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Rayleigh Wave in an Initially Stressed Transversely Isotropic Dissipative Half-Space

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ABSTRACT

The governing equations of a transversely isotropic dissipative medium are solved analytically to obtain the surface wave solutions. The appropriate solutions satisfy the required boundary conditions at the stress-free surface to obtain the frequency equation of Rayleigh wave. The numerical values of the non-dimensional speed of Rayleigh wave speed are computed for different values of frequency and initial stress parameter. The effects of transverse isotropy and initial stress parameter are observed on the Rayleigh wave speed.

Keywords: Transversely isotropic; Dissipative medium; Initial stress; Rayleigh wave; Frequency equation

1 INTRODUCTION

WE can understand the interior of the Earth better when we consider various additional parameters, e.g. porosity, initial stress, viscosity, dissipation, temperature, voids, diffusion, etc. Initial stresses in a medium are caused by various reasons such as creep, gravity, external forces, difference in temperatures, etc. The reflection of plane waves at free surface, interface and layers is important in estimating the correct arrival times of plane waves from the source. Various researchers studied the reflection and transmission problems at free surface, interfaces and in layered media. For example, Sinha [1] considered the problem of transmission of an elastic wave between two homogeneous half-space. Gupta [2] studied the reflection of plane waves from a linear transition layer in liquid media. Tooly, et al. [3] alalyzed the reflected and transmitted waves generated by a plane, monochromatic, compressional wave incident at a plane interface between two half-spaces. Gupta [4] obtained plane wave reflection coefficients in a situation where an inhomogeneous material forms a transition layer between two homogeneous, elastic half-spaces. Gupta [5] also studied the propagation of SH-waves in inhomogeneous media. Acharya [6] studied the reflection of compressional waves for an elastic, isotropic, inhomogeneous medium in which compressional and shear-wave equations are separable. Cerveny [7] derived the formulae for the reflection and transmission coefficients of plane elastic waves for a transition layer. Singh, et al. [8] studied the reflection and refraction of SH-waves at the plane boundary between two laterally and vertically heterogeneous solids. Singh [9] studied the reflection from insulated and isothermal stress-free surface of a thermoelastic solid half-space under hydrostatic initial stress. Sharma [10] considered the propagation of plane waves in a general anisotropic elastic medium in the presence of initial stress and studied the reflection at the free plane surface for partition of energy among the three reflected waves.

The study of propagation of plane waves and reflection phenomena in the presence of initial stresses as well as dissipation is interesting. Dey and Dutta [11] studied the propagation and attenuation of seismic body waves in

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initially stressed dissipative medium. Selim and Ahmed [12] studied the propagation and attenuation of seismic body waves in dissipative medium under initial and couple stresses.With the help of Biot [13] theory of incremental deformation, Selim [14] studied the reflection of plane waves at a free surface of an initially stressed dissipative medium. Recently, Singh and Arora [15] studied the reflection of plane waves from a free surface of an initially stressed transversely isotropic dissipative medium.

Biot [16] studied the Rayleigh wave under the influence of initial stresses. Guz [17] studied the surface waves in bodies with initial stress. Babich, et al. [18] presented a review on elastic waves in bodies with initial stress. Rayleigh wave the in presence of both initial stress and dissipation parameters is not studied yet in literature. In the present paper, we studied the problem on Rayleigh wave at a stress-free surface of an initially stressed transversely isotropic solid half-space with dissipation. The frequency equation of the Rayleigh wave is obtained and analyzed numerically to observe the effects of initial stresses, dissipation and frequency on the non-dimensional speed of the Rayleigh wave.

2 FORMULATION OF THE PROBLEM AND SOLUTION

Following Biot [13], the basic dynamical equations of motion in x-z plane for an infinite, initially stressed medium, in the absence of external body forces are,

$$\begin{aligned} \frac{\partial s_{11}}{\partial x}+\frac{\partial s_{13}}{\partial z}-P\frac{\partial \omega}{\partial z}&=\rho\frac{\partial^2 u}{\partial t^2}, \\ \frac{\partial s_{31}}{\partial x}+\frac{\partial s_{33}}{\partial z}-P\frac{\partial \omega}{\partial x}&=\rho\frac{\partial^2 w}{\partial t^2}, \end{aligned} \tag{1}$$

where ρ is the density, $\omega=\frac{1}{2}\left(\frac{\partial w}{\partial x}-\frac{\partial u}{\partial z}\right)$ is rotational component, s_{ij} (i, j = 1, 3) are incremental stress components, u and w are the displacement components. Following Biot [13], the stress-strain relations are

$$\begin{aligned} s_{11}&=\left(C_{11}+P\right)\frac{\partial u}{\partial x}+\left(C_{13}+P\right)\frac{\partial w}{\partial z}, \\ s_{13}&=s_{31}=C_{44}\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right), \\ s_{33}&=C_{33}\frac{\partial w}{\partial z}+C_{13}\frac{\partial u}{\partial x}, \end{aligned} \tag{2}$$

where C_{ij} are the incremental elastic coefficients. For dissipative medium, elastic coefficients are replaced by the following complex constants

$$\begin{aligned} &C_{11}\to C_{11}^R+iC_{11}^I, C_{13}\to C_{13}^R+iC_{13}^I, \\ &C_{33}\to C_{33}^R+iC_{33}^I, C_{44}\to C_{44}^R+iC_{44}^I, \end{aligned} \tag{3}$$

where, $i=\sqrt{-1}$, C_{11}^R, C_{11}^I, C_{13}^R, C_{13}^I, C_{33}^R, C_{33}^I, C_{44}^R, C_{44}^I are real. Following Fung [19], the stress and strain components in dissipative medium are,

$$s_{ij}=\overline{s_{ij}}\, e^{i\bar{\omega}t}, u_i=\overline{u_i}\, e^{i\bar{\omega}t}, \tag{4}$$

where (i, j = 1, 3) and $\bar{\omega}$ being the angular frequency. Using Eqs. (3) and (4), the Eq. (2) becomes,

$$\begin{aligned}
\bar{s}_{11} &= \left(C_{11}^{R} + iC_{11}^{I} + P\right)\frac{\partial u}{\partial x} + \left(C_{13}^{R} + iC_{13}^{I} + P\right)\frac{\partial w}{\partial z},\\
\bar{s}_{31} = \bar{s}_{13} &= \left(C_{44}^{R} + iC_{44}^{I}\right)\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right),\\
\bar{s}_{33} &= \left(C_{33}^{R} + iC_{33}^{I}\right)\frac{\partial w}{\partial z} + \left(C_{13}^{R} + iC_{13}^{I}\right)\frac{\partial u}{\partial x}.
\end{aligned} \tag{5}$$

With the help of Eq. (5), the Eq. (1) becomes

$$\begin{aligned}
&\left(C_{11}^{R} + P\right)\frac{\partial^2 \bar{u}}{\partial x^2} + \left(C_{13}^{R} + C_{44}^{R} + \frac{P}{2}\right)\frac{\partial^2 \bar{w}}{\partial x \partial z} + \left(C_{44}^{R} + \frac{P}{2}\right)\frac{\partial^2 \bar{u}}{\partial z^2}\\
&\quad + \rho \bar{\omega}^2 \bar{u} + i\left[C_{11}^{I}\frac{\partial^2 \bar{u}}{\partial x^2} + \left(C_{13}^{I} + C_{44}^{I}\right)\frac{\partial^2 \bar{w}}{\partial x \partial z} + C_{44}^{I}\frac{\partial^2 \bar{u}}{\partial z^2}\right] = 0,
\end{aligned} \tag{6}$$

$$\begin{aligned}
&C_{33}^{R}\frac{\partial^2 \bar{w}}{\partial z^2} + \left(C_{13}^{R} + C_{44}^{R} + \frac{P}{2}\right)\frac{\partial^2 \bar{u}}{\partial x \partial z} + \left(C_{44}^{R} - \frac{P}{2}\right)\frac{\partial^2 \bar{w}}{\partial x^2}\\
&\quad + \rho \bar{\omega}^2 \bar{w} + i\left[\left(C_{13}^{I} + C_{44}^{I}\right)\frac{\partial^2 \bar{u}}{\partial x \partial z} + C_{44}^{I}\frac{\partial^2 \bar{w}}{\partial x^2} + C_{33}^{I}\frac{\partial^2 \bar{w}}{\partial z^2}\right] = 0.
\end{aligned} \tag{7}$$

The surface wave solutions of Eqs. (6) and (7) are sought in the following form

$$u(x,z,t) = \phi(kz)e^{ik(x-ct)}, \qquad w(x,z,t) = \varphi(kz)e^{ik(x-ct)} \tag{8}$$

Making use of Eq. (8), the Eqs. (6) and (7) become

$$\left[a_1 + \rho c^2 + a_2 D^2\right]\phi(kz) + a_3 D\psi(kz) = 0, \tag{9}$$

$$a_3 D\phi(kz) + \left[a_4 + \rho c^2 + a_5 D^2\right]\psi(kz) = 0, \tag{10}$$

where $D = \frac{d}{dz}$ and

$$a_1 = -\left(C_{11}^{R} + iC_{11}^{I} + P\right), \quad a_2 = \left(C_{44}^{R} + iC_{44}^{I} + \frac{P}{2}\right), \quad a_3 = i\left[\left(C_{44}^{R} + C_{44}^{R} + \frac{P}{2}\right) + i\left(C_{13}^{I} + C_{44}^{I}\right)\right],$$
$$a_4 = -\left(C_{44}^{R} - \frac{P}{2} + iC_{44}^{I}\right), \quad a_5 = C_{33}^{R} + iC_{33}^{I}.$$

Eqs. (9) and (10) have non-trivial solution if

$$D^4 + LD^2 + M = 0 \tag{11}$$

where

$$L=\frac{a_2a_4+a_1a_5-a_3^2+\rho c^2(a_2+a_5)}{a_2a_5}, \qquad M=\frac{a_1a_4+\rho c^2(a_1+a_4)+\rho^2c^4}{a_2a_5}.$$

Let m_1^2, m_2^2 be the roots of corresponding auxiliary Eq.(11), then the general solutions of Eq. (11) are

$$u=\left[A_1e^{-ikm_1z}+A_2e^{-ikm_2z}+A_3e^{ikm_1z}+A_4e^{ikm_2z}\right]e^{ik(x-ct)} \tag{12}$$

$$w=\left[\varepsilon_1A_1e^{-ikm_1z}+\varepsilon_2A_2e^{-ikm_2z}+\varepsilon_1A_3e^{ikm_1z}+\varepsilon_2A_4e^{ikm_2z}\right]e^{ik(x-ct)} \tag{13}$$

where

$$\varepsilon_i=\frac{-\left[a_1+\rho c^2-a_2k^2m_i^2\right]}{ikm_ia_3}, \qquad (i=1, 2)$$

$$m_1^2+m_2^2=\frac{-L}{k^2}, \qquad m_1^2m_2^2=\frac{M}{k^4}$$

3 DERIVATION OF THE FREQUENCY EQUATION

We consider an initially stressed transversely isotropic dissipative solid half-space with x-axis as free surface and the negative z-axis is pointing into the half-space. The particular solutions in half space ($z \le 0$)

$$u=\left[B_1e^{ikm_1z}+B_2e^{ikm_2z}\right]e^{ik(x-ct)} \tag{14}$$

$$w=\left[\varepsilon_1B_1e^{ikm_1z}+\varepsilon_2B_2e^{ikm_2z}\right]e^{ik(x-ct)} \tag{15}$$

The boundary condition at the free surface z = 0 are

$$\Delta fx=0, \qquad \Delta fz=0 \tag{16}$$

where, $\Delta fx=s_{13}+e_{13}P$, $\Delta fz=s_{33}$.

The boundary conditions (16) are also written as:

$$\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right)=0, \tag{17}$$

$$\left(C_{33}^R+iC_{33}^I\right)\frac{\partial w}{\partial z}+\left(C_{13}^R+iC_{13}^I\right)\frac{\partial u}{\partial x}=0. \tag{18}$$

With help of Eqs. (14) and (15) into Eqs. (17) and (18), we get

$$B_1\left(m_1+\varepsilon_1\right)+B_2\left(m_2+\varepsilon_2\right)=0 \tag{19}$$

$$B_1\left[\left(C_{33}^R+iC_{33}^I\right)\varepsilon_1m_1+\left(C_{13}^R+iC_{13}^I\right)\right]+B_2\left[\left(C_{33}^R+iC_{33}^I\right)\varepsilon_2m_2+\left(C_{13}^R+iC_{13}^I\right)\right]=0 \tag{20}$$

Eqs. (19) and (20) have non trivial solution if

$$\left(m_1+\varepsilon_1\right)\left[\left(C_{33}^R+iC_{33}^I\right)\varepsilon_2 m_2+\left(C_{13}^R+iC_{13}^I\right)\right]-\left(m_2+\varepsilon_2\right)\left[\left(C_{33}^R+iC_{33}^I\right)\varepsilon_1 m_1+\left(C_{13}^R+iC_{13}^I\right)\right]=0, \tag{21}$$

which is the frequency equation of the Rayleigh wave in an initially stressed transversely isotropic dissipative medium. In absence of initial stresses and dissipation, we have

$$P=0 \quad , \quad C_{11}^I=0 \quad , \quad C_{13}^I=0 \quad , \quad C_{33}^I=0. \tag{22}$$

Then, the above analysis reduces for the case of Rayleigh wave in a transversely isotropic medium.

4 NUMERICAL RESULTS

For numerical purpose, a particular example of the Zinc material is chosen with the following physical constants,

$$\begin{aligned}
&\mathrm{C}_{11}^{\mathrm{R}}=1.628\times10^{10}\,\mathrm{N.m^{-2}}, \mathrm{C}_{33}^{\mathrm{R}}=1.562\times10^{10}\,\mathrm{N.m^{-2}}, \mathrm{C}_{13}^{\mathrm{R}}=0.508\times10^{10}\,\mathrm{N.m^{-2}},\\
&\mathrm{C}_{44}^{\mathrm{R}}=0.385\times10^{10}\,\mathrm{N.m^{-2}}, \mathrm{C}_{11}^{\mathrm{I}}=1.025\times10^{10}\,\mathrm{N.m^{-2}}, \mathrm{C}_{33}^{\mathrm{I}}=0.950\times10^{10}\,\mathrm{N.m^{-2}},\\
&\mathrm{C}_{13}^{\mathrm{I}}=0.425\times10^{10}\,\mathrm{N.m^{-2}}, \mathrm{C}_{44}^{\mathrm{I}}=0.325\times10^{10}\,\mathrm{N.m^{-2}}, \rho=7.14\times10^{3}\,\mathrm{kg.m^{-3}}.
\end{aligned}$$

The non-dimensional speeds ($\rho c^2/ c^R_{44}$), ($\rho c^2/ c^R_{11}$), ($\rho c^2/ c^R_{33}$) and ($\rho c^2/ c^R_{13}$) of the Rayleigh wave are computed for a certain range of the frequency and initial stress parameter in absence and presence of dissipation. The numerical values of non-dimensional speeds ($\rho c^2/ c^R_{44}$), ($\rho c^2/ c^R_{11}$), ($\rho c^2/ c^R_{33}$) and ($\rho c^2/ c^R_{13}$) are shown graphically versus frequency in Figs. 1, 3, 5 and 7. From Figs. 1, 3, 5 and 7, it is observed that the values of non-dimensional speeds increase very sharply in low frequency range. The comparison of solid and dotted curves in Figs. 1, 3, 5 and 7 shows the significant effect of dissipation on the non-dimensional speed of Rayleigh wave.The non-dimensional speeds of Rayleigh wave decrease in presence of dissipation for the chosen range of the frequency.

The numerical values of non-dimensional speeds ($\rho c^2/ c^R_{44}$), ($\rho c^2/ c^R_{11}$), ($\rho c^2/ c^R_{33}$) and ($\rho c^2/ c^R_{13}$) are shown graphically versus initial stress parameter ($P^* = P/c^R_{44}$) in Figs. 2, 4, 6 and 8. For the range $2 \le P^* \le 10$, the non-dimensional speeds of Rayleigh wave first increases slowly to its maximum and then decreases slowly. The comparison of solid and dotted curves in Figs. 2, 4, 6 and 8 shows the significant effect of dissipation on the non-dimensional speeds. The non-dimensional speeds of Rayleigh wave decrease in presence of dissipation for the chosen range of initial stress parameter.

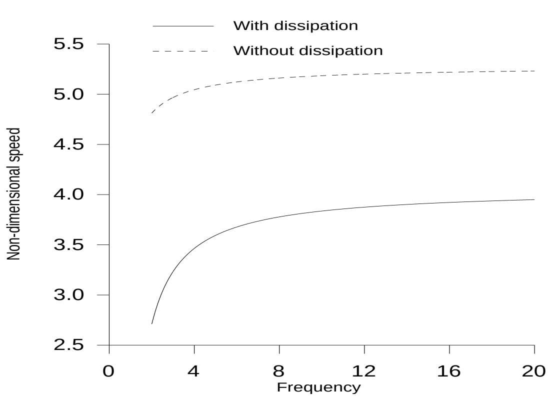

Fig. 1
Variation of the non-dimensional speed ($\rho c^2/ c^R_{44}$) versus frequency ω.

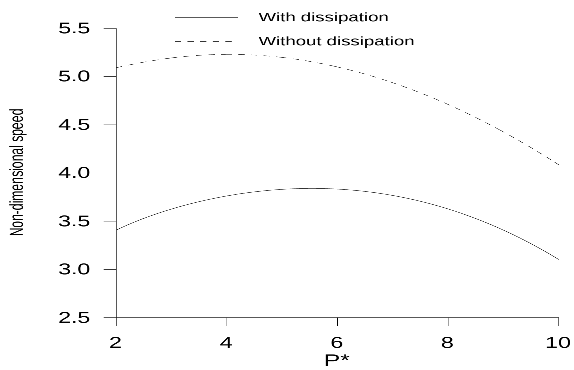

Fig. 2
Variation of the non-dimensional speed ($\rho c^2/ c^R_{44}$) versus initial stress ($P^* = P/c^R_{44}$).

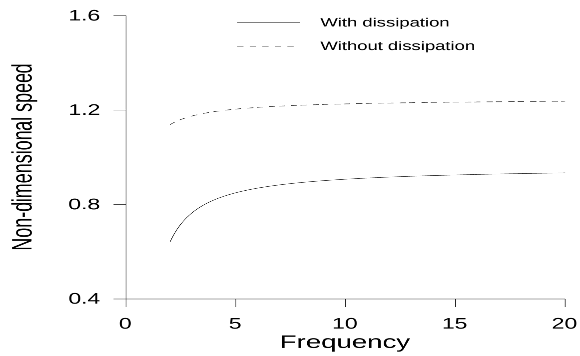

Fig. 3
Variation of the non-dimensional speed ($\rho c^2/ c^R_{11}$) versus frequency ω.

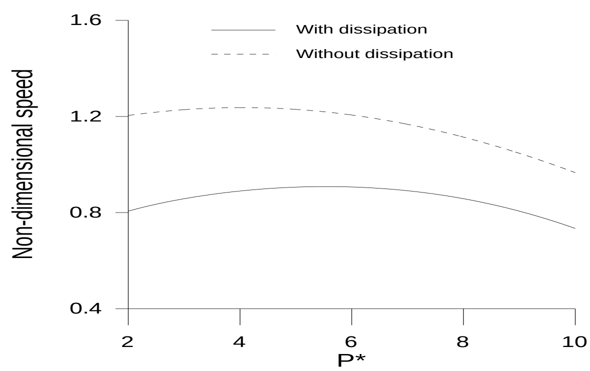

Fig. 4
Variation of the non-dimensional speed ($\rho c^2/ c^R_{11}$) versus initial stress ($P^* = P/c^R_{44}$).

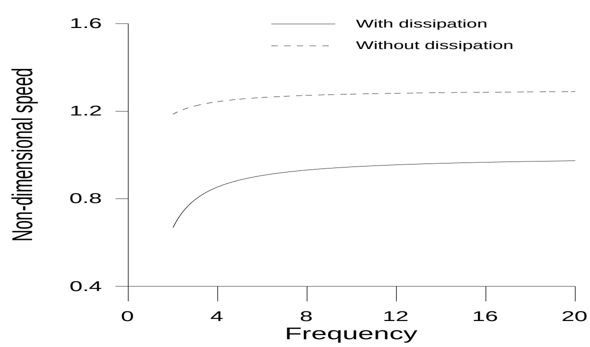

Fig. 5
Variation of the non-dimensional speed ($\rho c^2/ c^R_{33}$) versus frequency ω.

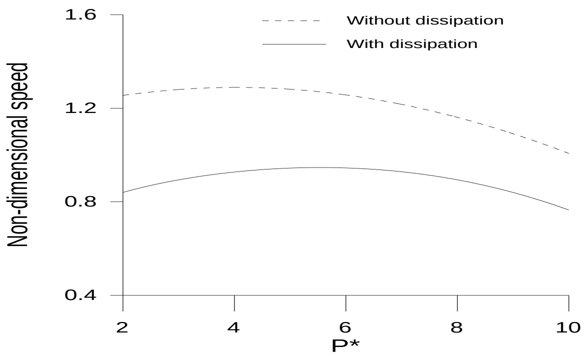

Fig. 6
Variation of the non-dimensional speed ($\rho c^2/ c^R_{33})$ versus initial stress ($P^* = P/c^R_{44}$).

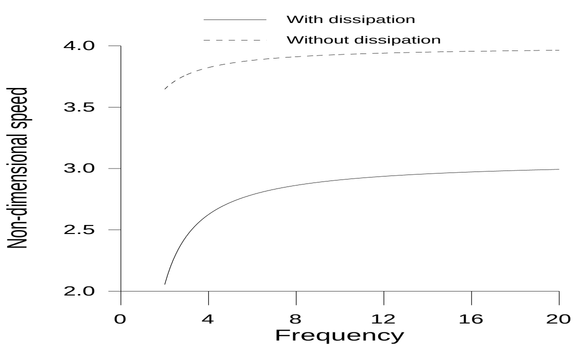

Fig. 7
Variation of the non-dimensional speed ($\rho c^2/ c^R_{13})$ versus frequency ω.

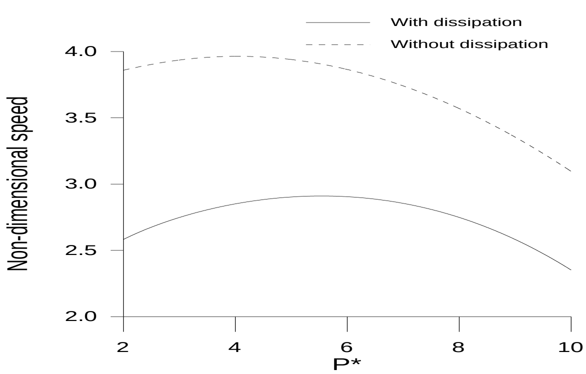

Fig. 8
Variation of the non-dimensional speed ($\rho c^2/ c^R_{13})$ versus initial stress ($P^* = P/c^R_{44}$).

5 CONCLUSIONS

The surface wave solutions of the governing equations of an initially stressed transversely isotropic dissipative medium are obtained in the half-space. With the help of these solutions and the required boundary conditions at the free surface of the half-space, the frequency equation of the Rayleigh wave is derived. A numerical example is chosen for numerical computations of the non-dimensional speed of Rayleigh wave. The effects of dissipation, initial stresses and frequency on the non-dimensional speed of Rayleigh wave are obtained significantly.

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