.59</td>
<td>3596.53</td>
<td>3564.44</td>
<td>3508.18</td>
</tr>
</tbody>
</table>

### Table 3
Variation of the first three natural frequency parameters of CGFO beam resting on the elastic foundation with various linear modulus ($k_i=1, k = k_0(l - \alpha i)$)

<table>
<thead>
<tr>
<th>$k_0$</th>
<th>$\alpha$</th>
<th>$\lambda_1$</th>
<th>$\lambda_2$</th>
<th>$\lambda_3$</th>
<th>$\lambda_4$</th>
<th>$\lambda_5$</th>
<th>$\lambda_6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>0</td>
<td>41.717</td>
<td>142.220</td>
<td>316.629</td>
<td>82.676</td>
<td>220.434</td>
<td>430.399</td>
</tr>
<tr>
<td></td>
<td>0.2</td>
<td>41.113</td>
<td>142.044</td>
<td>316.550</td>
<td>82.373</td>
<td>220.320</td>
<td>430.341</td>
</tr>
<tr>
<td></td>
<td>0.4</td>
<td>40.500</td>
<td>141.868</td>
<td>316.471</td>
<td>82.068</td>
<td>220.207</td>
<td>430.283</td>
</tr>
<tr>
<td></td>
<td>0.6</td>
<td>39.877</td>
<td>141.692</td>
<td>316.392</td>
<td>81.763</td>
<td>220.093</td>
<td>430.225</td>
</tr>
<tr>
<td>1000</td>
<td>0</td>
<td>47.332</td>
<td>143.967</td>
<td>317.418</td>
<td>85.646</td>
<td>221.565</td>
<td>430.980</td>
</tr>
<tr>
<td></td>
<td>0.2</td>
<td>46.263</td>
<td>143.619</td>
<td>317.260</td>
<td>85.060</td>
<td>221.339</td>
<td>430.864</td>
</tr>
<tr>
<td></td>
<td>0.4</td>
<td>45.167</td>
<td>143.271</td>
<td>317.103</td>
<td>84.470</td>
<td>221.113</td>
<td>430.748</td>
</tr>
<tr>
<td></td>
<td>0.6</td>
<td>44.042</td>
<td>142.923</td>
<td>316.945</td>
<td>83.875</td>
<td>220.887</td>
<td>430.632</td>
</tr>
<tr>
<td>2000</td>
<td>0</td>
<td>56.924</td>
<td>147.399</td>
<td>318.989</td>
<td>91.298</td>
<td>223.810</td>
<td>432.138</td>
</tr>
<tr>
<td></td>
<td>0.2</td>
<td>55.136</td>
<td>146.720</td>
<td>318.766</td>
<td>90.195</td>
<td>223.363</td>
<td>431.907</td>
</tr>
<tr>
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<td>0.4</td>
<td>53.284</td>
<td>146.039</td>
<td>318.362</td>
<td>89.078</td>
<td>222.915</td>
<td>431.675</td>
</tr>
<tr>
<td></td>
<td>0.6</td>
<td>51.359</td>
<td>145.356</td>
<td>318.048</td>
<td>87.946</td>
<td>222.467</td>
<td>431.444</td>
</tr>
</tbody>
</table>
Table 4
Variation of the first three natural frequency parameters of CGFO beam resting on the elastic foundation with various linear modulus ($k_1=1, k=k_0(1-x)$)

<table>
<thead>
<tr>
<th>$k_0$</th>
<th>$\beta$</th>
<th>s-s</th>
<th>c-c</th>
<th>c-s</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$\Omega_1$</td>
<td>$\Omega_2$</td>
<td>$\Omega_3$</td>
</tr>
<tr>
<td>500</td>
<td>0</td>
<td>41.177</td>
<td>142.220</td>
<td>316.629</td>
</tr>
<tr>
<td></td>
<td>0.2</td>
<td>41.377</td>
<td>142.107</td>
<td>316.578</td>
</tr>
<tr>
<td></td>
<td>0.4</td>
<td>41.033</td>
<td>141.994</td>
<td>316.526</td>
</tr>
<tr>
<td></td>
<td>0.6</td>
<td>40.686</td>
<td>141.882</td>
<td>316.474</td>
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<tr>
<td>1000</td>
<td>0</td>
<td>47.332</td>
<td>143.967</td>
<td>317.418</td>
</tr>
<tr>
<td></td>
<td>0.2</td>
<td>46.730</td>
<td>143.744</td>
<td>317.315</td>
</tr>
<tr>
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<td>0.4</td>
<td>46.119</td>
<td>143.521</td>
<td>317.211</td>
</tr>
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<td>45.498</td>
<td>143.299</td>
<td>317.108</td>
</tr>
<tr>
<td>2000</td>
<td>0</td>
<td>56.924</td>
<td>147.399</td>
<td>318.989</td>
</tr>
<tr>
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</tr>
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<td>0.6</td>
<td>53.838</td>
<td>146.094</td>
<td>318.373</td>
</tr>
</tbody>
</table>

Fig.3
Convergency of the normalized natural frequency.

Fig.4
Effect of Winkler elastic foundation coefficients on the natural frequency parameter of CGFO as well as composite laminated beams ($k_1=1$).

Fig.5
Variations of the natural frequency parameter of a CGFO beam resting on different kinds of Winkler elastic foundation ($k_1=1$).
5 CONCLUSIONS

In this research work, the GDQ method has been used to study free vibration analysis of continuous grading fiber orientation (CGFO) beam. We checked the effectiveness of this method in predicting free vibration behavior of a functionally graded fiber orientation beams by comparing its results for isotropic condition with corresponding numerical results in the literature. From this study, some conclusions can be made:

- It has been found that the convergence of the GDQ results is very fast. The numerical results obtained by using only ten grid points agree very well with those in the literature.
- It results frequency characteristics of the CGFO beam behave very much the same as that of discrete laminate one. The new and interesting results show that the natural frequency of the CGFO beam is smaller that of a discrete laminate composite one and tends to the discrete laminated beam with increasing layers.
- It results frequency characteristics of the CGFO beam resting on a constant Winkler elastic foundation is almost the same as of a composite beam with different fiber orientations for large values of Winkler elastic modulus ($k$), and fiber orientations has less effect on the natural frequency parameter.
- The kind of Winkler elastic foundation doesn't affect on the natural frequency parameter of CGFO beam for Winkler elastic constant ($k_0$), ranges 10< $k_0$<1000.
- It has been resulted the natural frequency parameter converges with increasing the shearing layer elastic coefficient
- It is noticed, the natural frequency parameter of a CGFO beam resting on a variable Winkler elastic foundation decreases from constant type to parabolic and then linear types.

REFERENCES

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