Journal of Solid Mechanics Vol. 4, No. 3 (2012) pp. 226-236

Measurement of Variation in Fracture Strength and Calculation of Stress Concentration Factor in Composite Laminates with Circular Hole

A.R. Ghasemi*, I. Razavian

Department of Mechanical Engineering, University of Kashan, Kashan, Iran

Received 13 June 2012; accepted 15 August 2012

ABSTRACT

In this research, residual strength and stress concentration factor of laminated composites with a circular open hole are studied analytically, numerically and experimentally. The numerical study was carried out using the finite element method. Moreover an analytical study was carried out with developing of point stress criterion. Mechanical testing was performed to determine the un-notched tensile properties and notched strength of composite laminates and characteristic length to reinforcement of the notched strength of composite laminates are determined. Results show that the influence of specimen dimension, notch size, lay ups and material properties are important on residual strength and stress concentration factor of laminated composite materials.

Keywords: Residual strength; Composite laminates; Characteristic length; Point stress; Stress concentration

1 INTRODUCTION

AVAILABILITY of accurate values for strength of composite laminates is very important while designing structures made from this type of materials. These ideal composite structures might be damaged by holes drilled to fasten parts with rivets and bolts [1,2]. All these discontinuities are the reasons for deviation of virtual lines representative of stress flow in material close to these zones. This phenomenon is called stress concentration. The behavior of composite laminates facing discontinuity is different compared to the isotropic materials in the same condition. This behavior depends on the lay-ups, material properties and geometry of the sample. Therefore, study of this phenomenon is an essential object in order to be aware of actual strength of composite structures.

The point stress criterion, the progressive damage model and the fracture mechanics are three main criteria to study strength of composite laminates with hole. Due to simplicity and accuracy, the point stress criterion is used widely in research and industry. The point stress criterion, first presented by Whitney and Nuismer [3], is an extension of the characteristic length criterion. Based on this criterion, the laminate fails when the stress in an identifiable distance from the hole reaches to the strength of the laminate without a hole. This description for failure may solve the problem of elastic stress analysis in predicting the nonlinear behavior for yield-damage of laminates at the hole edge. Later on Pipes et al. [4], and Tan [5] modified this criterion. All these modified models propose a characteristic length to identify the notched strength of composite laminates.

The progressive damage model was developed to predict the extent of damage and damage progression in notched composite laminates [6]. Because the evaluation of the progressive damage model was based on the finite element method, it is less attractive than other closed form criteria.

---

* Corresponding author. Tel.: +98 361 555 9930.
E-mail address: Ghasemi@kashanu.ac.ir (A.R. Ghasemi).

The model based on fracture mechanics, such as WEK model [7] and Mar-Lin criterion [8], and the damage zone model (DZM) [9] and the damage zone criterion (DZC) [10] are some criteria based on fracture mechanics. DZM or DZC model states that damage zone is approximated by a crack with cohesive stresses acting on its surfaces, and damage in the material is taken into account by reducing the cohesive stresses with increased crack opening, which in turn corresponds to increased separation of materials. Similarly, an effective crack growth model was developed using an iterative technique to predict the residual strength of composite laminates containing a hole or sharp notches [11].

In this study, point stress criterion is used to investigate stress concentration and residual strength in composite materials such as Carbon / Epoxy and Glass / Epoxy, where stress concentration factor is calculated for several diameter to width ratio, using analytical and numerical methods. In order to validate the results, fracture strength tests were carried out on the prepared experimental samples using tensile test before and after drilling, and characteristic length for reinforcement is calculated and the results are compared with theory.

2 THE STRESS CONCENTRATION IN ISOTROPIC MATERIALS

2.1 Exact solution

The stress distribution is not monotonic around any geometrical discontinuity in isotropic material, therefore the stress increases locally which is known as stress concentration. In the case of isotropic materials, the stress concentration factor has already been investigated analytically and numerically. This quantity is independent of the property of isotropic material and size of the sample depends only on geometrical parameters. There are following well-known Eq.(1) for tension and bending of isotropic materials:

$$k_T = \frac{\sigma_{\max}}{\sigma_{ave}} \rightarrow \begin{cases} \sigma_{ave} = P/A & \text{Tension} \\ \sigma_{ave} = MC/I & \text{Bending} \end{cases} \tag{1}$$

where σ_{ave} is the stress far from the discontinuity, $\sigma_{\max}$ is the maximum stress closed to the discontinuity, and k_T is defined as the stress concentration factor. P, A, M, C, I are force; area; bending moment, outer radius of cylinder, and moment of inertia. To establish and verify the used simulation method, in the first step, this factor is calculated for a rectangular plate with geometrical conditions presented in Table 1. The calculation is made based on both analytical and numerical methods. Subsequently, the obtained results are verified with available accurate values.

Table1

Constant elastic coefficient of material and geometrical dimensions of samples

Radius $R(mm)$	Width $W(mm)$	Thickness $t(mm)$	Poisson's Ratio υ	Elastic modulus $E\ (GPa)$
4	40	8	0.3	207

For this special problem, Eq. (2) is used widely as an analytical solution [13]:

$$K = 3 - 3.13(\frac{2r}{D}) + 3.36(\frac{2r}{D})^2 - 1.53(\frac{2r}{D})^3 \tag{2}$$

where r is the radius of the hole and D is the width of sample.

2.2 The finite element simulation

The ANSYS commercial software is used to simulate this problem numerically. A two dimensional element called Plane 82 which is physically and geometrically consistent for this problem is selected to mesh the model. The optimum number of element is used based on the convergence of the obtained results. The observed stress distribution proves disruption of stress around the hole, as expected (similar to Fig. 2). The stress far from the notch is 30 MPa, while this quantity at the notch place is 80.4 MPa, which is the evidence of local increase of stress at the notch place. The stress concentration factor is $k_T = \frac{80.4}{30} = 2.6$. A comparison between the results obtained numerically and analytically is presented in Table 2. Also the errors are calculated.

Table 2
Analytical and numerical obtained values for stress concentration factor

Analytical result	Numerical result	Accurate value	Error of analytical method	Error of numerical method
2.51	2.68	2.47	6.4%	7.8%

As can be seen from Table 2. , there is an acceptable agreement between the results. Therefore, applicability and accuracy of the FE simulation method used to obtain stress concentration factor is verified. Also the accuracy of the value calculated analytically is confirmed.

3 THE STRESS CONCENTRATION IN ORTHOTROPIC MATERIALS AND COMPOSITE LAMINATES

In this part of the study, the stress concentration in an orthotropic ply and composite laminates is investigated. Since the property of composite materials is quite different from isotropic materials, the behavior of this type of materials facing discontinuity is expected to be different, consequently. In the first step, the stress concentration factors in Carbon / Epoxy and Glass / Epoxy obtained analytically for several diameters to width ratios and with different lay-ups are compared. Then the FE analyses are used to calculate this quantity for the same problems. Finally, the results of the two different methods are compared with each other.

3.1 Exact solution

In this study, the point stress criterion is considered for the analytical approach [3]. According to this criterion for a unidirectional layer:

$$K_T^{\infty} = 1 + \left[2\left[\sqrt{\frac{E_{xx}}{E_{yy}}} - \upsilon_{xy} \right] + \frac{E_{xx}}{G_{xy}} \right]^{0.5} \tag{3}$$

where K_T^{∞} is the stress concentration factor for an infinite rectangular plate with a circular hole. E_{xx}, E_{yy}, and G_{xy} are axial, transversal, and shear modules, respectively. υ_{xy} is the major Poisson's ratio for the laminate. Since Eq.(3) is developed for an infinite plate, a correction factor is implemented to modify this equation for a finite plate [12]. Also, in the case of composite laminates, equivalent modules for the laminates are replaced in Eq.(3). With a given radius and width for the sample, the stress concentration factor may be calculated based on a modified form of Eq. (3) as:

$$\frac{K_T^{\infty}}{K_T} = \frac{3(1-{}^{2r}\!/_{W})}{2+(1-{}^{2r}\!/_{W})^3} + \frac{1}{2}\left(\frac{2r}{W}M\right)^6 (K_T^{\infty}-3) \times \left[1-\left(\frac{2r}{W}M\right)^2\right] ; \tag{4}$$

where K_T , K_T^{∞} are stress concentration factor at the hole edge for a finite width plate and an infinite plate,

respectively. Also, M is the finite width correction (FWC) factor [12] which depends on the geometry of the sample and is identified with Eq.(5):

$$M^2 = \frac{-1+\sqrt{1-8\left[(3(1-{}^{2r}\!/_{W}))/(2+(1-{}^{2r}\!/_{W})^3)-1\right]}}{2\,({}^{2r}\!/_{W})^2}. \tag{5}$$

A computer program is prepared to calculate stress concentration factor for orthotropic material and composite laminates based on the above-mentioned theory.

3.2 The finite element simulation

ANSYS [14] is the commercial software used to simulate this problem numerically. Since the composite is a laminated material, the Shell 99 [14] element is selected to mesh the model Fig. 1. Fig. 2 shows the stress distribution in the case of an orthotropic material.

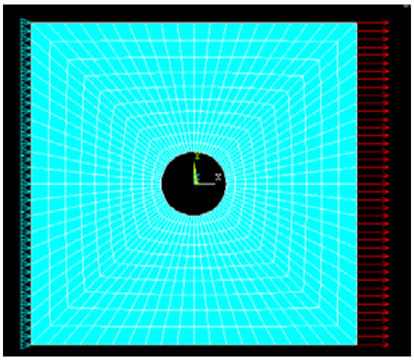


Fig. 1
The FE model; Load and boundary conditions.

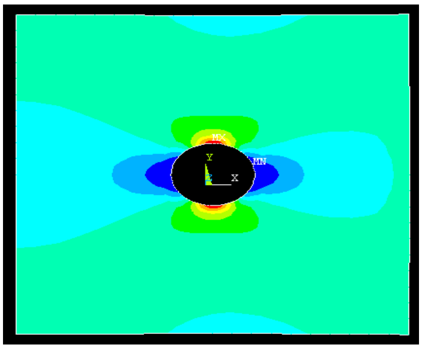
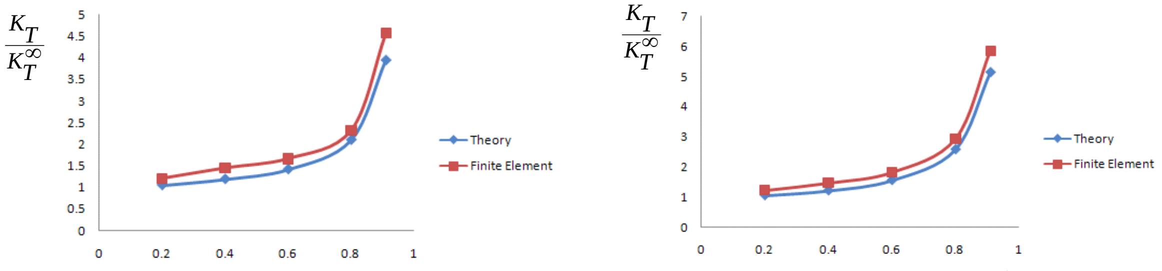


Fig. 2
Stress distribution obtained by the FE method.

For each case, the stress concentration factor is calculated considering stress close and far from the notch place. Several cases with different diameter to width ratios are studied for lay-up $\left[0^\circ\right]$ and two composites Carbon / Epoxy and Glass / Epoxy. Table 3. and Fig. 3 show the results. To study the effect of geometry, the case of lay-up $\left[0^\circ\right]$ is considered. $\frac{K_T^\infty}{K_T}$ varies slowly while the diameter to width ratio increases from 0.2 to 0.6. However, this variation increases rapidly for the range of 0.6 to 1.0. Moreover, the results reveal that the stress concentration factor is larger for Glass / Epoxy compared to Carbon / Epoxy.



(a) (b)

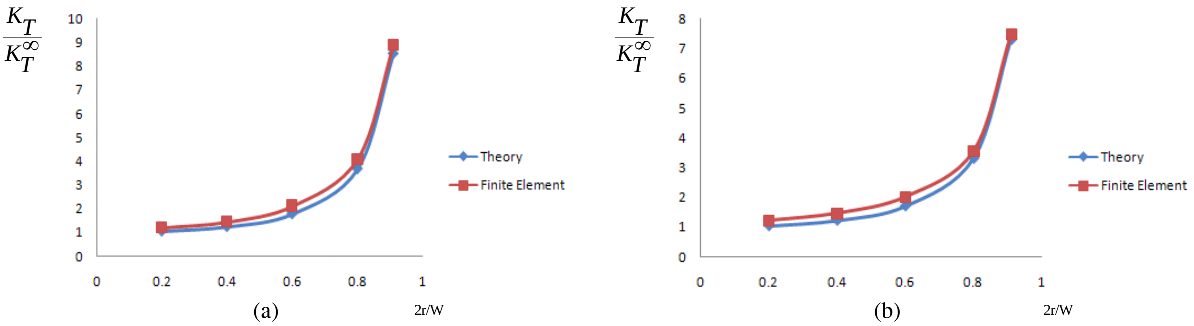
Fig. 3
Stress concentration factor for lay-up $\left[0^\circ\right]$, analytical and numerical. (a) Carbon / Epoxy , (b) Glass / Epoxy.

Table 3
Stress concentration factor for lay-up $\left[0^\circ\right]$

$\frac{2r}{W}$	M	K_T/K_T^∞					
		Carbon/Epoxy			Glass/Epoxy		
		Analytical	Numerical	Error	Analytical	Numerical	Error
0.2	1.44	1.05	1.22	13.9	1.05	1.22	14.4
0.4	1.35	1.19	1.46	18.4	1.21	1.47	17.6
0.6	1.23	1.41	1.67	15.3	1.55	1.82	14.7
0.8	1.11	2.10	2.33	9.9	2.58	2.93	12.0
0.91	1.05	3.94	4.57	13.7	5.15	5.85	11.9

The FE models are built for lay-up $\left[90^\circ\right]$ and several diameters to width ratios. The results are presented in Table 4. and Fig. 4.

As can be seen from Fig. 4, there is a smooth variation in stress concentration factor curve for the diameter to width ratio between 0.2 and 0.6. However this variation is high in the cases with ratios between 0.6 and 1.0. Also it is observed that for low values of the ratio, two curves show similar trend while larger stress concentration factors are obtained for larger values of diameter to width ratio in the case of Carbon / Epoxy.



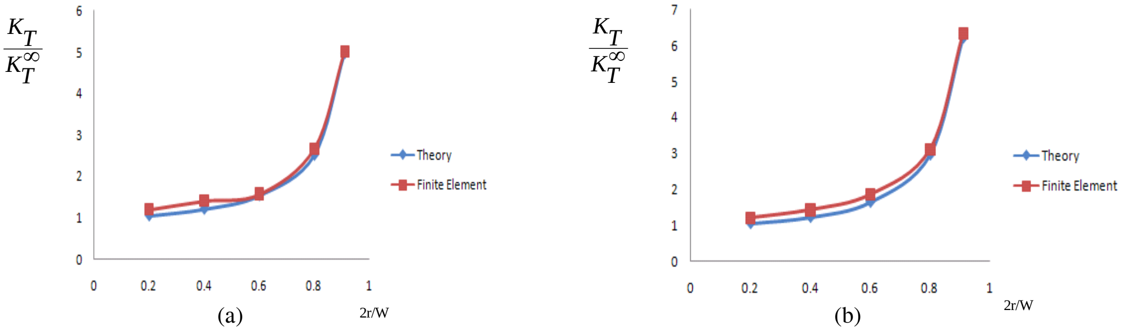
(a) (b)

Fig. 4
Stress concentration factor for lay-up $\left[90^\circ\right]$, analytical and numerical. (a) Carbon / Epoxy , (b) Glass / Epoxy.

Table 4
Stress concentration factor for lay-up $\left[90^\circ\right]$

$\frac{2r}{W}$	M	K_T/K_T^∞					
		Carbon/Epoxy			Glass/Epoxy		
		Analytical	Numerical	Error	Analytical	Numerical	Error
0.2	1.44	1.05	1.20	12.8	1.05	1.21	13.7
0.4	1.35	1.24	1.44	14.1	1.23	1.46	15.6
0.6	1.23	1.78	2.11	15.8	1.72	2.01	14.7
0.8	1.11	3.68	4.10	10.2	3.32	3.55	6.4
0.91	1.05	8.56	8.92	4.0	7.33	7.48	2.0

In addition to study the stress concentration factor for lay-ups $\left[0^\circ\right]$ and $\left[90^\circ\right]$, this quantity is investigated in the cases having different lay-ups. The Cross-Ply $\left[0^\circ_2/90^\circ_2\right]_S$ is studied for several diameters to width ratio Carbon / Epoxy and Glass / Epoxy. Table 5. and Fig.5 summarize the obtained results.



(a) (b)

Fig. 5
Stress concentration factor for cross-ply laminate $\left[0^\circ_2/90^\circ_2\right]_S$, analytical and numerical.(a) Carbon / Epoxy , (b) Glass / Epoxy.

Table 5
Stress concentration factor for cross-ply laminate $\left[0^\circ_2/90^\circ_2\right]_S$

$\frac{2r}{W}$	M	K_T/K_T^∞					
		Carbon/Epoxy			Glass/Epoxy		
		Analytical	Numerical	Error	Analytical	Numerical	Error
0.2	1.44	1.05	1.20	12.7	1.05	1.20	12.8
0.4	1.35	1.21	1.41	14.2	1.22	1.42	14.1
0.6	1.23	1.54	1.57	2.0	1.64	1.85	11.0
0.8	1.11	2.51	2.67	5.9	2.96	3.12	4.8
0.91	1.05	4.97	5.02	1.0	6.23	6.33	1.7

The stress concentration factor is larger in the case with Cross-Ply $\left[0_2/90_2\right]_S$ compared to the case with lay-up $\left[0^\circ\right]$, while this factor is smaller compared to lay-up $\left[90^\circ\right]$. In addition, the curves show larger values for Glass / Epoxy compared to Carbon / Epoxy.

In the next step, quasi-isotropic laminates $\left[0^\circ/\pm 45^\circ/90^\circ\right]_S$ is considered. Table 6 and Fig. 6 summarize the results. The result, show that the stress concentration factor in quasi-isotropic laminates is larger than cross-ply laminates and lay-up $\left[0^{\circ}\right]$. Unlike other laminates, for quasi-isotropic laminates this factor does not differ between laminates made from Glass / Epoxy compare to Carbon / Epoxy.

Table 6
Stress concentration factor for quasi-isotropic laminate $\left[0^{\circ}/\pm45^{\circ}/90^{\circ}\right]_S$

$\frac{2r}{W}$	M	K_T/K_T^{∞}					
		Carbon/Epoxy			Glass/Epoxy		
		Analytical	Numerical	Error	Analytical	Numerical	Error
0.2	1.44	1.05	1.19	12.1	1.05	1.18	11.4
0.4	1.35	1.23	1.43	13.6	1.23	1.41	12.7
0.6	1.23	1.72	1.86	7.7	1.72	1.94	11.4
0.8	1.11	3.35	3.54	5.5	3.35	3.62	7.5
0.91	1.05	7.41	7.65	3.1	7.41	7.71	3.9

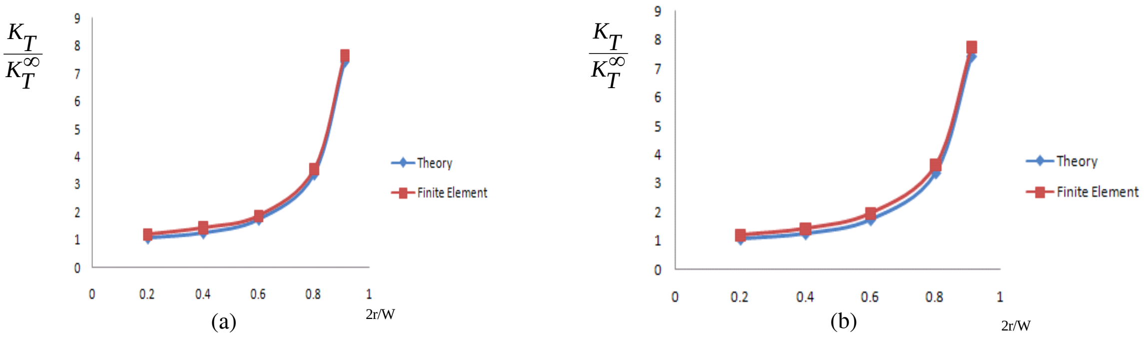


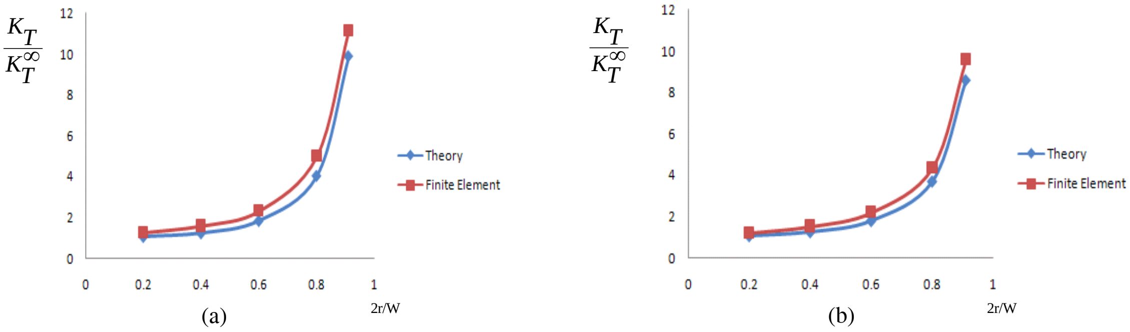
Fig. 6
stress concentration factor for quasi-isotropic laminate $\left[0^{\circ}/\pm45^{\circ}/90^{\circ}\right]_S$, analytical and numerical. (a) Carbon / Epoxy , (b) Glass / Epoxy.

As the last case, angle-ply laminates $\left[\pm45\right]_S$ is considered. Table 7. and Fig.7 summarize the results of stress concentration factor for several diameters to width ratios.

In brief, the stress concentration factor for laminates $\left[\pm45\right]_S$ is larger than lay-ups $\left[90^{\circ}\right]$ and $\left[0^{\circ}\right]$, as well as, cross-ply and quasi-isotropic laminates. In addition, for the case of laminates $\left[\pm45\right]_S$ this factor is smaller for Glass / Epoxy compare to Carbon / Epoxy.

Table 7
Stress concentration factor for angle-ply laminate $\left[\pm45\right]_S$

$\frac{2r}{W}$	M	$\frac{K_T}{K_T^{\infty}}$					
		Carbon/Epoxy			Glass/Epoxy		
		Analytical	Numerical	Error	Analytical	Numerical	Error
0.2	1.44	1.05	1.27	17.8	1.05	1.21	13.7
0.4	1.35	1.24	1.60	22.3	1.24	1.52	18.7
0.6	1.23	1.83	2.33	21.2	1.78	2.20	19.1
0.8	1.11	4.03	5.01	19.5	3.69	4.36	15.2
0.91	1.05	9.90	11.14	11.2	8.60	9.57	10.06



(a) (b)

Fig. 7
Stress concentration factor for angle-ply laminate $[\pm 45]_s$, analytical and numerical. (a) Carbon / Epoxy , (b) Glass / Epoxy.

4 EXPERIMENTAL STUDY OF FRACTURE STRENGTH

A few samples of woven glass fibers and epoxy resin are made by hand lay up to study the fracture strength of composite laminate experimentally. The manufacturing process and the method of experiment are according to the standard ASTM D3039[15]. The elastic constants of composite samples are measured with tensile and shear tests. Table 8. summarizes the result. Since the samples are made from woven fibers, the moduli in two major directions are equal.

Table 8
Elastic constants for composite samples

$E_1(GPa)$	$E_2(GPa)$	$E_6 = G_{12}(GPa)$	ν
14.7	14.7	2.1	0.3

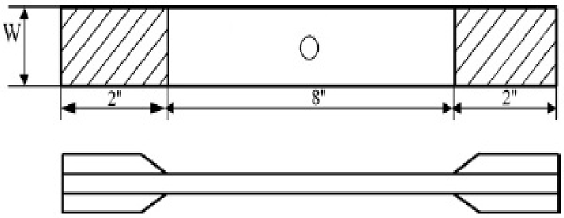


Fig. 8
Geometry and dimensions of specimens and location of hole.

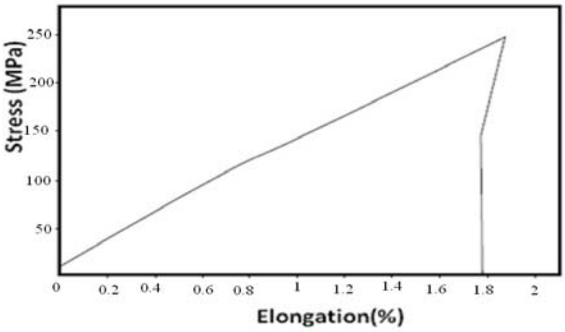


Fig. 9
Stress _ Elongation result test before hole drilling.

In the next step, to study fracture strength experimentally, the samples are drilled to make holes with different diameters. To reach the identified diameter, first a small hole is made with drilling machine. Then the sample is drilled again to obtain larger hole. This step may repeat several times. To avoid the delamination of layers around the hole, the drilling process is made with carbide or diamond tools with the rotating speed of 2200 RPM and 130 mm/sec feed. The diameters of the hole vary from ratio of diameter to width ranging between 0.2 to 0.6. Fig. 8 shows geometry and dimensions of specimen and Fig. 9 is a stress-elongation result test before hole drilling.

Fig.10 shows a few samples which are broken to two parts after tensile test. The evidence of brittle fracture can be seen in this picture. Table 9. summarizes the measured fracture strength before and after drilling. These are the average of five values measured for each test related to special diameter to width ratio.

Also Table 9. contains the ratio of measured strengths before and after drilling. The obtained result demonstrates that this strength ratio decreases with hole diameter from 0.7 to 0.32.

Fig. 10
Brittle fracture of samples after tensile test.

Table 9
The measured fracture strength of samples

$\frac{2r}{W}$	σ_N^{∞} (MPa)	σ_0 (MPa)	$\sigma_N^{\infty}/\sigma_0$
0.2	164.8	234.6	0.70
0.4	127.8	234.6	0.54
0.6	76.2	234.6	0.32

In this Table, σ_0 and σ_N^{∞} are the fracture strengths of composite laminates without and with hole, respectively and r is the radius of the hole. Although there are a few mathematical models to predict the residual strength of drilled composite laminate, the widely-used point stress model [16] with calculation of characteristic length is used in this study. In this method for the orthotropic panel with circular hole loaded with normal stress σ_y, when the fracture strength of a point at the distance of characteristic length d_0 from the hole edge reaches to fracture strength of same panel without hole, the failure is occurred [16].

$$\sigma_y(x,0)=\sigma_0 \qquad x=r+d_0 \tag{6}$$

For an infinite orthotropic panel loaded with uniformly distributed stress σ_y^{∞}, the stress at the point located in the distance x from hole edge in axial direction is [16]:

$$\sigma_y(x,0)=\left(\frac{\sigma_y^{\infty}}{2}\right)\left(2+\left(\frac{r}{x}\right)^2+3\left(\frac{r}{x}\right)^4-(K_T^{\infty}-3)\left[5\left(\frac{r}{x}\right)^6-7\left(\frac{r}{x}\right)^8\right]\right); \quad x\geq r \tag{7}$$

where K_T^{∞} is the stress concentration factor for an infinite panel with circular hole defined by Eq. (3). Substituting Eq.(3) in Eq.(7) and then into Eq.(6) this quantity is as [16]:

$$\frac{\sigma_N^{\infty}}{\sigma_0}=\frac{2}{(2+\varepsilon^2+3\varepsilon^4-(K_T^{\infty}-3)(5\varepsilon^6-7\varepsilon^8)} \qquad \varepsilon=\frac{r}{(r+d_0)} \tag{8}$$

With application of above equations and experimentally measured data in Table 9. the residual strength of composite laminates is calculated. Then the characteristic length is calculated to reinforce the area around the hole and overcome the effect of stress concentration. Fig.11 shows the calculated values as a function of sample geometrical parameter. The characteristic length is maximum for the case with diameter to width ratio of 0.4.

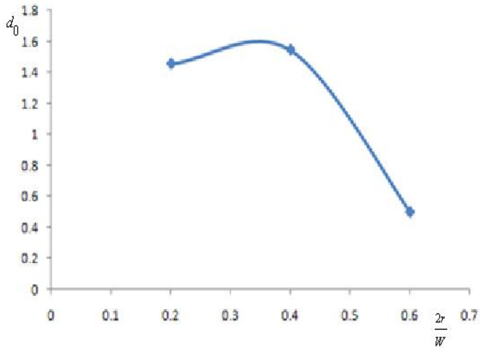


Fig. 11
Calculated characteristic length for different diameter to width ratio.

5 CONCLUSIONS

The result obtained by FE simulation of drilled panels made from isotropic, orthotropic and laminated composite plate demonstrates that stress concentration factor increases with diameter to width ratio. The point stress method is used to study this factor analytically. Since this method is introduced for orthotropic ply, its application is extended to composite laminates by implementing equivalent modulus. The agreement observed in comparison of the results obtained analytically and numerically proves applicability of the FE method in simulation of the behavior of orthotropic materials and composite laminates, and also it verifies the applicability and accuracy of implementing equivalent modulus in analytical methods. This study estimates 30% drop in strength for the samples with least diameter to width ratio, while this drop increases to 70% for larger ratios. This study describes a method to calculate characteristic length by implementing the ratio of strengths before and after drilling. If the composite structure is reinforced in the region located at characteristic length from the hole edge, there is a great opportunity to overcome the effect of stress concentration around the hole.

ACKNOWLEDGEMENTS

The authors are grateful to the University of Kashan for supporting this work by Grant No. 158495/02.

REFERENCES

[1] Rooijen R.V., Sinke J., De-Veris T. J., Zwaag S.V.D., 2004, Property optimization in fiber metal laminates, *Journal of Applied Composite Materials* **11**(2): 63-76.

[2] Hagenbeek M., Hengel C., Bosker O.J., Vermeeren C.A.J.R., 2003, Static properties of fiber metal laminates, *Journal Of Applied Composite Materials* **10**(4): 207-222.

[3] Nuismer R.J., Whitney J.M., 1976, Uniaxial fracture of composite laminates containing stress concentrations, *Fracture Mechanics of Composites ASTM STP* **593**:117-142.

[4] Pipes R.B., Wetherhold R.C., Gillespie J.W., 1979, Notched strength of composite materials, *Journal of Composite Materials* **13**: 148-160.

[5] Tan S.C., 1987, Laminated composites containing an elliptical opening. Approximate stress Analyses and fracture models, *Journal of Composite Materials* **21**: 925-948.

[6] Chang F.K., Chang K.Y., 1987, A progressive damage model for laminated composites containing stress concentrations, *Journal of Composite Materials* **21**: 834-855.

[7] Waddoups M.E., Eisenmann J.R., Kaminski B.E., 1971, Macroscopic fracture mechanics of advanced composite materials, *Journal of Composite Materials* **5**: 446-454.

[8] Mar J.W., Lin K.Y., 1979, Characterization of splitting process in graphite/epoxy composites, *Journal of Composite Materials* **13**: 278-287.

[9] Backlund J., 1981, Fracture analysis of notched composites, *Computer and Structure* **13**: 145-154.

[10] Ericsson I., Aronsson C.G., 1990, Strength of tensile loaded graphite/ epoxy laminates containing cracks, open and filled holes, *Journal of Composite Materials* **24**: 456-482.

[11] Shin C.S., Wang C.M., 2004, An improved cohesive zone model for residual notched strength prediction of composite laminates with different orthotropic lay-up, *Journal of Composite Materials* **38**(9): 713-736.

[12] Khatibi A., Ye L., 1997, Residual strength simulation of fibre reinforced metal laminates containing a circular hole, *Journal of Composite Materials* **31**(9): 142-163.

[13] Spotts M. F., 1998, *Design of Machine Elements*, Prentice Hall.

[14] Ansys Help System, Analysis Guide and Theory Reference, ver10.

[15] ASTM, 2003, *Standard Test Method For Tensile Properties of Polymer Matrix Composite Materials*, Designation D 3039.

[16] Wu G., Tan Y., Yang J.M., 2007, Evaluation of residual strength of notched fiber metal laminates, *Journal of Materials Science and Engineering*, *Part A* **457**: 338-349.