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Closed-form Solution of Dynamic Displacement for SLGS Under Moving the Nanoparticle on Visco-Pasternak Foundation

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ABSTRACT
In this paper, forced vibration analysis of a single-layered graphene sheet (SLGS) under moving a nanoparticle is carried out using the non-local elasticity theory of orthotropic plate. The SLGS under moving the nanoparticle is placed in the elastic and viscoelastic foundation which are simulated as a Pasternak and Visco-Pasternak medium, respectively. Movement of the nanoparticle is considered as a linear movement with constant velocity from an edge to another edge of graphene sheet. Using the non-linear Von Kármán strain-displacement relations and Hamilton's principle, the governing differential equations of motion are derived. The differential equation of motion for all edges simply supported boundary condition is solved by an analytical method and therefore, the dynamic displacement of SLGS is presented as a closed-form solution of that. The influences of medium stiffness (Winkler, Pasternak and damper modulus parameter), nonlocal parameter, aspect ratio, mechanical properties of graphene sheet, time and velocity parameter on dimensionless displacement (dynamic displacement to static displacement of SLGS) are studied. The results indicate that, as the values of stiffness modulus parameter increase, the maximum dynamic displacement of SLGS decreases. Therefore, the results are in good agreement with the previous researches.

Keywords: Graphene sheet; Visco-Pasternak medium;, Moving nanoparticle; Closed-form solution; Non-local elasticity theory

1 INTRODUCTION

IN early 2007, the United Nations reported that nanotechnology, which then accounted for approximately 0.1% of the global manufacturing economy, would grow to 14% of the market by 2014. Nanotechnology is a field of applied science concerned with the control of matter at dimensions of roughly 1 to 100 nm (nm). At the particle size of 1 to 100 nm, nano-scale materials may have different molecular organizations and properties than the same chemical substances in a larger size. Nano-sized chemicals can have different properties due to [1]: 1- Increased relative surface area per unit mass, which can increase physical strength and chemical reactivity. 2- In some cases, the dominance of quantum effects at the nanometer size, which changes basic material properties.

Vibration analysis of isotropic and orthotropic plates using the classical theory of elasticity (generalized Hook's law) is stated for various theory of plates in many books [2-4].

The non-local elasticity theory was proposed and developed by Eringen [5-8] to consider small scale effect in the continuum model of nano-structures. In recent years, studies about the vibration of nano-structures using the non-local theory of elasticity are included many researches due to superior vibration characteristics of them. Pradhan and

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Phadikar [9] studied the non-local vibration of single and double layered nano-plates using the classical and first-order shear deformation (FSDT) theories. The governing differential equations of motion are solved by Navier's approach for simply supported boundary condition. Murmu and Adhikari investigated non-local vibration of bonded double nano-plate systems [10] and instability analysis of double nano-beam systems [11] and the governing equations of motion in terms of displacements are solved by the new analytical method.

The foundation of sheets, nano-beams and nano-tubes can be assumed as linear (Winkler and Pasternak) elastic and viscoelastic (Visco-Winkler and Visco-Pasternak) medium or nonlinear elastic medium. Pradhan and Kumar [12] have carried out vibration analysis of the orthotropic SLGS embedded in a Pasternak elastic medium. The normal forces are considered at the Winkler elastic medium although the shear forces are added also in the Pasternak elastic medium. Ghorbanpour and et all [13] studied non-local vibration of a coupled system of DLGSs by Visco-Pasternak medium. The differential equation of motion are solved by the differential quadrature method (DQM) [14, 15]. Forced vibration of graphene sheets can be assumed under the moving nanoparticle. Kiani carried out Forced vibration of carbon nano-tube [16] and plates [17-19] subjected to a moving nanoparticle and also, Simsek [20] investigated forced vibration of coupled system of carbon nano-tubes under the moving nano-particle. Ghorbanpour and et all [21] studied forced vibration of BNNTs subjected to the moving nanoparticle.

In this study, the closed-form solution of dynamic displacement for SLGS subjected to the moving nanoparticle is presented by the analytical method for Pasternak and Visco-Pasternak foundation.

2 MATHEMATICAL MODEL

A schematic diagram of SLGS subjected to a moving nanoparticle on Pasternak and visco-Pasternak foundation is illustrated in Fig.1.

The Hamilton's principle is used for deriving the governing differential equations of motion that is given as:

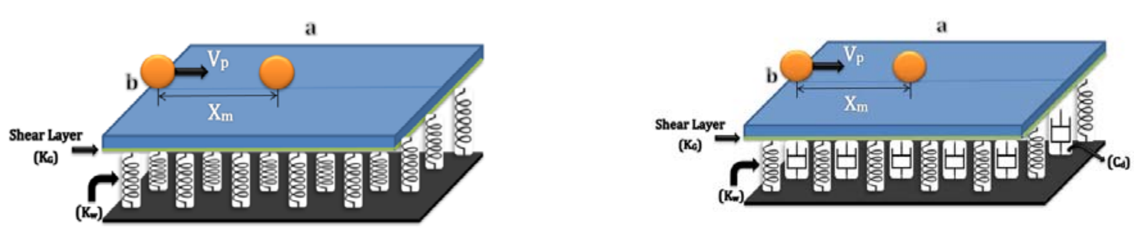
$$\int_0^t \left(\delta T - \delta V + \delta W\right) dt = 0 \tag{1}$$

where δT, δV and δW are the virtual kinetic energy, the virtual strain energy and the virtual work done by external applied forces, respectively.

The governing differential equation of motion is derived using the Hamilton's principle for a SLGS which is presented as [13]:

$$\begin{aligned} &-D_{11}\frac{\partial^4 w_0}{\partial x^4} - 2(D_{12}+D_{66})\frac{\partial^4 w_0}{\partial^2 x \partial^2 y} - D_{22}\frac{\partial^4 w_0}{\partial y^4} + q = I_0\frac{\partial^2 w_0}{\partial t^2} - I_2\frac{\partial^2}{\partial t^2}\nabla^2\left(w_0\right) \\ &-\mu\nabla^2\left[I_0\frac{\partial^2 w_0}{\partial t^2} - I_2\frac{\partial^2}{\partial t^2}\nabla^2\left(w_0\right) - q\right] \end{aligned} \tag{2}$$

where q and w_0 are transverse loading and displacement, respectively. Also, D_{ij} is defined as:



(a) (b)

Fig. 1
A SLGS subjected to a moving nanoparticle on (a) Pasternak foundation (b) Visco-Pasternak foundation.

$$D_{ij} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \tilde{Q}_{ij} z^2 dz \tag{3}$$

$$\begin{aligned}
\tilde{Q}_{11} &= Q_{11}C^4 + 2\left(Q_{12} + Q_{66}\right)S^2C^2 + Q_{22}S^4 \\
\tilde{Q}_{12} &= \left(Q_{11} + Q_{22} - 4Q_{66}\right)S^2C^2 + Q_{12}\left(S^4 + C^4\right) \\
\tilde{Q}_{22} &= Q_{11}S^4 + 2\left(Q_{12} + Q_{66}\right)S^2C^2 + Q_{22}C^4 \\
\tilde{Q}_{16} &= \left(Q_{11} - Q_{12} - 2Q_{66}\right)SC^3 + \left(Q_{12} - Q_{22} + 2Q_{66}\right)CS^3 \\
\tilde{Q}_{26} &= \left(Q_{11} - Q_{12} - 2Q_{66}\right)CS^3 + \left(Q_{12} - Q_{22} + 2Q_{66}\right)SC^3 \\
\tilde{Q}_{66} &= \left(Q_{11} + Q_{22} - 2Q_{66} - 2Q_{12}\right)S^2C^2 + Q_{66}\left(S^4 + C^4\right)
\end{aligned} \tag{4}$$

and also,

$$Q_{11} = \frac{E_1}{1-\nu_{12}\nu_{21}} \qquad Q_{12} = \frac{E_1\nu_{12}}{1-\nu_{12}\nu_{21}} \qquad Q_{21} = \frac{E_2\nu_{21}}{1-\nu_{12}\nu_{21}} \qquad Q_{22} = \frac{E_2}{1-\nu_{12}\nu_{21}} \quad Q_{66} = G_{12} = \frac{E_1}{2(1-\nu_{12})} \tag{5}$$

$$\mathrm{S} = \sin\theta \qquad\qquad C = \cos\theta \tag{6}$$

where E_1 and E_2 are the Young's modules in directions 1 and 2, ν_{12} and ν_{21} denote the Poisson's ratios and G_{12} is the shear modulus. The structure of the orthotropic graphene sheet is known as armchair (ϑ=0) and zigzag (ϑ=90). Here, the structure of orthotropic SLGS is assumed as armchair structure.

$$I_0, I_1, I_2 = \int_{-\frac{h}{2}}^{\frac{h}{2}} \rho_0(1, h, h^2)dz \tag{7}$$

where I_0, I_1, I_2 are mass moments of inertia and ρ_0 denotes the density of the material. Here, the transverse loading on SLGS is applied by two items:

1- By the foundation (Pasternak and Visco-Pasternak medium) [13]
2- By the moving nanoparticle the transverse loading by the moving nanoparticle on SLGS is obtained by [20]:

$$q_0 = P\delta(x - x_m) \tag{8}$$

where δ is the Delta Dirac function and q_0 is the transverse loading of moving nanoparticle.

$$x_m = V_p t \tag{9}$$

where t is the time of arrival nanoparticle at location x_m with constant velocity (V_p).

2.1 The SLGS under moving nanoparticle on Pasternak foundation

By substituting transverse loading of moving nanoparticle Eq.(8) and foundation [13] into Eq.(2), the governing differential equation of motion for SLGS under the moving nanoparticle on Pasternak foundation is derived as:

$$-D_{11}\frac{\partial^4 W_0}{\partial x^4}-2(D_{12}+D_{66})\frac{\partial^4 W_0}{\partial x^2\partial y^2}-D_{22}\frac{\partial^4 W_0}{\partial y^4}+\left(K_G+\mu K_w+I_2\frac{\partial^2}{\partial t^2}\right)\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)W_0-K_wW_0$$
$$-\mu K_G\left(\frac{\partial^4}{\partial x^4}+2\frac{\partial^4}{\partial x^2\partial y^2}+\frac{\partial^4}{\partial y^4}\right)W_0-I_0\frac{\partial^2 W_0}{\partial t^2}+\mu\frac{\partial^2}{\partial t^2}I_0\left(\frac{\partial^2 W_0}{\partial x^2}+\frac{\partial^2 W_0}{\partial y^2}\right)-\mu I_2\frac{\partial^2}{\partial t^2}\nabla^2\left(\frac{\partial^2 W_0}{\partial x^2}+\frac{\partial^2 W_0}{\partial y^2}\right) \tag{10}$$
$$=-P\delta\left(x-x_m\right)+\mu P\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\delta\left(x-x_m\right)$$

Here, the dynamic displacement of SLGS for all edges simply supported (SSSS) boundary condition using the separation variables is assumed as [9]:

$$W_0\left(x,y,t\right)=\sum_{i=1}^{n}\sum_{j=1}^{m}\sin\left(\frac{i\pi x}{a}\right)\sin\left(\frac{j\pi y}{b}\right)q_n^{(i,j)}(t) \tag{11}$$

Substituting Eq.(11) into Eq.(10), the governing differential equation of motion in terms of i and j can be obtained as:

$$A_1^{(i,j)}(t)\sin\left(\frac{i\pi y}{a}\right)\sin\left(\frac{j\pi y}{b}\right)=f(x,t) \tag{12}$$

where

$$A_1^{(i,j)}(t)=K^{(i,j)}q_n^{(i,j)}\left(t\right)+m^{(i,j)}\ddot{q}_n^{(i,j)}(t) \tag{13}$$

Hence, the initial stiffness matrix of system is obtained as:

$$K^{(i,j)}=-D_{11}\left(\frac{i\pi}{a}\right)^4-2(D_{12}+D_{66})\left(\frac{i\pi}{a}\right)^2\left(\frac{j\pi}{b}\right)^2-D_{22}\left(\frac{j\pi}{b}\right)^4-K_w-\left(K_G+\mu K_w\right)B^{(i,j)}-\mu K_GE^{(i,j)} \tag{14}$$

So, the initial mass matrix of system is derived as:

$$m^{(i,j)}=\mu\left[-I_0B^{(i,j)}+I_2E^{(i,j)}\right]-I_0-I_2B^{(i,j)} \tag{15}$$

and also, the initial forced loading is obtained by:

$$f(x,t)=\mu P\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\delta\left(x-x_m\right)-P\delta\left(x-x_m\right) \tag{16}$$

where K_w and K_G are the Winkler and Pasternak modulus parameters, respectively. Also, $B^{(i,j)}$ and $E^{(i,j)}$ are derived by:

$$B^{(i,j)}=\left(\frac{i\pi}{a}\right)^2+\left(\frac{j\pi}{b}\right)^2 \tag{17}$$

$$E^{(i,j)}=\left(\frac{i\pi}{a}\right)^4+\left(\frac{j\pi}{b}\right)^4+2\left(\frac{i\pi}{a}\right)^2\left(\frac{j\pi}{b}\right)^2 \tag{18}$$

Therefore, the final forced loading, stiffness and mass matrix by multiplying two sides of Eq.(13) by *sin(iπx/a)sin(jπy/b)* and double integrating through the length and width of sheet are obtained as:

$$\int_0^a\int_0^b A_1^{(i,j)}(t)\left(\sin\left(\frac{i\pi x}{a}\right)\sin\left(\frac{j\pi y}{b}\right)\right)^2 dxdy = \int_0^a\int_0^b f(x,t)dxdy \tag{19}$$

Double integrations into Eq.(19) are solved Using the below equations:

$$\int_{x_1}^{x_2} g(x)\delta^{(n)}(x-x_m)dx = \begin{cases}(-1)^n g^{(n)}(x) & if\ x_1 < x_m < x_2\\ 0 & Otherwise\end{cases} \tag{20}$$

$$\int_0^a\int_0^b\left(\sin\left(\frac{i\pi x}{a}\right)\sin\left(\frac{j\pi y}{b}\right)\right)^2 dxdy = \begin{cases}0.25ab & \begin{cases}i=m\\ j=n\end{cases}\\ 0 & \begin{cases}i\neq m\\ j\neq n\end{cases}\end{cases} \tag{21}$$

Therefore, the forced loading and the differential equation of motion in terms of *i* and *j* are derived:

$$\int_0^a\int_0^b f(x,t)dxdy = \int_0^a\int_0^b\left(-P\delta(x-x_m)+\mu P\nabla^2\delta(x-x_m)\right)dxdy = \frac{-2Pb\left(1+\mu\left(\frac{i\pi}{a}\right)^2\right)}{j\pi}\sin\left(\frac{i\pi x_m}{a}\right) \tag{22}$$

$$K_n^{(i,j)}q_n^{(i,j)}(t)+m_n^{(i,j)}\ddot{q}_n^{(i,j)} = f_n^{(i,j)}(t) \tag{23}$$

where

$$f_n^{(i,j)}(t) = -\frac{2Pb}{j\pi}\left(1+\mu\left(\frac{i\pi}{a}\right)^2\right)\sin\left(\frac{i\pi V_p t}{a}\right) \tag{24}$$

$$K_n^{(i,j)} = 0.25K^{(i,j)}ab \tag{25}$$

$$m_n^{(i,j)} = 0.25m^{(i,j)}ab \tag{26}$$

Therefore,

$$\ddot{q}_n^{(i,j)} + \left(\omega_n^{(i,j)}\right)^2 q_n^{(i,j)}(t) = \tilde{f}_n^{(i,j)}(t) \tag{27}$$

where

$$\tilde{f}_n^{(i,j)}(t) = \frac{f_n^{(i,j)}(t)}{m_n^{(i,j)}} \tag{28}$$

$$\omega_n^{(i,j)} = \sqrt{\frac{K_n^{(i,j)}}{m_n^{(i,j)}}} \tag{29}$$

$q_n^{(i,j)}$ will obtain by taking Laplace of two sides of Eq. (27),

$$L\left[\ddot{q}_n^{(i,j)} + \left(\omega_n^{(i,j)}\right)^2 q_n^{(i,j)}(t)\right] = L\left(\tilde{f}_n^{(i,j)}(t)\right) \tag{30}$$

$$\Rightarrow\left[s^2 Q_n^{(i,j)}(s) + \left(\omega_n^{(i,j)}\right)^2 Q_n^{(i,j)}(s)\right] = L\left(\tilde{f}_n^{(i,j)}(t)\right) \tag{31}$$

$$L\left[\ddot{q}_n^{(i,j)} + \left(\omega_n^{(i,j)}\right)^2 q_n^{(i,j)}(t)\right] = L\left(\tilde{f}_n^{(i,j)}(t)\right) \tag{32}$$

Therefore,

$$Q_n^{(i,j)}(s) = \frac{L\left(\tilde{f}_n^{(i,j)}(t)\right)}{\left(s^2 + \left(\omega_n^{(i,j)}\right)^2\right)} \tag{33}$$

$$\Rightarrow q_n^{(i,j)}(t) = L^{-1}(Q_n^{(i,j)}(s)) = L^{-1}\left\{\frac{L\left(\tilde{f}_n^{(i,j)}(t)\right)}{\left(s^2 + \left(\omega_n^{(i,j)}\right)^2\right)}\right\} = L^{-1}\left(\frac{1}{s^2 + \left(\omega_n^{(i,j)}\right)^2}\right) * \tilde{f}_n^{(i,j)}(t) \tag{34}$$

Therefore, $q_n^{(i,j)}$ is derived by:

$$\Rightarrow q_n^{(i,j)}(t) = \frac{1}{\omega_n^{(i,j)}} \int_0^t \tilde{f}_n^{(i,j)}(\tau).sin\omega_n^{(i,j)}(t-\tau)\,d\tau \tag{35}$$

$$q_n^{(i,j)}(t) = \int_0^t B_1^{(i,j)} \sin\left(\omega_f^{(i)}\tau\right) \sin\omega_n^{(i,j)}(t-\tau)\,d\tau \tag{36}$$

where

$$\omega_f^{(i)} = \frac{i\pi V_p}{a} \tag{37}$$

$$B_1^{(i,j)} = \frac{-2Pb\left(1+\mu\left(\frac{i\pi}{a}\right)^2\right)}{j\pi m_n^{(i,j)}\omega_n^{(i,j)}} \tag{38}$$

where $\omega_f^{(i)}$ can be defined as the forced frequency of moving nanoparticle. The velocity and time parameter is defined as:

$$T = \frac{x_m}{a} \tag{39}$$

$$r = \frac{\omega_f^{(i)}}{\omega_n^{(i,j)}} \tag{40}$$

Therefore, t in term of time parameter is derived as:

$$\overset{V_p = \frac{r\omega_n^{(i,j)}a}{i\pi}}{\Rightarrow} \quad t = \frac{aT}{\frac{r\omega_n^{(i,j)}a}{i\pi}} \tag{41}$$

Here, two cases for velocity parameter (r) are considered:

1. $r=1$
2. $r\neq 1$

Therefore, for case of $r\neq 1$:

$$q_n^{(i,j)}\left(t\right) = B_1^{(i,j)} A_2^{(i,j)}(t) \tag{42}$$

where

$$A_2^{(i,j)}\left(t\right)=\frac{\left(\omega_f^{(i)}t\right)\sin\left(\omega_n^{(i,j)}t\right)-\omega_n^{(i,j)}sin\left(\omega_f^{(i)}t\right)}{\left(\omega_f^{(i)}t\right)^2-\left(\omega_n^{(i,j)}\right)^2} \tag{43}$$

Therefore, by substituting $q_n^{(i,j)}$ from Eq.(42) into Eq.(11), the closed-form solution of dynamic displacement of SLGS for case of r$\neq$1 is obtained by:

$$W_0\left(x,y,t\right)=\sum_{i=1}^{m}\sum_{j=1}^{n}B_1^{(i,j)}A_2^{(i,j)}\left(t\right)\sin\left(\frac{i\pi x}{a}\right)\sin\left(\frac{j\pi y}{b}\right) \tag{44}$$

also, for case of r=1:

$$q_n^{(i,j)}\left(t\right)=\int_0^t B_1^{(i,j)}\sin\left(\omega_n^{(i,j)}\tau\right)sin\omega_n^{(i,j)}\left(t-\tau\right)d\tau \tag{45}$$

Therefore,

$$q_n^{(i,j)}\left(t\right)=B_2^{(i,j)}A_3^{(i,j)}(t) \tag{46}$$

where

$$A_3^{(i,j)}(t)=\omega_n^{(i,j)}tcos\left(\omega_n^{(i,j)}t\right)-\sin\left(\omega_n^{(i,j)}t\right) \tag{47}$$

$$B_2^{(i,j)} = \frac{Pb\left(1+\mu\left(\frac{i\pi}{a}\right)^2\right)}{j\pi m_n^{(i,j)}\left(\omega_n^{(i,j)}\right)^2} \tag{48}$$

Therefore, the closed-form solution of dynamic displacement of the SLGS for case of r=1 is derived by:

$$W_0\left(x,y,t\right)=\sum_{i=1}^{m}\sum_{j=1}^{n}B_2^{(i,j)}A_3^{(i,j)}(t)sin\left(\frac{i\pi x}{a}\right)sin\left(\frac{j\pi y}{b}\right) \tag{49}$$

2.2 The SLGS under moving nanoarticle on visco- Pasternak foundation

The differential equation of motion for SLGS subjected to the moving nanoparticle on Visco-Pasternak foundation by the Hamilton's principle is derived as:

$$\begin{aligned}&-D_{11}\frac{\partial^4 W_0}{\partial x^4}-2(D_{12}+D_{66})\frac{\partial^4 W_0}{\partial x^2\partial y^2}-D_{22}\frac{\partial^4 W_0}{\partial y^4}+\left(K_G+\mu K_w+\mu I_0\frac{\partial^2}{\partial t^2}+I_2\frac{\partial^2}{\partial t^2}\right)\left(\frac{\partial^2 W_0}{\partial x^2}+\frac{\partial^2 W_0}{\partial y^2}\right)\\&+\mu C_d(\frac{\partial^2\dot{W}_0}{\partial x^2}+\frac{\partial^2\dot{W}_0}{\partial y^2})-K_wW_0-C_d\dot{W}-I_0\frac{\partial^2 W_0}{\partial t^2}-\mu\left(K_G+I_2\frac{\partial^2}{\partial t^2}\right)\left(\frac{\partial^4 W_0}{\partial x^4}+2\frac{\partial^4 W_0}{\partial x^2\partial y^2}+\frac{\partial^4 W_0}{\partial y^4}\right)\\&=-P\delta\left(x-x_m\right)+\mu P\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\delta\left(x-x_m\right)\end{aligned} \tag{50}$$

The dynamic displacement of SLGS for all edges simply supported (SSSS) boundary condition is assumed as [9]:

$$W_0\left(x,y,t\right)=\sum_{i=1}^{n}\sum_{j=1}^{m}\sin\left(\frac{i\pi x}{a}\right)\sin\left(\frac{j\pi y}{b}\right)q_d^{(i,j)}(t) \tag{51}$$

Substituting Eq. (51) into Eq.(50), the governing differential equation of motion can be obtained as:

$$A_4^{(i,j)}(t)\sin\left(\frac{i\pi y}{a}\right)\sin\left(\frac{j\pi y}{b}\right)= f(x,t) \tag{52}$$

where

$$A_4^{(i,j)}(t)=K^{(i,j)}q_d^{(i,j)}(t)+C^{(i,j)}\dot{q}_d^{(i,j)}(t)+m^{(i,j)}\ddot{q}_d^{(i,j)}(t) \tag{53}$$

Hence, the initial stiffness matrix of system is obtained by:

$$K^{(i,j)}=-D_{11}\left(\frac{i\pi}{a}\right)^4-2(D_{12}+D_{66})\left(\frac{i\pi}{a}\right)^2\left(\frac{j\pi}{b}\right)^2-D_{22}\left(\frac{j\pi}{b}\right)^4-K_w-K_GB^{(i,j)}-\mu K_wB^{(i,j)}-\mu K_GE^{(i,j)} \tag{54}$$

and the initial mass matrix of system is derived as:

$$m^{(i,j)}=\mu\left[-I_0B^{(i,j)}+I_2E^{(i,j)}\right]-I_0-I_2B^{(i,j)} \tag{55}$$

The initial damper matrix and forced loading are obtained by Eq.(56) and Eq.(57), respectively.

$$C^{(i,j)}=-C_d-\mu C_dB^{(i,j)} \tag{56}$$

$$f(x,t)=-P\delta\left(x-x_m\right)+\mu P\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\delta\left(x-x_m\right) \tag{57}$$

where C_d are the damper modulus parameter. The final forced loading, stiffness, damper and mass matrix by multiplying two sides of Eq.(52) by *sin(iπx/a)sin(jπy/b)* and double integrating through the length and width of sheet are obtained as:

$$\int_0^a\int_0^b A_4^{(i,j)}(t)\left(\sin\left(\frac{i\pi x}{a}\right)\sin\left(\frac{j\pi y}{b}\right)\right)^2dxdy=\int_0^a\int_0^b f(x,t)dxdy \tag{58}$$

Furthermore, the final forced loading, stiffness, damper and mass matrix of system can be converted to:

$$f_d^{(i,j)}\left(t\right)=-\frac{2Pb}{j\pi}\left(1+\mu\left(\frac{i\pi}{a}\right)^2\right)\sin\left(\frac{i\pi V_pt}{a}\right) \tag{59}$$

$$K_d^{(i,j)}=0.25K^{(i,j)}ab \tag{60}$$

$$C_d^{(i,j)}=0.25C^{(i,j)}ab \tag{61}$$

$$m_d^{(i,j)}=0.25m^{(i,j)}ab \tag{62}$$

$q_d^{(i,j)}(t)$ (time parameter of dynamic displacement) is derived as:

$$q_d^{(i,j)}(t) = X_d^{(i,j)}(t)\sin\left(\frac{i\pi V_p}{a}t + \varphi_d^{(i,j)}\right) \tag{63}$$

where

$$X_d^{(i,j)} = \frac{-\frac{2Pb}{j\pi}\left(1+\mu\left(\frac{i\pi}{a}\right)^2\right)}{\sqrt{\left(K_d^{(i,j)} - m_d^{(i,j)}\left(\frac{i\pi V_p}{a}\right)^2\right)^2 + \left(C_d^{(i,j)}\left(\frac{i\pi V_p}{a}\right)\right)^2}} \tag{64}$$

and also,

$$\varphi_d^{(i,j)} = \tan^{-1}\left(\frac{C_d^{(i,j)}\left(\frac{i\pi V_p}{a}\right)}{K_d^{(i,j)} - m_d^{(i,j)}\left(\frac{i\pi V_p}{a}\right)^2}\right) \tag{65}$$

where, the natural frequency of SLGS vibration is obtained by:

$$\omega_n^{(i,j)} = \sqrt{\frac{K_d^{(i,j)}}{m_d^{(i,j)}}} \tag{66}$$

and also, the damping frequency is derived as:

$$\omega_d^{(i,j)} = -\xi_d^{(i,j)}\omega_n^{(i,j)} + i\left(\sqrt{1-(\xi_d^{(i,j)})^2}\right)\omega_n^{(i,j)} \tag{67}$$

where $\xi_d^{(i,j)}$ is the damping parameter and is derived as:

$$\xi_d^{(i,j)} = \frac{C_d^{(i,j)}}{2m_d^{(i,j)}\omega_n^{(i,j)}} \tag{68}$$

If the velocity parameter becomes equal to 1, the natural frequency of vibration will be equal to forced frequency of system. Furthermore, the closed-form solution of dynamic displacement of SLGS under moving the nanoparticle on the visco-Pasternak medium for $r \neq 1$ by substituting Eq.(63) into Eq.(51) is derived as:

$$W_0\left(x,y,t\right) = \sum_{i=1}^{n}\sum_{j=1}^{m}\frac{L^{(i,j)}}{\sqrt{F^{(i,j)}}}R^{(i,j)}(t) \tag{69}$$

where

$$F^{(i,j)} = \left(K_d^{(i,j)} - m_d^{(i,j)}\left(\omega_f^{(i)}\right)^2\right)^2 + \left(C_d^{(i,j)}\omega_f^{(i)}\right)^2 \tag{70}$$

$$H^{(i,j)} = \frac{C_d^{(i,j)}\omega_f^{(i)}}{K_d^{(i,j)} - m_d^{(i,j)}\left(\omega_f^{(i)}\right)^2} \tag{71}$$

$$L^{(i,j)} = -\frac{2Pb}{j\pi}\left(1+\mu\left(\frac{i\pi}{a}\right)^2\right) \tag{72}$$

$$R^{(i,j)}(t)=\sin\left(\omega_f^{(i)}t+\tan^{-1}H^{(i,j)}\right)sin\left(\frac{i\pi x}{a}\right)sin\left(\frac{j\pi y}{b}\right) \tag{73}$$

The closed-form solution of dynamic displacement for r=1 is obtained by:

$$W_0\left(x,y,t\right)=\sum_{i=1}^{n}\sum_{j=1}^{m}\frac{L^{(i,j)}}{\sqrt{N^{(i,j)}}}P^{(i,j)}(t) \tag{74}$$

where

$$N^{(i,j)}=\left(K_n-m_n\left(\omega_n^{(i,j)}\right)^2\right)^2+\left(C_n\omega_n^{(i,j)}\right)^2 \tag{75}$$

$$S^{(i,j)}=\left(\frac{C_n\omega_n^{(i,j)}}{K_n-m_n\left(\omega_n^{(i,j)}\right)^2}\right) \tag{76}$$

$$P^{(i,j)}(t)=\sin\left(\omega_n^{(i,j)}t+\tan^{-1}S^{(i,j)}\right)\sin\left(\frac{i\pi x}{a}\right)\sin\left(\frac{j\pi y}{b}\right) \tag{77}$$

3 RESULTS AND DISCUSSIONS

The values of non-local parameter, Winkler, Pasternak and damper modulus parameter and the values of mechanical properties of orthotropic and isotropic graphene sheet are taken according to [13]. The maximum static displacement of plate subjected to a concentrated force on the middle of plate is derived according to [3]:

$$W_{st}^{\max}=\frac{4P}{\pi^4abD}\sum_{i=1}^{n}\sum_{j=1}^{m}\frac{1}{\left(\left(\frac{i}{a}\right)^2+\left(\frac{j}{b}\right)^2\right)^2} \tag{78}$$

where

$$D=\frac{Eh^3}{12(1-\vartheta^2)} \tag{79}$$

The natural frequency of system is obtained by Eq.(66). The frequency ratio of SLGS is derived by:

$$FR=\frac{\left(\omega_n^{(i,j)}\right)^{non-local}}{\left(\omega_n^{(i,j)}\right)^{local}} \tag{80}$$

where, the non-local and local natural frequency are derived by substituting $\mu\neq0$ and μ=0 into Eq.(66), respectively. As regards validation of our work, the SLGS frequency ratio can be calculated from Eq.(80), and considering K_w=K_G=C_d=0. However, the obtained results for the selected values of non-local parameter are listed in Table 1.
As can be seen, frequency ratio decreases with increasing non-local parameter.

Table 1
Comparison of the results in this paper and other papers

Nonlocal Parameter (μ) nm^2	Reference [12, 22] ω^{nl}/ω^{l}	Reference [9] ω^{nl}/ω^{l}	Reference [13] ω^{nl}/ω^{l}	Present paper ω^{nl}/ω^{l}
0	1.0000	1.0000	1.000000	1.0000
1	0.9139	0.9139	0.913865	0.9139
2	0.8467	0.8468	0.846733	0.8468
3	0.7925	0.7926	0.792509	0.7926

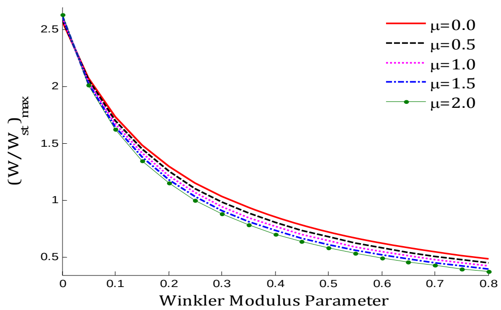


Fig. 2
Effect of non-local parameter on dynamic displacement.

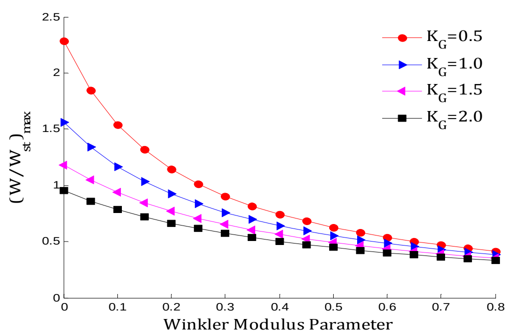
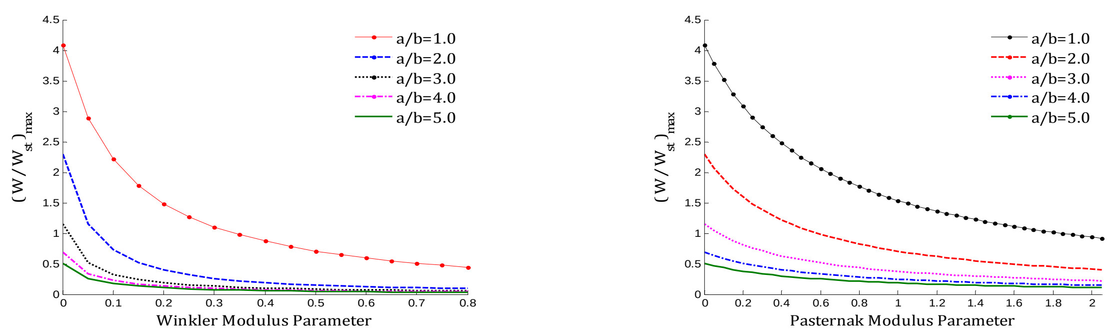


Fig. 3
Effect of Winkler and Pasternak modulus parameter on dynamic displacement, simultaneously.



(a) (b)

Fig. 4
The maximum dynamic displacement of graphene sheet versus stiffness modulus parameter for various values aspect ratios (a) Winkler modulus parameter (b) Pasternak modulus parameter.

The maximum dynamic displacement to static displacement of SLGS with respect to Winkler modulus parameter for various values of non-local parameter is plotted in Fig.2. It can be found that, as the values of non-local parameter increase, the values of maximum dynamic displacement of SLGS decrease. The maximum dynamic displacement to static displacement of SLGS versus Winkler modulus parameter for various values of Pasternak modulus parameter is shown in Fig.3. It can be observed that, as the values of Winkler modulus parameter increase, the values of maximum dynamic displacements for all values of Pasternak modulus parameter decrease and this reduction is especially significant for the lower values of Pasternak coefficient. This is due to the fact that increasing Winkler and Pasternak coefficients increase the sheet stiffness. Therefore, the changes value of maximum dynamic displacements of SLGS for highest values of Winkler and Pasternak modulus parameter are going to zero. The maximum dynamic displacement to static displacement of SLGS with respect to the stiffness parameter for various values of aspect ratio (a/b) is presented in Fig.4. As the Winkler modulus parameter increases, the dynamic displacements decrease for all values of aspect ratios. It is observed that, as values of aspect ratio increase, the maximum dynamic displacement decreases. Therefore, as the values of aspect ratios increase, the effect of Winkler and Pasternak modulus parameter on the maximum dynamic displacement decreases. This is due to the effect of foundation stiffness on the larger SLGS reduces. By comparing cases *a* and *b* in Fig.4, it can be found that the effect of Winkler modulus parameter on maximum dynamic displacement of SLGS is higher than the Pasternak modulus parameter.

The dynamic displacement to static displacement of SLGS with respect to the time parameter (T) is illustrated for various values of Winkler modulus parameter and velocity parameter (r) in Fig.5. In this figure, changes of dynamic displacement in term of time parameter for various values of velocity parameter are presented, and it is observed that, changes of dynamic displacement for various values of velocity parameter are different with each other. It can be seen, as the values of velocity parameter increase, the dynamic displacement in during the time parameter can't be reduced to zero.

The maximum dynamic displacement to static displacement of SLGS versus the damper modulus parameter for various values of non-local parameter is presented in Fig.6. The results indicate that, as the damper modulus parameter increases, the maximum dynamic displacements decrease for all values of non-local parameter.

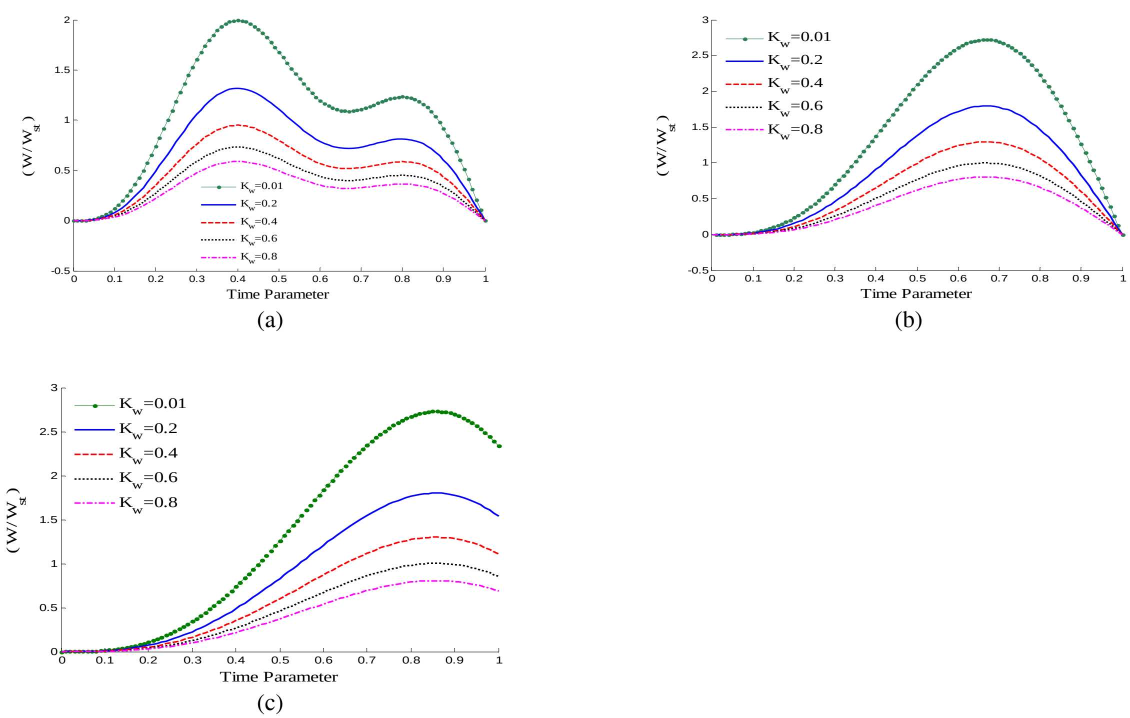


Fig. 5
The maximum dynamic displacement of SLGS versus Time parameter for various values of Winkler modulus parameter (a) r=0.25 (b) r=0.5 (c) r=0.75.

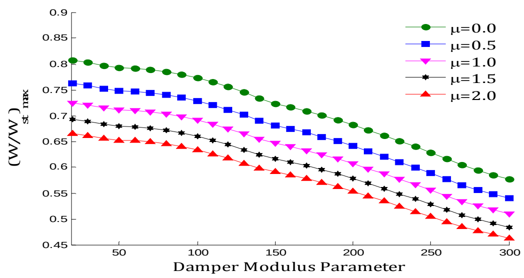


Fig. 6
The maximum dynamic displacement of graphene sheet versus damper modulus parameter.

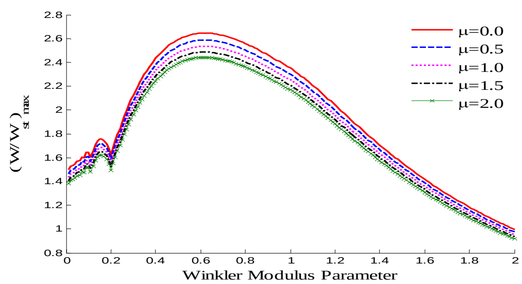


Fig. 7
Dynamic displacement of graphene sheet versus velocity parameter for various nonlocal parameters.

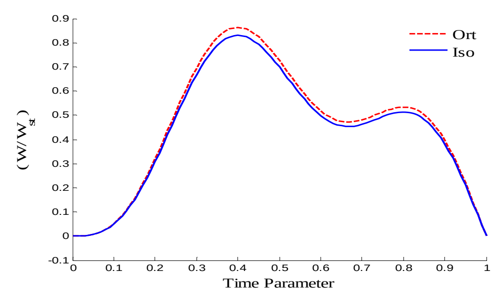


Fig. 8
Dynamic displacement of graphene sheet versus velocity parameter for various nonlocal parameters (r=0.25).

The maximum dynamic displacement to static displacement (dimensionless displacement) of SLGS with respect to velocity parameter for various values of non-local parameter is presented in Fig.7. It can be found that, as the values of velocity parameter increase, the maximum dynamic displacements changes for all values of non-local parameter. The dynamic displacement to static displacement of SLGS with respect to the time parameter (T) is plotted in Fig.8 for orthotropic and isotropic structure of SLGS. It can be seen that, the dynamic displacement of orthotropic SLGS is higher than isotropic that in during time parameter. Perhaps, It is due to that the isotropic structure of SLGS is stiffer than the orthotropic structure of that.

4 CONCLUSION

In this paper, forced vibration of SLGS under the moving nanoparticle studied using the non-local elasticity theory of orthotropic plate. The medium of SLGS assumed viscoelastic medium that is simulated as a Visco-Pasternak foundation. The results indicate that, the maximum dynamic displacement of SLGS decreases as the values of nan-local parameter increase. In addition, the maximum dynamic displacement decreases as the values of the Winkler, Pasternak and damper modulus parameter increase and also, the results indicate that the effect of Winkler modulus parameter on dynamic displacement of SLGS is higher than the Pasternak modulus parameter and damper modulus parameter, respectively. Also, as the values of aspect ratio increase, the maximum dynamic displacement decreases and the effect of foundation stiffness by increasing aspect ratio of SLGS on that reduces. In addition, the results showed that, the dynamic displacement of isotropic SLGS is higher than the orthotropic of one.

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