

Modifying Stress-Strain Curves Using Optimization and Finite Elements Simulation Methods

A. Rezaei Pour Almasi^{1,*}, F. Fariba², S. Rasoli³

¹Department of Mechanical Engineering, Khoy Branch, Islamic Azad University, Khoy, Iran

²Department of Mechanical Engineering, Hamedan Branch, Islamic Azad University, Hamedan, Iran

³Young Researchers and Elite Club, Hamedan Branch, Islamic Azad university, Hamedan, Iran

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ABSTRACT

Modifying stress-strain curves is one of the important topics in mechanical engineering and materials science. Real stress-strain curves should be modified after necking point as stress becomes three-dimensional after creation of throat, and consequently, equivalent stress should be used instead of axial one. Also, distribution of triple stresses across throat section is not uniform anymore, and it is not possible to calculate the stress through dividing force value by surface area. Methods presented to modify these curves mainly have some defects which enter the error resulting from simplifying assumptions into the results. Entrance of stress analysis softwares into mechanical engineering has caused use of finite elements methods in order to modify stress-strain curves. As you know, being as an input for stress analysis software, as one of the applications of these curves, has a direct effect on simulation results. Optimization methods have been developed and extended in engineering sciences. Modifying stress-strain curves may be an application of these methods. Considering the sample shape resulting from tension test as the basis in this research, we have changed the modified stress-strain curve in a way that the shape resulting from simulation coincides with the sample resulting from the test. Accordingly, the stress-strain curve has been modified, and the results have been verified, using results obtained from normal methods such as Bridgeman method.

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1 INTRODUCTION

STRESS-STRAIN curves are widely used in mechanical engineering and metal-deforming industries. In fact, these curves are an indication of properties of materials. The curves are extracted using tension test on standard samples. But, attention should be paid to some important points when extracting these curves. Usually, test machines present the displacement-force relation for the sample. This curve is converted into engineering stress-strain curves by applying the existing equations. Then, real stress-strain curves are extracted, using some relations named conversion equations. After a certain strain, the sample undergoes being straitened. Creation of throat is due to existence of holes in the sample, but in fact increase in stress caused by reduction of surface area is larger and stronger than hardening effect resulting from hardening strain, thus causing creation of throat. After creation of throat, stress which was uniform and one-dimensional before creation of throat will become three-dimensional and

* Corresponding author. Tel.: +98 9148570710.

E-mail address: almasiamir67@yahoo.com (A. Rezaei Pour Almasi).

have a non-uniform distribution. The stress-strain curves resulting from conversion equations are not true for the time after creation of throat. In order to obtain true stress-strain curves, the sample should be photographed using specific cameras, the current diameter of the sample should be measured, and finally, stress-strain equations should be used. These curves are true stress-strain curves which are also true for the time after creation of throat. But these curves should be modified for the time after throat as there are three dimensions for stress after creation of throat. In this circumstance, effective stress should be drawn versus effective strain. The obtained curves are named Modified Curves.

Modified curves are of great importance in metal-deforming industries with large strains as well as in finite elements simulations. Researchers have presented some methods for modifying stress-strain curves. Bridgeman [1] is the first analyst who has carried out a comprehensive research in this regard. In 1944, he introduced a classic equation based on simplifying assumptions, through which modified curve can be extracted by calculating modification factor. Ha has also presented distribution equations of three-dimensional stresses and strains in throat section. Bridgeman equations are presented in terms of relation of curvature radius of the sample in the throat place to radius of the sample in the same place point $\left(\frac{R}{a}\right)$. So, these values should be measured during the test. This

operation is a bit hard and may cause some error to enter the calculations. After Bridgman, other researchers named Davidenkove [2] and Sibel [3] separately presented some equations as modification factor. Their equations, along with Bridgman's are known as Classic Equations. Among them, Bridgman equation has been more commonly used than the others', and on the other hand, has presented stress-strain distribution in form of some separate equations. The main problem while using the classic equations is to determine and measure curvature radius of the sample during the test. In order to obviate the problem, in 1981, a researcher named Leroy [4] proved that relation of curvature radius to radius of sample for a broad range of metals can be represented by an equation in terms of strain and starting strain of throat with a 25 percent error. With this equation presented, the relation can be found without any need for related measures, and the classic equations, then, can be used. This method is able to present an estimate of the modified curve. After entrance of finite element method into mechanical engineering, some analysts tried to model the phenomenon of throat using finite elements coding, and consequently, present the modified curve as well as distribution of stress and strain in the throat section. Among them, Nidelman [5] modeled the phenomenon of throat by solving boundary value problems and using plasticity theories through coding based on finite elements. He also obtained the distribution of stress and strain in the throat section during some certain strains. Using finite elements method in large deformations and coding, in 1988, Michael Brunig [6] analyzed a circular sample under tension in several conditions for the whole sample as well as along the sample. Making the force diagram as a basis from test and simulation, he modified the stress-strain curve, and indicated that a sample can be modeled applying its specific boundary conditions along the sample. In 2004, Nordesan & Redance [7] modeled the phenomenon of throat for a thin rectangular-section plate, using coding, finite elements method, and strain gradient plasticity theory which was firstly introduced by a researcher in 2001. In their research, they the lag between maximum load point and creation of throat, as a basis for their solution. Use of stress analysis softwares in investigating phenomenon of throat was firstly started in 2004 by Pinokoc [8]. Using ABAQUS software and introducing a method named Inverse Method, and with the help of coding, he modified the stress-strain curve. In 2003, Tang [9] carried out a throat analysis in a rod under simple tension using a hardening strain model coupled with failure model, and showed the effect of considering the failure model in making results be real. Since phenomenon of throat is not separate from failure mechanics, combining model of material with failure models can make results more real.

In order to modify the stress-strain curve, Yun Ling [10], in 2004, introduced a specific function as the stress-strain relationship after creation of throat by using ABAQUS simulation. The function has a percentage of Halamoon function and a percentage of line equation named Weight Percentage, and the goal of modifying the curve is to determine this value. Considering force curve as a basis, deviation from the test, and simulation, he has determined the weight percentage.

Through thoroughly studying necking process in a steel sample, Mirone [11] has introduced a group of functions which are of a constant value for a lot of steels, and has accordingly presented some analytical equations for modifying stress-strain curves. During several research projects, Majzubi [14] has made valuable efforts to determine constants of model for materials. In order to determine constants of model for popular materials, he has already used some optimization methods such as Genetic and Neural Algorithms, making shape of throat profile as a basis for his solution.

2 GOVERNING THEORY AND EQUATION

Results obtained from tension test are usually extracted in the form of force-displacement diagrams. They are converted into engineering stress-strain diagrams by using engineering strain and stress equations. Real stress-strain diagram can be obtained using conversion equations from engineering stress and strain equations. These equations are as follows [11]:

$$\sigma = S(1+e) \quad \varepsilon = Ln(1+e) \quad (1)$$

where ε and e are engineering and real strains, and S , σ are engineering and real stresses, respectively. It should be noted that these relationships are true for the time before sample has reached the throat point, and after reaching this point, they are not to be used in the problem. In this condition, diameter of the sample is carefully measured during the test by using some equipment such as high-speed cameras, and thus, stress-strain relationship is determined using the following equation [11]:

$$\varepsilon = Ln \frac{L_f}{L_0} = Ln \frac{A_0}{A} = Ln \frac{\pi d_0^2}{\pi d^2} = 2Ln \frac{d_0}{d} \quad (2)$$

where A_0 and A are area of throat section before and at the considered time in the throat point, respectively. Strain equation has been extracted based on the assumption that the volume of the sample remains constant during the plastic deformation. Now, the curve obtained from the above equation is the real stress-strain diagram based on measurement. This is a true diagram that should be modified after the throat point as stress will have components in three directions due to the creation of throat in the sample. A fundamental question is that which component of stress and strain should be considered as the basis in the stress-strain curve? In the case stress and strain have several components, effective stress-strain curve should be drawn. The curve determines yield area. Such a diagram is named modified curve. Effective stress-strain equations are as following [10-11]:

$$\sigma_e = \frac{1}{\sqrt{2}} \left[(\sigma_x - \sigma_y)^2 + (\sigma_x - \sigma_z)^2 + (\sigma_z - \sigma_y)^2 + 6(\tau_{xy}^2 + \tau_{xz}^2 + \tau_{zy}^2) \right]^{1/2} \quad (3)$$

$$d\varepsilon_e = \frac{\sqrt{2}}{3} \left[(d\varepsilon_x - d\varepsilon_y)^2 + (d\varepsilon_x - d\varepsilon_z)^2 + (d\varepsilon_z - d\varepsilon_y)^2 + 6d\gamma_{xy}^2 + 6d\gamma_{zy}^2 + 6d\gamma_{xz}^2 \right]^{1/2} \quad (4)$$

Assuming that shear stresses are insignificant in the tested sample during the test, the equations are transformed into the following:

$$\sigma_e = \frac{1}{\sqrt{2}} \left[(\sigma_z - \sigma_r)^2 + (\sigma_z - \sigma_\theta)^2 + (\sigma_\theta - \sigma_r)^2 \right]^{1/2} \quad (5)$$

$$d\varepsilon_e = \frac{\sqrt{2}}{3} \left[(d\varepsilon_z - d\varepsilon_r)^2 + (d\varepsilon_z - d\varepsilon_\theta)^2 + (d\varepsilon_\theta - d\varepsilon_r)^2 \right]^{1/2} \quad (6)$$

where z , r and θ are triple coordinates in longitude, radial, and peripheral directions. Assuming that the volume of the sample remains constant during the tension test on the circular sample will result in:

$$\varepsilon_z + \varepsilon_r + \varepsilon_\theta = 0 \quad (7)$$

In the classic method, strain components in radial and peripheral directions are considered to be equal to each other. So:

$$\varepsilon_z = -2\varepsilon_r = -2\varepsilon_\theta \quad (8)$$

In this test, longitude and effective strains are also considered as equal to each other. The problem can be proved analytically as follows:

$$d\varepsilon_e = \frac{\sqrt{2}}{3} \left[(d\varepsilon_z - d\varepsilon_r)^2 + (d\varepsilon_z - d\varepsilon_\theta)^2 + (d\varepsilon_\theta - r)^2 \right]^{1/2} = \frac{\sqrt{2}}{3} \left[2.(d\varepsilon_z - (-0.5d\varepsilon_z))^2 \right]^{1/2} \quad (9)$$

And:

$$d\varepsilon_e = d\varepsilon_z \quad (10)$$

In fact, increase in imposed stress under unstable conditions is more than increase in hardness of the sample due to the hardening strain existing in it, and accordingly at this time, the sample becomes unstable in its weaker part and its diameter is decreased. In such circumstances, not only stress becomes three-dimensional, but also distribution of stress and strain in the throat section gets uniform, as seen in the simulation process.

3 THE TEST AND OBTAINED RESULTS

In this research, experiments have been conducted on a widely used steel namely ST 304. The sample is a circular one according to E8 standard [13], and is stretched by Instron tension test apparatus with the tension speed of 5 millimeter per minute at room temperature.

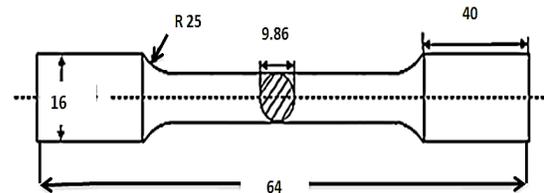


Fig. 1

Instron test apparatus, along with imaging equipment and dimensions of the sample.



Fig. 2

Some images recorded during the test.

For the measurements needed, required images has been taken from the sample during the test through manual image processing by installing HANDY CAM –SONY MODEL: DCR-HC32E model of Sony video camera, with the speed of 30 frame per second and required equipment such as light resource. These images represent diameter and throat profile of the sample. Here, it has been tried to make the measurements by the least equipment. Resolution and quality of the images recorded have been improved using graphical softwares, and surface of the sample as well as the points made on it has been located using softwares such as GATE DATA and DIGITIZER.

Some points are made on the samples so that strain points can be determined through measuring amount of displacement. The force-displacement diagram obtained from the test is as follows:

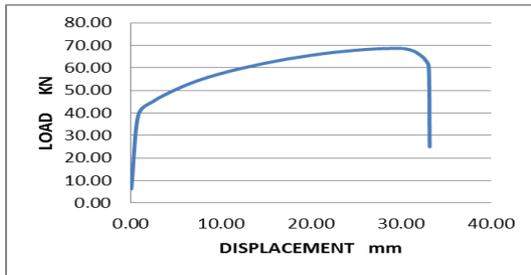


Fig. 3
Force-displacement and engineering stress-strain diagrams.

The real stress-strain diagram can be obtained from conversion equations (Eqs. (1)). This diagram is shown in the figure, but given that these equations are true only for the time before creation of throat, it cannot be used at the times after happening of throat. Considering real stress-strain equations, the current diameter of the sample should be measured during the test in order to calculate the current area and determine the real stress-strain equations as following:

$$\sigma = \frac{P}{A} \quad , \quad \epsilon = 2Ln \frac{d_0}{d} \tag{11}$$

Measurement of current diameter of the sample is done using the obtained images. The image below shows a comparison between the area of the surface obtained from theoretical equations and that obtained from the pictorial measurement, as well as between the results obtained from them. Given that these two values are equal to each other at the time before creation of the throat, accuracy of the measurement technique used in this experiment is verified.

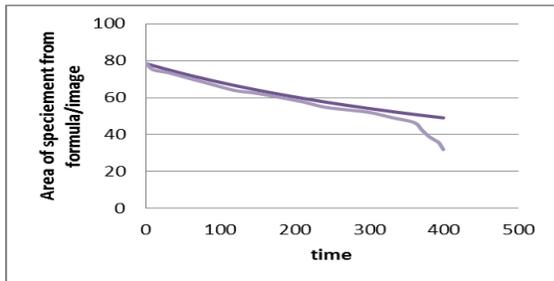


Fig. 4
Changes in area of the surface obtained from the equation and the measurement.

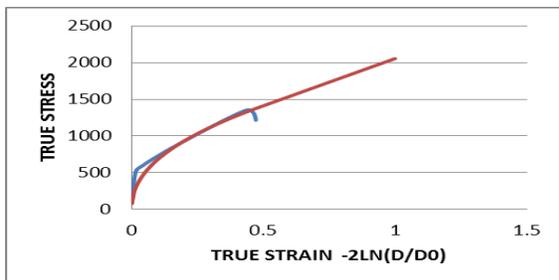


Fig. 5
True stress-strain diagram obtained from the conversion equations and the measurement.

In practice, real diagrams obtained from measurement are introduced as stress-strain relationships. For the material under study, this diagram can be introduced by two different functions for the times before and after creation of throat. If the curve is drawn only for the time before creation of throat and Halamom function is crossed over it, the constants of this function can be determined. Also considering the diagram obtained, it can be shown that

the stress-strain equation for this type of steel after creation of throat is of a linear type. So, the stress-strain equation for this material is as follows:

$$\begin{cases} \sigma = K \varepsilon^n & \varepsilon < \varepsilon_N, K = 1913, n = 0.43 \\ \sigma = A\varepsilon + B & \varepsilon > \varepsilon_N, A = 1263, B = 782 \end{cases} \quad (12)$$

when trying to determine the constants for the model of the material, it is possible to obtain more accurate values by drawing logarithmic diagram of stress versus strain and matching the line equation to it. Drawing the logarithmic stress-strain diagram, the following curved is obtained:

$$\text{Ln}\sigma = \text{Ln}(K) + n \cdot \text{Ln}(\varepsilon) \quad (13)$$

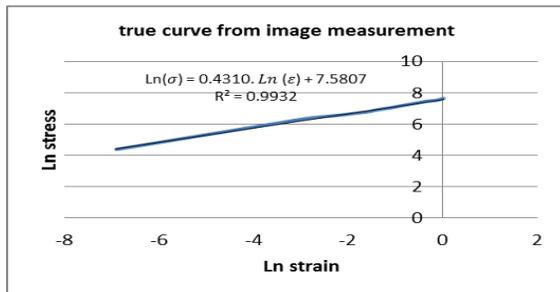


Fig. 6
Logarithmic diagram of stress-strain.

Accordingly, n is the slope of the line. Finally, values of the constants will be obtained as following:

$$n = 0.43 \quad K = 1913 \quad (14)$$

Through processing the images obtained from the cameras during the test and using the precise measurements, the following versus-time data have been extracted from the throat:

Table 1

Values obtained from the images, including diameter in the throat and diameter out of the throat

Time	Start	60 s	120 s	210 s	320 s	360 s	370 s	380 s	Fracture
Diameter in throat section	9.86	9.3	9.05	8.5	8.04	7.58	7.27	6.36	6.14
Diameter out of throat region	9.86	9.3	9.05	8.5	8.04	8.09	8.08	8.08	8
The length of throat region						14	12.69	13.07	20.9
Region									

4 FINITE ELEMENTS SIMULATION

Here we have used ABAQUS analysis software so as to simulate the quasi-static tension experiment on the ST304 steel sample. Principles related to large deformations have been used in this analysis. Due to the existing axial symmetry, only an eighth of the sample has been modeled and analyzed. The model has 250 elements which are of four-node axial symmetry type namely CAX4R in the software.

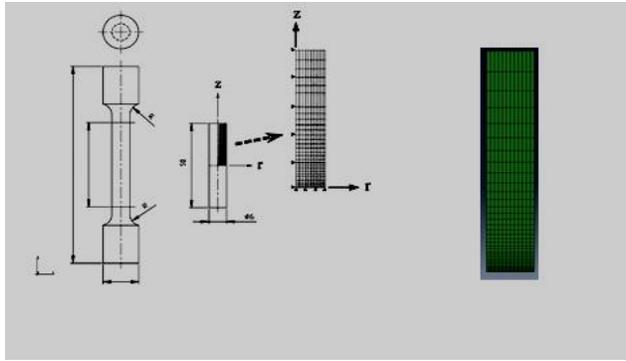


Fig. 7
Type of element-sorting in the sample under study.

The elements have been sorted in order of size and from small to large ones. This type of element-sorting causes elements to be smaller in the region close to the throat and causes the number of them to be larger. But this number is gradually decreased. In order to make the research more precise, a failure model known as Porous Metal Plasticity has been used. Based on Gerson model, this model has been considered by Hung-Yang Yeh [12] in 2003. In this model, material is considered as having some holes in it, and relative density is considered to be more than 0.9 percent. It has three constants, which are input to the software as follows:

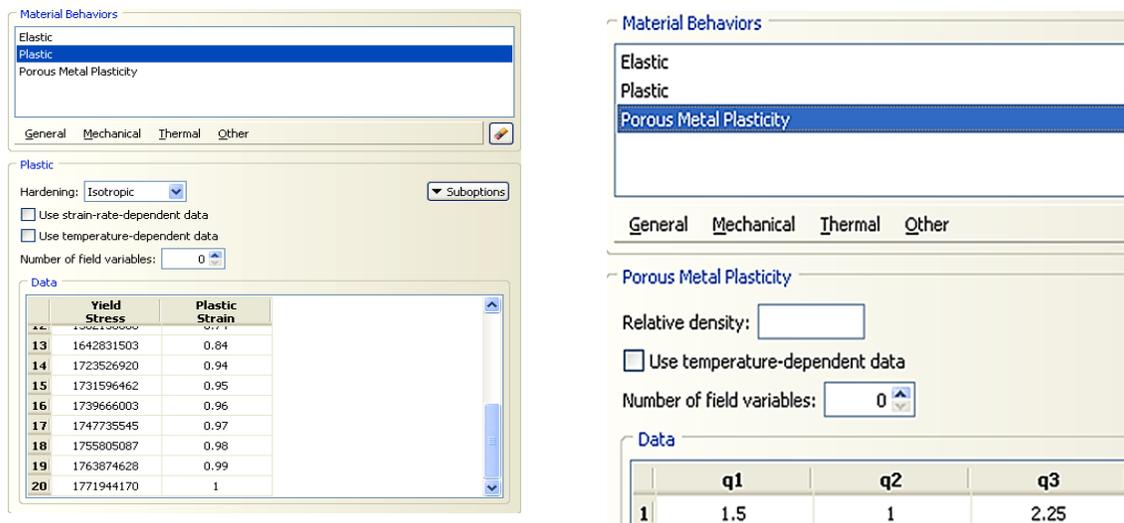


Fig. 8
Inputting failure mechanic characteristics to the software.

The material has been also considered as homogeneous and isotropic.

5 OPTIMIZATION METHOD

Given that stress-strain behavior of the material is linear after creation of throat, the following definition can be used:

$$\begin{cases} \sigma = K \varepsilon^n & \varepsilon < \varepsilon_N, K = 1913, n = 0.43 \\ \sigma = A\varepsilon + B & \varepsilon > \varepsilon_N \end{cases} \quad (15)$$

The purpose of modifying the stress-strain curve is to determine A and B values which are constants of the line equation. In principle, the followings are carried out to make an optimization in this regard:

1. Carrying out tension test and extracting basis sizes: here, the basis is three different diameters on the sample and in the throat. These three include the minimum diameter, and the diameters at the distance of 10 millimeter and 20 millimeter from it, as shown in the following figure:

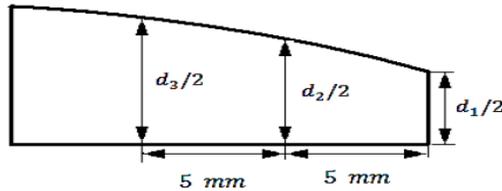


Fig. 9
Basis diameters.

These diameters have been measured using imaging method.

2. Simulating the tension test similar to the experiment: in this case, given the points already remarked about simulation, inputting the values for stress and strain according to the diagram, the stress-strain curve is modified, and these three diameters are determined through the simulation results after running the software. Based on the number of the unknown parameters, which is 2, simulation process should be carried out for seven times, and diameters in question should be extracted. First, the arbitrary values for A and B, which are corresponding to a stress-strain curve, are input to the software, and diameters are extracted then.

3. Target function is defined as follows:

$$OBJ = \sqrt{(d_{1m} - d_{1t})^2 + (d_{2m} - d_{2t})^2 + (d_{3m} - d_{3t})^2} \quad (16)$$

where d_{1m} is the diameter in the throat region obtained from the simulation, and d_{1t} is the diameter in the same region obtained from the test. Values for three diameters obtained from the test and simulation exist in the target function. If these values are exactly equal to each other, the function will be equal to zero. Purpose of optimization is to extract some values for the unknown parameters that minimize the target function.

Several values for the presented target function can be extracted putting the values obtained in the Eq.(17), as shown in the following Table. 2.

Table 2

Values for A and B and diameters obtained from the simulation

Item	A	B	$d_{1m} - d_{1t}$	$d_{2m} - d_{2t}$	$d_{3m} - d_{3t}$	OBJ
0	1293	761				
1	1045	864	0.88	0.8	-0.5	1.290116
2	870	938	-1.3	0.74	-0.1	1.4992
3	1203	798	-0.04	0.6	-0.18	0.627694
4	1100	841	1.42	1.2	-0.1	1.861827
5	905	923	-1.13	0.72	-0.02	1.340037
6	607	1050	1.61	1.2	-0.38	2.043649
7	703.5	1008.5	1.652	1.356	-0.1	2.139589

4. Based on optimization theory principles, target function can be defined in another way as following [14]:

$$OBJ(x, y) = a_1 + a_2x + a_3y + a_4xy + a_5x^2 + a_6y^2 + a_7x^2y^2 \quad (17)$$

In this equation, x and y are values of A and B, respectively. The equation also has 7 constants from a_1 through a_7 , which should be determined. That is, the equation can be rewritten for this problem as follows:

$$OBJ(x, y) = a_1 + a_2A + a_3B + a_4AB + a_5A^2 + a_6B^2 + a_7A^2B^2 \tag{18}$$

Putting the values listed and obtained from simulation in the above table results in seven equations, and the seven constants existing in the above equation can be obtained by solving them. These seven equations and seven unknown parameters have been solved using Assmath mathematic operation software, and its constants have been extracted as well:

Table 3
Values of the constants existing in the equation

a_1	-40413.385
a_2	-32.535
a_3	135.642
a_4	-7.1419
a_5	0.0147
a_6	-0.07985
a_7	9.50E-10

5. Putting these constants in the target function equation, an equation in terms of A and B or x and y is achieved as follows:

$$OBJ(x, y) = -40413.385 - 32.532x + 135.64y - 0.000071419xy + 0.0147x^2 - 0.07985y^2 + (9.5E - 10)x^2y^2 \tag{19}$$

Now in the last step in this method, some values of the two unknown parameters should be determined, for which the target function would be minimum or close to zero. These values are extracted using Genetic Algorithm as following:

Table 4
Values obtained from optimization and value of the target function related to those values

A	785.341
B	970.51
OBJ	3.96E-05

These two values are corresponding to the stress-strain equation after creation of throat. Accordingly, the stress-strain equation obtained from this method is presented as follows:

$$\begin{cases} \sigma = K\varepsilon^n & \varepsilon < \varepsilon_N, K = 1913, n = 0.43 \\ \sigma = A\varepsilon + B & \varepsilon > \varepsilon_N, A = 785, B = 970 \end{cases} \tag{20}$$

The stress-strain curve obtained from these values is the modified curve. In order to simulate the process using ABAQUS software, from now on, it is possible to use the following data, which are modified version of the stress-strain values obtained from optimization method, and to be sure that the results properly coincide with the reality.

It is noteworthy to remind that in this specific material, the stress-strain relationship is linear after the throat is created, and modifying the stress-strain curve after creation of the throat has converted into a problem to determine the aforementioned line equation.

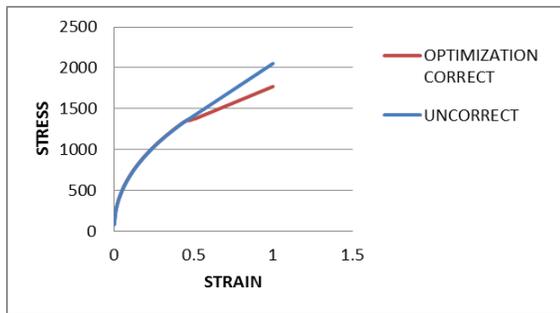


Fig. 10
The curve modified by using optimization.

6 MODIFYING THE STRESS-STRAIN CURVE USING CONVENTIONAL METHODS

In order to investigate the accuracy of the method, it is necessary to extract the modified curves from the other methods, and compare the results obtained with those obtained from the recommended method.

6.1 Bridgman-Leroy method

Leroy [4] has solved the problem existing in the Bridgman equation by introducing an equation for determining the relation of the curvature radius to the radius of the sample. This equation is valid for a broad range of metals, and along with Bridgman method presents a method to modify the stress-strain curve:

$$K = \left[\left(1 + \frac{2R}{a} \right) \ln \left(1 + \frac{a}{2R} \right) \right]^{-1} \quad \frac{a}{R} = 1.1(\varepsilon_P - \varepsilon_N) \quad (21)$$

where ε_N is starting strain of the throat that is equal to 0.43 for the problem in question in this research. ε_P is plastic or effective strain. Determining the value of ... in each strain, putting it in the equation, and applying it to the unmodified stress, finally the values $A=758.5$ and $B=1000$ are obtained for the stress-strain equation at the times after creation of throat.

6.2 Bridgman method [1]

Among the methods known as Classic, Bridgman method is the most commonly used one, and at the same time, is the most valid one. In this method, first throat profile should be photographed in some points after creation of the throat, and its curvature radius should be measured in the throat point. To do this in the material under study, this operation is done only at the time just before the failure. For this purpose, first the throat profile obtained from the images is extracted from the images recorded, and a quadratic curve is passed over it. This equation is as following:

$$r = 0.105z^2 - 0.035z + 3.1 \quad (22)$$

where z and r are longitudinal and radial components, respectively. The throat profile is as follows:

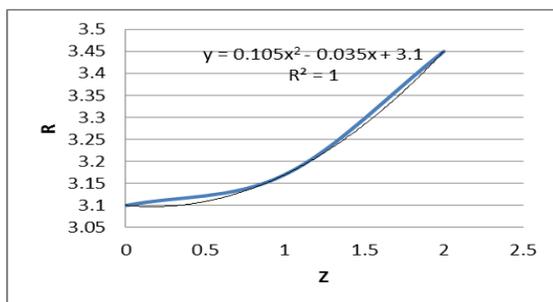


Fig. 11
The throat profile curve obtained from the images recorded during the test, at the point just before the failure, along with the equation passed over it.

Using mathematic equation of curvature radius, simplifying it, and applying the following equations, curvature radius is finally obtained:

$$\frac{1}{R} = \frac{d^2r}{dz^2} = 0.21 \quad R = 4.7 \quad \frac{R}{a} = 1.556 \tag{23}$$

Calculating this value, and putting it in the equation, the following modification factor is achieved:

$$K = \left[\left(1 + \frac{2R}{a} \right) \text{Ln} \left(1 + \frac{a}{2R} \right) \right]^{-1} = 0.872 \tag{24}$$

This is the value of modification factor at the time just before the failure. Multiplying this value by the value of unmodified stress at that time, the modified stress is calculated. Ultimately, values of the constants related to the noted line equation are obtained as following:

$$A = 790 \quad B = 1000 \tag{25}$$

7 CONCLUSIONS

The following table shows the values for the constants of the line equation for several methods:

Table 5
Values related to several methods of modifying the curve

	OPTIMIZATION	BRIDG. LEROY	BRIDG.
A	785	758.5	791
B	970	1000	998
Fracture stress	1778	1762	1783
Correction in frac.	0.88	0.86	0.872

As shown in the table, the results obtained from the optimization method are close to those obtained from the other existing methods. It can be better shown by drawing the modified stress-strain curve using different methods:

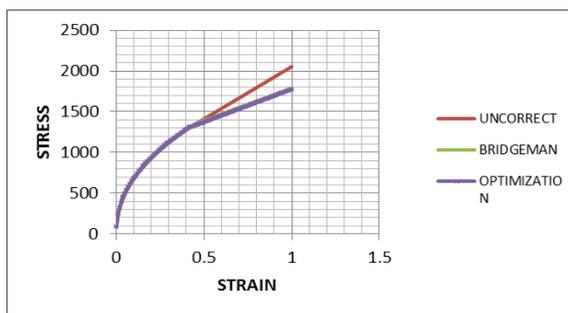


Fig. 12
The stress-strain curves obtained from optimization method and the other methods.

As seen in the results, the results obtained from this method are very close to those obtained from Bridgman method. This method, however, has not the problems related to Bridgman approach. According to Bridgman, it is necessary to precisely measure throat profile. This profile is extracted by photographing. Then, an equation is passed over it through some computer operations. The equation should be determined so that it passes on the points close to

the throat, and in fact, should more overlaps with the profile at the points close to the throat. Then, making derivation from the equation and putting it in the curvature radius equation, this radius is calculated.

Problems related to this method appear in practical uses. While extracting Bridgman equation, some assumptions have been considered, reducing its accuracy.

The method suggested, along with image processing operation using a normal video camera may properly modify stress-strain curves with a high degree of accuracy.

This method can be used as a useful procedure.

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