Journal of Solid Mechanics Vol. 4, No. 2 (2012) pp. 159-169

Dynamic Analysis of a Nano-Plate Carrying a Moving Nanoparticle Considering Eelectrostatic and Casimir Forces

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Received 25 February 2012; accepted 27 April 2012

ABSTRACT
This paper reports an analytical method to show the effect of electrostatic and Casimir forces on the pull-in instability and vibration of single nano-plate (SNP) carrying a moving nanoparticle. Governing equations for nonlocal forced vibration of the SNP under a moving nanoparticle considering electrostatic and Casimir forces are derived by using Hamilton's principle for the case when two ends are simply supported. The problem is solved by using the analytically and the time integration methods. The detailed parametric study is considered, focusing on the remarkable effects of the nanoparticle position, nonlocal parameters, nano-plate length, mode number, electric voltage of the Casimir parameter, and dielectric spacer with an initial gap g_0 on vibration of SNP.

Keywords: Casimir force; Vibration analysis; Moving nanoparticle; Mode number

1 INTRODUCTION

IRECENT years a lot of attentions in the field of nano-structural carbon material controllers and nano electromechanical systems have been seen. Electrostatically nano-plate structures have used into many nano applications, such as sensors and mass transport [1]. In different designs, there are two plates conducted with each other by dielectric material fills the gap between them.
Sometimes up the gap is relatively large and the electrostatic model is formulated incorporating higher order correction of electrostatic forces [2].

Applying voltage difference between the two plates is consequence in the deflection of the plates [3]. The applying voltage and the gap between two plates are called pull-in voltage and displacement, respectively [4].

The Casimir effect is the striking force between couples of parallel conducting plates that arises from quantum fluctuations in the ground state of the electromagnetic field [5]. The effect of temperature on the Casimir force is down for a system of two different parallel plates [6]. Silva et al. investigated that in the case of high-temperature limited the repulsive character of the Casimir force between the plates is purely entropic. The effect of Casimir force on bending and vibration of an electrostatically actuated circular plate are investigated by Wang et al. [7] .He showed that the proposed method is accurate and stable which is an effective method to analyze the deformation and free vibration of a circular electrostatically actuated microplate.

Considering the effect of in-plane prestress is quite useful in determining the design parameters of these systems [8]. Some researchers have paid attention to the dynamics and vibration of microplates [8]. Researched the vibrational behavior of electrostatically actuated rectangular microplates subjected to nonlinear squeeze film damping and in-plane forces is investigated by the combined finite element method and finite difference method. D.

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Spinello [9] investigated small vibration of a parallel array of identical microplates predeformed by an electric field and closed-form expressions for the array's modal properties. Vibrations of a fixed–fixed narrow microbeam electrostatically actuated by applying a voltage difference to it and a parallel rigid conductor that its gaps between the two conductors are comparable to the beam's thickness are analyzed. Batra et al. [10] showed that fundamental frequency of the beam may first increase with increasing applied voltage, before suddenly dropping at the pull-in voltage.

It is reported that the study of nonlocal vibration and instability of bonded double-nanosystems are increased. Murmu and Adhikari [11] studied transverse vibration of two single-layered graphene sheets enclosed by polymer matrix. Their research highlighted the effect of small-scale or nonlocal effects considerably influence on the transverse vibration of nonlocal vibration of bonded double-nano-plates. Murmu and Adhikari [12] measured the axial instability of double-nanobeam-systems. Results reveal that the small-scale effects substantially influence the instability (or buckling) of double-nanobeam-systems. Ghorbanpour Arani et al. [13] analyzed vibration the coupled system of double- layered grapheme sheets (CS-DLGSs) embedded in Visco Pasternak foundation is carried out using the nonlocal elasticity theory of orthotropic plate. Their results indicated that the frequency ratio of the CS-DLGSs is more than the single-layered grapheme sheet (SLGS). Further, the motion of nanoparticles in nanotubes is investigated with rapid speed and a lot of attentions.

Motivated by these considerations, analysis of single nano-plate caring the moving nanoparticle that connected with rectangular plate by dielectric medium is investigated. The numerically results are obtained by using the analytically and the time integration methods. The effects of nanoparticle position, nonlocal parameters, nano-plate length, mode number, electric voltage on the Casimir parameter and dielectric spacer with an initial gap on vibration of double- graphene sheet are investigated.

2 MATHEMATICAL MODEL

For considering capability of atomic feature of the nanostructures it is used from component Eringen's nonlocal elasticity theory. Referring to theory as expressed, it is possible to represent the governing equations for an equivalent differential form as [13]:

$$\left(1-\mu\nabla^2\right)\sigma_{ij} = t_{ij} \tag{1}$$

where ∇^2 and μ are Laplacian operator and nonlocal coefficient, respectively. t_{ij} is the macroscopic or local stress tensor and σ_{ij} is the nonlocal stress tensor.

The schematic view of single nano-plate system (SNPs) under moving nano particle studied is shown in Fig. 1.

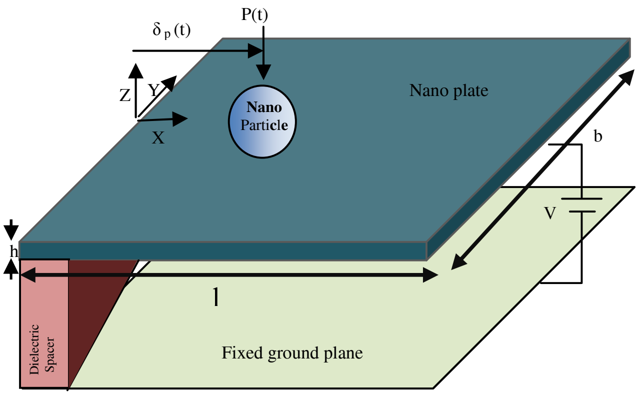

Fig.1
Schematic of single nano plate connected to applied DC voltage and moving nano particle.

The single nano-plate is coupled by dielectric medium to fixed grand plate with an initial gap g. The cross sectional area and thickness for single nano-plate are A and h, respectively. This system is subjected to axisymmetric mechanical, electrical loading and moving a nano particle on the single nano-plate with constant load P .

Based on classical plate theory, the displacement components (u_x, u_y, u_w) along the axes (x, y, z) can be as [13]:

$$u_x = -z\frac{\partial w}{\partial x}, \tag{2}$$

$$u_y = -z\frac{\partial w}{\partial y}, \tag{3}$$

$$u_z = w(x, y, t), \tag{4}$$

where w is the transverse displacement. The strain–displacement relations appropriate basis on the Eqs. (2, 3 and 4), can be written as [13]:

$$\varepsilon_{xx} = \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 - z\frac{\partial^2 w}{\partial x^2}, \tag{5}$$

$$\varepsilon_{yy} = \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 - z\frac{\partial^2 w}{\partial y^2}, \tag{6}$$

$$\gamma_{xy} = \frac{\partial w}{\partial x}\frac{\partial w}{\partial y} - 2z\frac{\partial^2 w}{\partial x \partial y} \tag{7}$$

$$\varepsilon_z = \gamma_{xz} = \gamma_{yz} = 0 \tag{8}$$

The plane stress constitutive relations can be express as [13]:

$$\sigma_{xx} = \frac{E}{1-\nu}\left(\varepsilon_{xx} + \nu\varepsilon_{yy}\right) \tag{9}$$

$$\sigma_{yy} = \frac{E}{1-\nu}\left(\varepsilon_{yy} + \nu\varepsilon_{xx}\right) \tag{10}$$

$$\sigma_{xy} = \frac{E}{1+\nu}\gamma_{xy}, \tag{11}$$

The stress resultant-displacement relations per unit length in the middle surface can be written as [13]:

$$N = \int_{-h/2}^{h/2} \sigma dz \qquad M = \int_{-h/2}^{h/2} z\sigma dz \tag{12}$$

The strain energy π_s , kinetic energy π_k and total work are given by [13]:

$$\pi_{si} = \frac{1}{2}\int\left(\sigma_{xx}\varepsilon_{xx} + \sigma_{yy}\varepsilon_{yy} + \sigma_{xy}\gamma_{xy}\right)dV \tag{13}$$

$$\pi_{ki} = \frac{1}{2}\int \rho\left(\left(\frac{\partial u}{\partial t}\right)^2 + \left(\frac{\partial v}{\partial t}\right)^2 + \left(\frac{\partial w}{\partial t}\right)^2\right)dv \tag{14}$$

$$\Omega_T = \frac{1}{2}\int P\delta\left(x - x_p\right)wdv + \frac{1}{2}\int\left(F_e + F_C\right)dv \tag{15}$$

In this study, for applying the voltage the effects of deflection dependent to distributed electrostatic force F_e and Casimir force F_C are assumed. These forces can be expressed as [3]:

$$F_e = \frac{\varepsilon_0 V^2}{2\left(g - w\right)^2} \tag{16}$$

$$F_C = \frac{\hbar C\pi^2}{240\left(g - w\right)^4} \tag{17}$$

where ε_0, V, g, $\hbar$ and C are the permittivity in vacuum, applied DC voltage, the gap between nano-plate and fixed grand plate, plank 's constant and the speed of light in vacuum.

By using the Hamilton's principle, higher order governing equations of motion for the dynamic version of the principle of virtual work are given [13]:

$$\delta\int_0^t\left(\pi_k - \pi_s + \Omega\right)dt = 0 \tag{18}$$

Substituting Eqs. (13), (14) and (15) into Eq. (18), integrating it by part and setting the coefficients δw equal to zero, equations of motion can be derived.

$$\delta w\ :\ \frac{\partial^2 M_{xx}}{\partial x^2} + 2\frac{\partial^2 M_{xy}}{\partial x\,\partial y} + \frac{\partial^2 M_{yy}}{\partial y^2} + \left(1 - (e_0 a)^2\nabla^2\right)F = \left(1 - (e_0 a)^2\nabla^2\right)\left\{\rho h\frac{\partial^2 w}{\partial t^2} - \frac{\rho h^3}{12}\left(\frac{\partial^4 w}{\partial x^2\partial t^2} + \frac{\partial^4 w}{\partial y^2\partial t^2}\right)\right. \tag{19}$$

where F and ρ are total transverse load and density for SNPs, respectively.

In this paper, the material of single nano-plate is assumed to be linear elastic and only the static deflection of the system is measured. The SNPs is connected to fixed grand plate by dielectric spacer and two its sides are simply supported. The nano-plate is subjected to a moving particle with constant load and neglecting the friction force between nano-plate and nanoparticle.

By using Eq. (19), the strain displacement relation for SNPs and the structural boundary conditions can be express as:

$$\frac{-Eh^3}{12\left(1 - \nu^2\right)}\left\{\frac{\partial^4 w}{\partial x^4} + 2\nu\frac{\partial^4 w}{\partial x^2\delta y^2} + \frac{\partial^4 w}{\partial y^4}\right\} + \left(1 - (e_0 a)^2\nabla^2\right)F = \left(1 - (e_0 a)^2\nabla^2\right)\left\{\rho h\frac{\partial^2 w}{\partial t^2} - \frac{\rho h^3}{12}\left(\frac{\partial^4 w}{\partial x^2\partial t^2} + \frac{\partial^4 w}{\partial y^2\partial t^2}\right)\right\} \tag{20}$$

$$w(0,t) = w(L,t) = 0 \tag{21}$$

$$\frac{\partial^2 w(0,t)}{\partial x^2} = \frac{\partial^2 w(L,t)}{\partial x^2} = 0 \tag{22}$$

The total external work due to electrostatic and Casimir forces and moving load can be written to these simply forms as [1, 3]:

$$F = -P\delta\left(x - x_p\right) + F_e + F_C \tag{23}$$

In order to obtain the analytical solution for electrostatic and Casimir forces and by using $W = \frac{w}{g}$, the set Tailor will be used [3]:

$$F_e = \frac{\varepsilon_0 V^2}{2\left(g-w\right)^2} = \frac{\varepsilon_0 V^2}{2g^2}\left(1 - \frac{w}{g}\right)^{-2} = \frac{\varepsilon_0 V^2}{2g^2}\left(1 + 2W + 3W^2 + ...\right) \tag{24}$$

and

$$F_C = \frac{\hbar C\pi^2}{240\left(g-w\right)^4} = \frac{\hbar C\pi^2}{240g^4}\left(1 - \frac{w}{g}\right)^{-4} = \frac{\hbar C\pi^2}{240g^4}\left(1 + 4W + 10W^2 + ...\right) \tag{25}$$

The essential dimensionless parameters can be expressed as:

$$\begin{array}{lllll} e_n = \dfrac{e_0 a}{L}, & \xi = \dfrac{h}{L}, & \varsigma = \dfrac{x}{L}, & \bar{t} = \dfrac{t}{L}\sqrt{\dfrac{E}{\rho}}, & \beta = m\pi \\ u = \dfrac{w_i}{h}, & \overline{F_e} = \dfrac{F_e}{E}, & \overline{F_e} = \dfrac{F_C}{E}, & \Gamma = \dfrac{y}{L}, & s = \dfrac{b}{L}, \\ \theta = \dfrac{n\pi}{s} & \bar{g} = \dfrac{g}{L}, & \alpha_e = \dfrac{\hbar C\pi^2}{240g^4E}, & \alpha_C = \dfrac{\varepsilon_0 V^2}{2g^2E}, & \bar{P} = \dfrac{P}{E}, \end{array} \tag{26}$$

Substituting Eqs (26) into Eq. (20), the dimensionless equations of motion could be rewritten as:

$$\begin{aligned} &\xi\frac{\partial^2 u}{\partial \bar{t}^2} - \frac{\xi^4}{12}\left(\frac{\partial^4 u}{\partial\varsigma^2\partial\bar{t}^2} + \frac{\partial^4 u}{\partial\Gamma^2\partial\bar{t}^2}\right) - e_n^2\left(\begin{array}{l} \xi\dfrac{\partial^4}{\partial\varsigma^2\partial\bar{t}^2} - \dfrac{\xi^4}{12}\left(\dfrac{\partial^6 u}{\partial\varsigma^4\partial\bar{t}^2} + \dfrac{\partial^6 u}{\partial\varsigma^2\partial\Gamma^2\partial\bar{t}^2}\right) + \\ \xi\dfrac{\partial^4 u}{\partial\Gamma^2\partial\bar{t}^2} - \dfrac{\xi^4}{12}\left(\dfrac{\partial^6 u}{\partial\Gamma^2\partial\varsigma^2\partial\bar{t}^2} + \dfrac{\partial^6 u}{\partial\Gamma^4\partial\bar{t}^2}\right) \end{array}\right) - \\ &\frac{\xi^3}{12\left(1-\nu^2\right)}\left(\frac{\partial^4 u}{\partial\varsigma^4} + 2\nu\frac{\partial^4 u}{\partial\varsigma^2\partial\Gamma^2} + \frac{\partial^4 u}{\partial\Gamma^4}\right) + \overline{F_C} + \overline{F_e} + e_n^2\overline{\nabla^2}\left(\overline{F_C} + \overline{F_e}\right) = \overline{P}\overline{\delta\left(x - x_p\right)} + e_n^2\overline{\nabla^2 P\delta\left(x - x_p\right)} \end{aligned} \tag{27}$$

where

$$\overline{1 - \left(e_0 a\right)^2 F_e} = \alpha_e\left[\left(1 + 2\xi\frac{u}{\bar{g}} + ...\right) - 2e_n^2\frac{\xi}{\bar{g}}\left(\frac{\partial^2 u}{\partial\varsigma^2} + \frac{\partial^2 u}{\partial\Gamma^2}\right) + ...\right] \tag{28}$$

$$\overline{1 - \left(e_0 a\right)^2 F_{C2}} = \alpha_C\left[\left(1 + 4\xi\frac{u_2}{\bar{g}} + 10W^2 + ...\right) - 4e_n^2\frac{\xi}{\bar{g}}\left(\frac{\partial^2 u_2}{\partial\varsigma^2} + \frac{\partial^2 u_2}{\partial\Gamma^2}\right) + ...\right] \tag{29}$$

3 ANALYTICAL SOLUTIONS

The single nano-plate consider with the length L and width b respectively. The Navier method is suggested as follows [1]:

$$\psi\left(\varsigma,\Gamma,\bar{t}\right)=\sum_{i=1}^{\infty}\sin\left(\beta\varsigma\right)\cos\left(\theta\Gamma\right)r\left(\bar{t}\right) \tag{30}$$

where m, n and $r\left(t\right)$ are half wave number and unknown time function is depended to generalized coordinates.

Substituting Eq. (30) into Eq. (20) then multiplying to Eq. (26) without the effect of time function, double integrating it over the area $\left(0,L\right)$ and $\left(0,b\right)$ and pay attention to the property of Dirac-delta function [1]:

$$\int_{x_1}^{x_2} f(x)\delta\left(x-x_0\right)dx=f(x_0)\quad x_1\langle x\langle x_2 \tag{31}$$

yields

$$\begin{aligned}&\left\{\begin{array}{l}\dfrac{\xi^3}{12\left(1-\nu^2\right)}\left(\beta^4+2\nu\beta^2\theta^2+\theta^4\right)-\alpha_C\dfrac{2\xi}{\bar{g}}-\\[2ex] 2e_n^2\dfrac{\xi}{\bar{g}}\alpha_C\left(\beta^2+\theta^2\right)-4\alpha_e\dfrac{\xi}{\bar{g}}\left(1+e_n^2\left(\beta^2+\theta^2\right)\right)\end{array}\right\}r\left(\bar{t}\right)+\\ &\left\{\xi+\frac{\xi^4}{12}\left(\beta^2+\theta^2\right)-e_n^2\left(-\xi\left(\beta^2+\theta^2\right)-\frac{\xi^4}{12}\left(\beta^4+2\beta^2\theta^2+\theta^4\right)\right)\right\}\ddot{r}\left(\bar{t}\right)=\\ &\left(\alpha_e+\alpha_C\right)\sin\beta\varsigma_p\cos\theta\Gamma_p+\bar{P}\left[1+e_n^2\left(\beta^2+\theta^2\right)\right]\sin\beta\varsigma_p\cos\theta\Gamma_p\end{aligned} \tag{32}$$

After that arrangement Eq.(32)the equation of motion could be expressed in a matrix form as:

$$\left[k\right]r(\bar{t})+\left[M\right]\ddot{r}(\bar{t})=f \tag{33}$$

where k and M are the stiffness and the mass matrixes. Thus, the differential equation can be obtained as follows

$$\ddot{r}(t)+\omega^2 r(t)=F(t) \tag{34}$$

where

$$\begin{aligned}\omega^2=&\left\{\begin{array}{l}\dfrac{\xi^3}{12\left(1-\nu^2\right)}\left(\beta^4+2\nu\beta^2\theta^2+\theta^4\right)-\alpha_C\dfrac{2\xi}{\bar{g}}-\\[2ex] 2e_n^2\dfrac{\xi}{\bar{g}}\alpha_C\left(\beta^2+\theta^2\right)-4\alpha_e\dfrac{\xi}{\bar{g}}\left(1+e_n^2\left(\beta^2+\theta^2\right)\right)\end{array}\right\}\div\\ &\left\{\xi+\frac{\xi^4}{12}\left(\beta^2+\theta^2\right)-e_n^2\left(-\xi\left(\beta^2+\theta^2\right)-\frac{\xi^4}{12}\left(\beta^4+2\beta^2\theta^2+\theta^4\right)\right)\right\}\end{aligned} \tag{35}$$

$$F(t)=\left[\left(\alpha_e+\alpha_C\right)\sin\beta\varsigma_p\cos\theta\Gamma_p+\overline{P}\left[1+e_n^2\left(\beta^2+\theta^2\right)\right]\sin\beta\varsigma_p\cos\theta\Gamma_p\right]\div$$
$$\left\{\xi+\frac{\xi^4}{12}\left(\beta^2+\theta^2\right)-e_n^2\left(-\xi\left(\beta^2+\theta^2\right)-\frac{\xi^4}{12}\left(\beta^4+2\beta^2\theta^2+\theta^4\right)\right)\right\} \tag{36}$$

Assume that nano particle is moving on the axial axes, therefore $\Gamma_p=0$
The solution of Eq. (34) with homogeneous initial conditions is given as [1]:

$$r(t)=\frac{1}{\omega}\int_0^t F(\tau)\sin\omega\left(t-\tau\right)d\tau \tag{37}$$

Substituting Eqs. (37) into (30), the dynamic deflection of the nano-plate can be obtained.

4 RESULTS AND DISCUSSION

In this paper ,the effects of small scale, nano-plate length, mode number, electrostatic and Casimir forces and on the SNPs under moving nanoparticle are investigated. The subsequent parameters are taken to calculate the numerical results [3, 13]:

$$\begin{aligned}&\varepsilon_0=8.854\times10^{-12}C^2N^{-1}m^{-2};\\&C=2.998\times10^8 m/s;\\&H=1.055\times10^{-34}Js;\\&\nu=0.3;\\&E=1.02Tpa;\\&P=10\times10^{-9};\\&h=0.075nm;\\&L=10nm\end{aligned} \tag{38}$$

The effect of moving nanoparticle on the SNPs is shown in Fig.2. When nanoparticle is at the left and right hand side, the deflection of SNPs is zero and the maximum deflection is occurred at mid distance of the nano-plate. Hence, the study of the system behavior under applied DC voltage to the single nano-plate, Casimir and electrostatic forces, aspect ratio and different L/b ratio are investigated at mid span of the SNPs. This figure compares dimension deflection of SNPs which are obtained by analytical method and nonlocal parameters for the state of particle in mid distance of the nano-plate. As can be seen in this figure, increasing in the values of nonlocal parameters, lead to decrease in the magnitudes of the dimension deflection of SNPs.

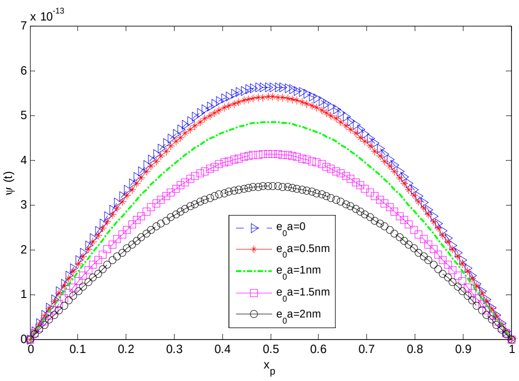

Fig.2
Variation of the SNP deflection as function of position of nano particle (x_p) for various values of nonlocal parameters .

The effect of nano-plate length on deflection of SNPs is shown in Fig. 3. It is essential to mention that deflection of mid span of SNPs increases by increasing the ratio of nano-plate width to its length (b/L). By using the square nano-plate system, the pull-in instability increases. It is due to the fact that the narrow nano-plate has higher instability than square nano-plate.

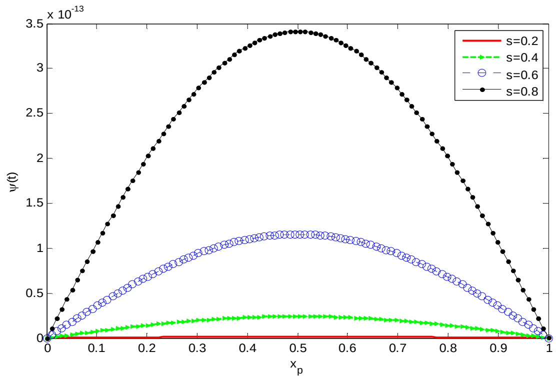

Fig.3
The effect of the various value of s (*b/L*) on deflection of SNPs as function of position of nano particle (x_p).

The aspect ratio effect on pull-in deflection in various applied half wave numbers is depicted in Fig. 4. It can be seen that the deflection of nano-plate decreases with the increase of the half wave number values. Also, it is important to note that the deflection of SNPs converges to zero by increasing the aspect values.

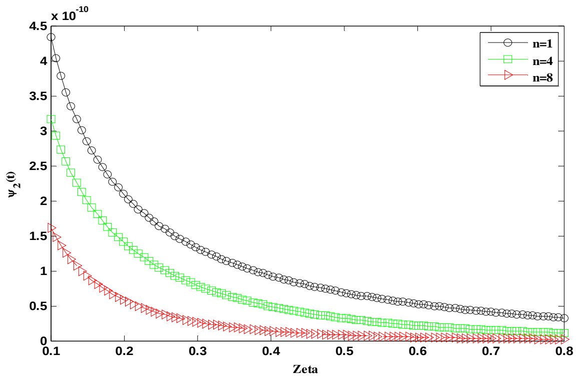

Fig.4
Variation of the SNP deflection as function aspect ratio (*h/L*) for various values of *n*.

The influence of the aspect ratio on pull-in deflection for selected values of nonlocal parameters is demonstrated in Fig.5. It is interesting to note that increase in the nonlocal parameter values lead to increase in the effect of aspect ratio on pull-in deflection. Also, the deflection of nano-plate is decreased by increasing the aspect ratio. It is assumed that by increasing the thickness of SNP, deflection of system is ignored.

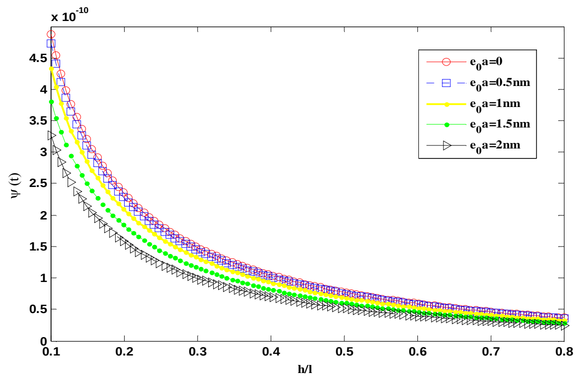

Fig.5
The influence of the nonlocal parameters on the deflection of SNPs as function of aspect ratio (*h/L*).

The effect of geometric on deflection behavior is investigated in Fig.6. This figure shows that by increasing the ratio of gap to width, the deflection magnitudes increase.

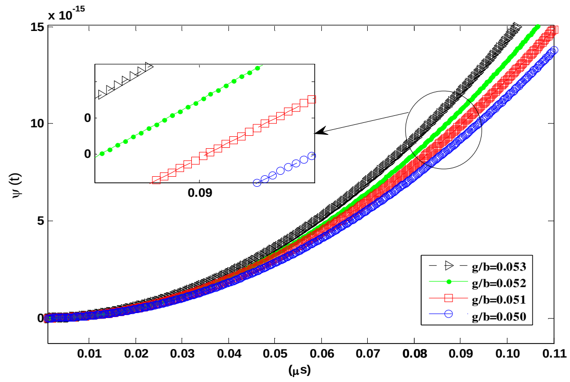

Fig.6
Variation of the deflection of SNPs as function of time for various value of ratio of gap to width.

Figs.7 and 8 show variation of deflection with time history undergoing selected values of nonlocal parameters and Casimir force. In Fig. 7 the speed of SNP collapses on to the fixed electrode increases when nonlocal parameter values increase. It should be noted that in Fig. 8, increase in the Casimir forces values lead to increase of the collapses on to the fixed electrode.

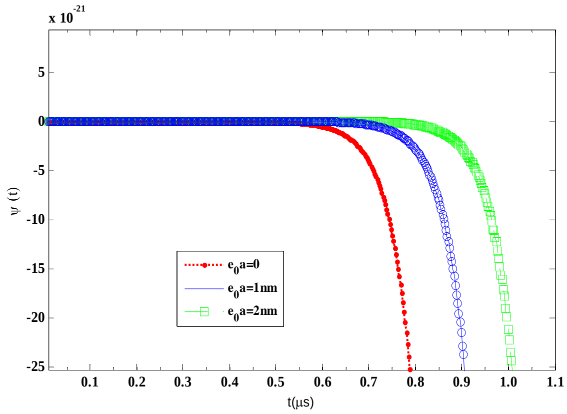

Fig.7
Non dimensional deflection of SNP as function of time for various values of nonlocal parameters.

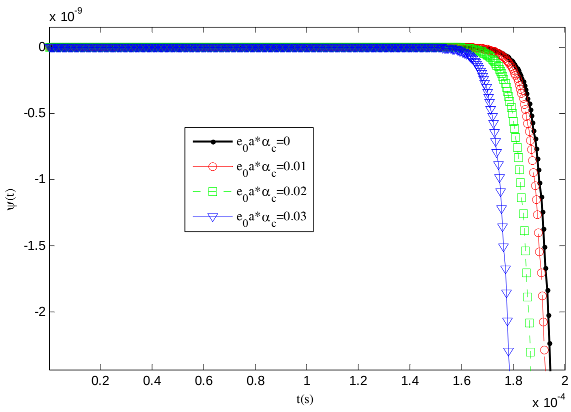

Fig.8
The effect of the Casimir force on the deflection of SNPs as function of time at $e_0a=2nm$.

In order to validate this work, a simplified analysis suggested by R. C. Batra et al. [14] on the Reduced-order models for microelectromechanical rectangular and circular plates incorporating the Casimir force is proposed. Fig. 9 shows the comparison of the critical Casimir force parameter on the aspect ratio between present work and those obtained by [14]. As it is demonstrated, there is no remarkable deviation for Casimir force parameter values and demonstrates good agreement with obtained results by [14].

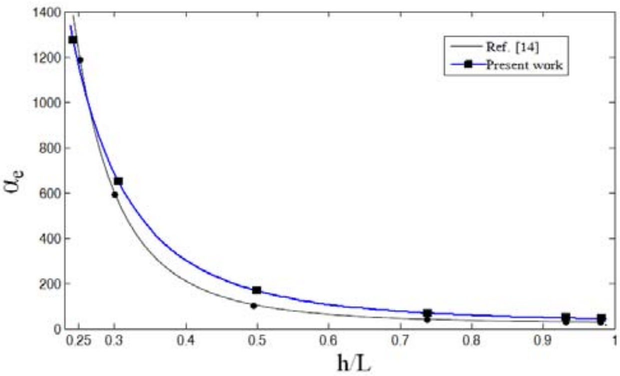

Fig.9
The comparison of the Casimir force parameter effect against aspect ratio between present study and Batra et al. [14].

5 CONCLUSION

In this work, an analytical method is used to show the effect of electrostatic and Casimir forces on the pull-in instability and vibration of SNP carrying a moving nanoparticle. The main motivations of this paper are using SNP in presence of electrostatic and Casimir forces and the nonzero nonlocal stress in an equivalent differential form under a moving nanoparticle. As it is presented, when nanoparticle is at the left and right hand side, the deflection of SNPs is zero and the maximum deflection is occurred at mid distance of the nano-plate. Also, increasing in the values of nonlocal parameters, leads to decrease in the magnitudes of the dimension deflection of SNPs. Furthermore, it is shown that deflection of mid span of SNPs increases by increasing the ratio of nano-plate width to its length (b/L). The effect of the ratio of gap to width on the deflection of the SNP is demonstrated as well. It can be seen that the speed of SNP collapses on to the fixed electrode for selected magnitudes of nonlocal parameters and Casimir forces was investigated.

ACKNOWLEDGMENTS

The authors are grateful to University of Kashan for supporting this work by Grant No. 65475/34. This research was supported by the Thermoelasticity Center of Excellence, Mechanical Engineering Department, Amirkabir University of Technology.

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