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Free Vibration Analysis of Moderately Thick Functionally Graded Plates with Multiple Circular and Square Cutouts Using Finite Element Method

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ABSTRACT
A simple formulation for studying the free vibration of shear-deformable functionally graded plates of different shapes with different cutouts using the finite element method is presented. The aim is to fill the void in the available literature with respect to the free vibration results of functionally graded plates of different shapes with different cutouts. The material properties of the plates are assumed to vary according to a power law distribution in terms of the volume fraction of the constituents. Validation of the formulation is done with the help of convergence studies with respect to the number of nodes and the results are compared with those from past investigations available only for simpler problems. In this paper rectangular, trapezoidal and circular plates with cutouts are studied and the effects of volume fraction index, thickness ratio and different external boundary conditions on the natural frequencies of plates are studied.

Keywords: Functionally graded materials; Free vibration; Circular/square/trapezoidal plates; Circular/square cutouts

1 INTRODUCTION

FUNCTIONALLY graded materials (FGMs) are a class of composites that have continuous variation of material properties from one surface to another and thus eliminate the stress concentration found in laminated composites. A typical FGM is made from a mixture of ceramic and metal. These materials are often isotropic but nonhomogeneous. The reason for interest in FGMs is that it may be possible to create certain types of FGM structures capable of adapting to operating conditions. The increase in FGM applications requires accurate models to predict their responses. A critical review of more recent works on the static, vibration and stability analysis of functionally graded (FG) plates can be found in the paper of Jha et al. [1]. Since the shear deformation has significant effects on the responses of FG plates, shear deformation theories such as first-order shear deformation theory (FSDT) and higher-order shear deformation theories (HSDTs) should be used to analyze FG plates.

The FSDT accounts for the shear deformation effects by linear variation for in-plane displacements and requires a shear correction factor, whereas the HSDTs account for the shear deformation effects by higher-order variations for in-plane displacements or both in-plane and transverse displacements. For example, Reddy [2-3] developed a third-order shear deformation theory (TSDT) with cubic variations for in-plane displacements. Xiang et al. [4-5] proposed a n-order shear deformation theory in which Reddy's theory can be considered as a specific case.

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The methods employed in the paper included a higher order shear deformation theory and two novel solutions for FGM structures. According to this paper, the application of the normal deformation theory may be justified if the in-plane size to thickness is equal to or smaller than 5.

Researchers have also turned their attention to the vibration and dynamic response of FGM's structures [6-7]. Chen et al [8] presented exact solutions for free vibration analysis of rectangular plates using Bessel functions with three edges conditions. Liew et al [9] studied the free vibration analysis of functionally graded plates using the element-free Kp-Ritz method. They studied the free vibration analysis of four types of functionally graded rectangular and skew plates. Hiroyuki Matsunaga [10] presented in his paper, the analysis of natural frequencies and buckling of FGM's plates by taking into account the effects of transverse shear and normal deformations and rotary inertia. Atashipour et al [11] presented a new exact closed- form procedure to solve free vibration analysis of FGM's rectangular thick plates based on the Reddy's third-order shear deformation plate theory.

For plates with cutouts, Chai [12] presented finite element and some experimental results on the free vibration of symmetric composite plates with central hole. Sakiyama and Huang [13] proposed an approximate method for analyzing the free vibration of square plate with different cutouts. Liu et al [14] studied static and free vibration analyses of composite plates with different cutouts via a linearly conforming radial point interpolation method. Maziar and Iman [16] studied the effect of relative distance of cutouts and size of cutouts on natural frequencies of FG plates with cutouts. The free vibration of functionally graded nonuniform straight-sided plates with circular and non-circular cutouts has been investigated by [18]. From the review of the above literature, it is observed that very little research and analysis work has been done yet on the natural frequencies of the FG plates with cutouts. The study presents here, the effect of volume fraction index, thickness ratio and different external boundary conditions on the natural frequencies of FG (Al/Al_2O_3) plates (such as rectangular, trapezoidal and circular) with cutouts.

The aim of this paper is to develop a simple first order shear deformation theory for the free vibration analysis of FG plates. The first order shear deformation theory is used to incorporate the effects of transverse shear deformation and rotary inertia. Equations of motion are derived from Hamilton's principle. Numerical examples are presented to verify the accuracy of the present theory. The accuracy and versatility of the algorithm are demonstrated via different examples for the functionally graded plates. This work, thus, aims to study the free vibration problem of Functionally Graded Plates with Multiple Circular and Square Cutouts which appears to have not been studied as yet.

2 FUNCTIONALLY GRADED MATERIAL PROPERTIES

A functionally graded material plate as shown in Fig. 1 is considered to be a plate of uniform thickness that is made of ceramic and metal. The material property is to be graded through the thickness according to a Power-Law distribution that is:

$$P(z) = P_m + (P_c - P_m)V_f \tag{a}$$

$$V_f = \left(\frac{z}{h}\right)^n \quad n \geq 0) \tag{b}$$

where P represents the effective material property, P_c and P_m denotes the ceramic and metal properties respectively, V_f is the volume fraction of the ceramic, h is the thickness of the plate, $0 \leq z \leq h$ and n is the volume fraction index.

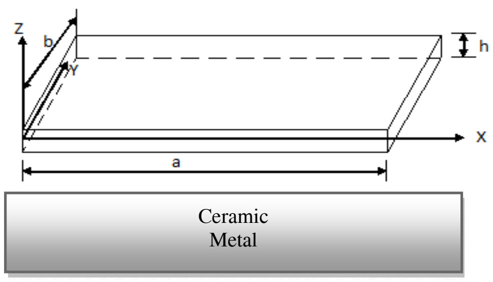

Fig. 1
Functionally graded plate.

2.1 Functionally graded plate elements

The finite element method is used for the vibration analysis of functionally graded plates. SOLID 185 is used for the modeling of general 3-D solid structures as shown in Fig. 2. It allows for prism and tetrahedral degenerations when used in irregular regions. The element is defined by eight nodes having three degree of freedom at each node. More than 2000 nodes might be used in calculation work.

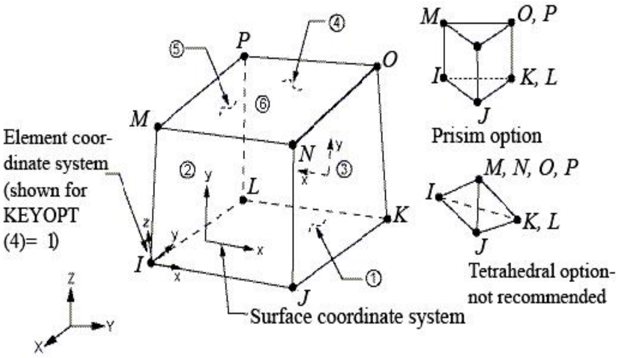

Fig. 2
Solid 185.

3 MATHEMATICAL FORMULATION

Fig. 1 shows the geometry of a functionally graded plate. Considering the first order shear deformation theory, the displacement fields are expressed as follows [19].

$$\left.\begin{aligned} u(x,y,z,t) &= u_0(x,y,t) + z\varphi_x(x,y,t) \\ v(x,y,z,t) &= v_0(x,y,t) + z\varphi_y(x,y,t) \\ w(x,y,z,t) &= w_0(x,y,t) \end{aligned}\right\} \tag{1}$$

where ($u_0, v_0, w_0, \phi_x, \phi_y$) are unknown functions to be determined. As before, (u_0, v_0, w_0) denote the displacements of a point on the plane $z = 0$; Note that

$$\frac{\partial u}{\partial z} = \phi_x, \frac{\partial v}{\partial z} = \phi_y \tag{2}$$

which indicate that ϕ_x and ϕ_y are the rotations of a transverse normal about the *y*- and *x*- axes, respectively as shown in Fig. 1.

The strain displacement relations can be expressed as follows.

In-plane strains at the mid-plane are:

$$\in_x^0 = \frac{\partial u_0}{\partial x} \tag{3}$$

$$\in_y^0 = \frac{\partial v_0}{\partial y} \tag{4}$$

$$\in_{xy}^0 = \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} \tag{5}$$

The curvatures are:

$$k_x^0 = \frac{\partial \phi_x}{\partial x} \tag{6}$$

$$k_y^0 = \frac{\partial \phi_y}{\partial y} \tag{7}$$

$$k_{xy}^0 = \frac{\partial \phi_x}{\partial y} + \frac{\partial \phi_y}{\partial x} \tag{8}$$

The shear strains in xz and yz planes are:

$$\varepsilon_{xz} = \phi_x + \frac{\partial w_0}{\partial x} \tag{9}$$

$$\varepsilon_{yz} = \phi_y + \frac{\partial w_0}{\partial y} \tag{10}$$

The strain components at any point can thus be expressed as,

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{yz} \\ \varepsilon_{xz} \\ \varepsilon_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_{xx}^{(0)} \\ \varepsilon_{yy}^{(0)} \\ \varepsilon_{yz}^{(0)} \\ \varepsilon_{xz}^{(0)} \\ \varepsilon_{xy}^{(0)} \end{Bmatrix} + z \begin{Bmatrix} k_{xx}^{(0)} \\ k_{yy}^{(0)} \\ 0 \\ 0 \\ k_{xy}^{(0)} \end{Bmatrix} \qquad \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{yz} \\ \varepsilon_{xz} \\ \varepsilon_{xy} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u_0}{\partial x} \\ \frac{\partial v_0}{\partial y} \\ \phi_y + \frac{\partial w_0}{\partial y} \\ \phi_x + \frac{\partial w_0}{\partial x} \\ \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} \end{Bmatrix} + z \begin{Bmatrix} \frac{\partial \phi_x}{\partial x} \\ \frac{\partial \phi_y}{\partial y} \\ 0 \\ 0 \\ \frac{\partial \phi_x}{\partial y} + \frac{\partial \phi_y}{\partial x} \end{Bmatrix} \tag{11}$$

4 NUMERICAL RESULTS AND DISCUSSION

The present study gives the free vibration results of moderately thick functionally graded plates with multiple circular and square cutouts. The boundary conditions considered here are various combinations of support conditions. The effects of boundary conditions, thickness ratio and aspect ratio are studied.

The present study is validated by comparison with the results available in the literature. The default parameter values of the functionally graded plates are as follows.

The default material properties used in the present study are as follows [18]:

$$E_c = 380 \text{ GPa}, \nu_c = 0.3, \rho_c = 3800 \text{Kg/m}^3, \alpha_c = 7.4\text{x}10^{-6} /^{\text{o}} \text{C}, K_c = 65 \text{ W/mk}$$
$$E_m = 70 \text{ GPa}, \nu_m = 0.3, \rho_m = 2700 \text{Kg/m}^3, \alpha_m = 7.4\text{x}10^{-6} /^{\text{o}} \text{C}, K_m = 65 \text{ W/mk}$$

In order to show the accuracy of methodology used for free vibration analysis of FG plates with cutouts, the fundamental natural frequencies of different plates (such as rectangular, trapezoidal and circular) with different cutouts (such as circular and non circular) are compared with the solutions presented by Maziar et al [16].

The material properties used in the convergence study are as follows [16]:

$$E_c = 70 \text{ GPa}, \nu_c = 0.3, \rho_c = 3800 \text{Kg/m}^3, \alpha_c = 7.4\text{x}10^{-6} /^{\text{o}} \text{C}, K_c = 65 \text{ W/mk}$$
$$E_m = 70 \text{ GPa}, \nu_m = 0.3, \rho_m = 2700 \text{Kg/m}^3, \alpha_m = 7.4\text{x}10^{-6} /^{\text{o}} \text{C}, K_m = 65 \text{ W/mk}$$

4.1 Isotropic plates

Convergence study of free vibration analysis of a simply supported square plate with a square hole is analyzed in Table 1. The geometry and material properties of plate are a=10, size ratio d/a=0.5, thickness ratio h/a=0.01, Young modulus E=200 GPa and Density ρ=8000 Kg/m^3. Convergence study of free vibration analysis of a simply supported square plate with a circular cut-out is analyzed in Table 2. The geometry and material properties of plate are a=10, radius to length ratio r/a=0.1, thickness ratio h/a=0.01, Young modulus E=200 GPa and density ρ=8000 Kg/m^3.

Table 3 shows the first eight fundamental frequencies for the different values of volume fraction index and it is clearly observed that by increasing the volume fraction index fundamental frequencies get decreases and Table 4 shows the effect of radius to length ratio on the fundamental frequencies of FG square plate with circular cut-outs. The result shows that the first fundamental frequency increases by increasing the radius to length ratio.

Table 1
Convergence of non-dimensional fundamental frequencies of isotropic square plate with square cut-out at the centre (simply supported for external boundaries)

M=N	$\bar{\omega}=\left[\dfrac{\rho h\omega^2 a^4}{D(1-\nu^2)}\right]^{1/4}$							
	i=1	2	3	4	5	6	7	8
4	5.6164	9.0338	9.6914	13.5788	16.8677	20.9517	21.4739	23.0141
8	5.0967	6.9922	7.0800	9.4111	9.8836	12.0658	12.1565	13.7822
10	5.0290	6.7807	6.7852	8.9400	9.5327	11.5087	11.5130	12.7709
12	5.0497	6.6769	6.7185	8.9439	9.1978	11.3475	11.4694	13.0727
14	5.0141	6.6311	6.6390	8.8821	9.0313	11.2022	11.2095	12.8474
16	4.9816	6.5422	6.5483	8.7488	8.8831	10.9980	11.0089	12.5498
18	4.9724	6.5156	6.5207	8.6994	8.8295	10.9186	10.9278	12.4726
20	4.9688	6.5073	6.5087	8.6869	8.8076	10.8938	10.9030	12.4544
22	4.9627	6.4923	6.4944	8.6666	8.7724	10.8476	10.8538	12.3808
[14]	4.9717	6.4810	6.4821	8.5509	8.8656	10.720	10.767	12.045
[17]	4.839	6.435	6.440	8.492	8.875	10.81	10.83	12.29

Table 2
Convergence of non-dimensional fundamental frequencies of isotropic square plate with circular cut-out at the centre (simply supported for external boundaries)

M=N	$\bar{\omega}=\left[\dfrac{\rho h\omega^2 a^4}{D(1-\nu^2)}\right]^{1/4}$							
	i=1	2	3	4	5	6	7	8
4	4.7060	8.1368	8.1867	9.9678	12.7068	12.9062	14.0269	14.3334
8	4.8037	7.9793	8.1344	10.6479	11.6635	12.2793	14.1800	14.6732
10	4.5427	7.3067	7.3108	9.3510	10.3677	10.6530	12.0104	12.0195
12	4.5506	7.3067	7.3254	9.3938	10.4106	10.8662	12.2459	12.2784
14	4.5167	7.1959	7.2095	9.1565	10.1966	10.5088	11.7618	11.7849
16	4.5090	7.1656	7.1668	9.0888	10.1199	10.4271	11.6410	11.6625
18	4.5035	7.1501	7.1516	9.0494	10.0964	10.4056	11.5998	11.6115
20	4.5003	7.1370	7.1382	9.0234	10.0669	10.3672	11.5485	11.5555
22	4.4985	7.1312	7.1315	9.0123	10.0524	10.3517	11.5306	11.5319
[14]	6.149	8.577	8.634	10.42	11.41	11.84	12.83	12.84
[17]	6.240	8.457	8.462	10.23	11.72	12.30	13.04	13.04

Table 3
Non-dimensionalized frequencies of FG square plate Al/Al_2O_3 with circular cut-outs (fully clamped for external boundaries *a*/*b*=1, *h*/*a*=0.1, *r*/a=0.1)

	$\overline{\omega}=(\omega a^2/h)\sqrt{\rho_c/E_c}$							
n	*i*=1	2	3	4	5	6	7	8
0	10.3922	18.6947	18.7431	27.0971	31.945	34.5005	38.6781	38.7893
1	6.7675	12.1735	12.2056	17.6455	20.8027	22.4665	25.1871	25.2593
5	5.3563	9.6351	9.6603	13.9661	16.4649	17.7818	19.9350	19.9922
200	5.2847	9.5063	9.5315	13.7795	16.245	17.543	19.6686	19.7252

Table 4
Effect of radius to length ratio on the natural frequencies of FG square plate with circular cut-outs. (fully clamped for external boundaries, *h/a* =0.1, *a/b* = 1, *n* = 1)

Mode	*r* /*a*			
	0.1	0.2	0.25	0.3
1	6.7675	8.3471	10.2182	13.3020
2	12.1735	11.4107	12.3620	14.5994
3	12.2056	11.4183	12.3827	14.6176
4	17.6455	16.4718	16.2538	16.7916
5	20.8027	19.1408	19.3117	21.2237
6	22.4665	24.6926	24.5035	24.9747
7	25.1871	24.7095	24.527	25.0130
8	25.2593	27.3918	29.484	30.3411
9	25.9366	30.0263	32.8261	32.2939
10	25.9417	30.0276	32.9492	36.6964

4.2 Rectangular plates

Rectangular plates with multiple cutouts are shown in Fig. 3.

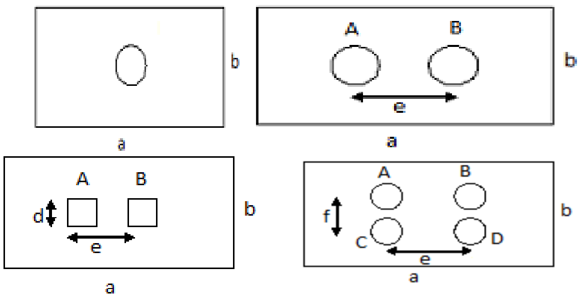

Fig. 3
Rectangular plates with multiple cutouts.

Table 5
Comparison of natural frequencies of FG rectangular plate with two circular cutouts (fully clamped for external boundaries)

	Mode									
e/b	1	2	3	4	5	6	7	8	9	10
0.5	512.55	648.11	805.35	1098.0	1251.9	1287.6	1466.3	1560.2	1801.6	1979.4
[15]	516.59	642.85	826.80	1147.6	1287.4	1288.8	1526.9	1598.1	1817.7	-
0.6	507.54	650.74	792.53	1107.1	1203.2	1213.2	1447.1	1584.9	1748.7	1904.3
[15]	510.71	654.29	823.05	1153.4	1247.1	1278.8	1525.7	1671.5	1818.6	-
0.7	503.58	642.15	785.82	1091.9	1110.9	1165.3	1440.4	1598.7	1735.0	1903.6
[16]	503.90	663.07	832.46	1157.3	1183.6	1269.4	1519.3	1725.6	1817.4	-

Table 6
Comparison of natural frequencies of FG rectangular plate with four circular cutouts (fully clamped for external boundaries)

	Mode									
	1	2	3	4	5	6	7	8	9	10
e/b										
0.7	487.08	620.94	756.67	1081.0	1250.0	1348.1	1387.3	1538.4	1765.9	1879.2
[16]	486.40	633.55	801.23	1136.3	1251.1	1435.4	1489.5	1679.3	1890.0	2035.2

Table 7
Comparison of natural frequencies of FG rectangular plate with two square cutouts (clamped-free for external boundaries)

	Mode									
	1	2	3	4	5	6	7	8	9	10
e/b										
0.7	473.62	496.04	602.99	769.94	1017.0	1242.4	1275.0	1364.4	1402.5	1435.0
[16]	470.74	476.14	541.35	837.44	922.06	1087.4	1095.6	1136.3	1245.3	1375.9

Table 8
Variation of natural frequencies with the volume fraction index *n* for Al/Al_2O_3 FG rectangular plates with two circular holes (fully clamped for external boundaries)

		Mode									
		1	2	3	4	5	6	7	8	9	10
h/b	n										
0.04	0	947.42	1225.6	1627.8	2258.9	2270.6	2492.6	2906.7	3125.1	3475.6	3920.0
	0.5	820.79	1017.3	1352.4	1872.3	1909.3	2023.5	2460.9	2597.4	2938.5	3252.9
	1	656.11	838.38	1102.9	1529.8	1568.4	1694.0	2002.3	2126.4	2450.5	2677.3
	2	667.44	819.91	1066.8	1449.8	1521.0	1614.8	2032.1	2103.1	2390.5	2423.1
0.06	0	1323.9	1707.5	2231.2	3069.7	3103.8	3312.5	3940.6	4114.2	4231.2	4685.3
	0.5	1077.4	1349.1	1764.8	2425.3	2516.1	2644.4	3211.7	3335.5	3736.2	3778.9
	1	887.06	1082.7	1435.1	1971.8	2028.5	2156.3	2610.6	2739.5	3125.4	3383.6
	2	729.59	918.40	1204.7	1666.3	1690.1	1787.9	2140.3	2343.7	2613.6	2827.6
0.08	0	1675.9	2132.4	2796.0	3710.6	3855.2	4067.8	4133.9	4829.2	5179.5	5663.5
	0.5	1292.7	1621.6	2120.1	2910.3	2934.5	3131.2	3743.8	3782.8	4035.7	4504.5
	1	1005.6	1274.2	1683.8	2316.4	2333.1	2516.1	2974.8	3217.0	3412.0	3557.2
	2	878.81	1081.5	1423.5	1947.1	1961.4	2101.3	2495.3	2693.0	2884.1	2966.0

Table 5 shows the comparison of natural frequencies of FG rectangular plate of side ratio a/b=2 having two holes of radius ratio r/b=0.15 and centre to centre distance ratio e/b=0.5, 0.6 and 0.7. Thickness of the plate is h=0.05. Table 6 shows the comparison of natural frequencies of FG rectangular plate having same side ratio and thickness with four holes of radius ratio r/b=0.1 and centre to centre distance ratio e/b=0.7 and f/b=0.4. Table 7 shows the comparison of natural frequencies of FG rectangular plate of same side ratio, thickness with two square cutouts of side ratio d/b=0.1 and e/b=0.7. The variation of natural frequencies with the volume fraction exponent for Al/Al_2O_3 FG rectangular plates of side ratio a/b=2 and thickness ratio h/b=0.04, 0.06 and 0.08, having two holes of radius ratio r/b=0.1 and centre to centre distance ratio e/b=0.8 are shown in tables 8 and 9 respectively. The results for first ten modes are computed. For the FG plates with fully clamped external boundary condition, the frequencies in all ten modes decrease as the volume fraction index increases. This is expected, because a large volume fraction index means that a plate has a smaller ceramic component and thus its stiffness is reduced. Similar results are also observed for the clamped- clamped and clamped- free for external boundary conditions. From Tables 8 and 9, it is also observed that natural frequencies slowly improve as the thickness ratio h/b increases from 0.04 to 0.06.

Table 10 shows that the variation of natural frequencies for first five modes of Al/Al_2O_3 FG rectangular plate (h/b=0.04) with two holes (r/b=0.1) for fully clamped, clamped-free and clamped-simply supported external boundary conditions are quite close to each other.

Table 9
Variation of natural frequencies with the volume fraction index *n* for Al/Al_2O_3 FG rectangular plates with two circular holes (clamped-free for external boundaries)

		Mode									
		1	2	3	4	5	6	7	8	9	10
h/b	n										
0.04	0	865.00	915.63	1063.9	1378.6	1783.1	2216.2	2310.9	2398.9	2484.0	2767.8
	0.5	716.37	742.00	919.51	1155.7	1490.2	1826.6	1869.8	1986.8	2108.8	2297.8
	1	582.40	607.83	720.84	637.92	1208.1	1504.5	1553.2	1616.4	1712.3	1899.8
	2	567.73	582.81	743.11	943.25	1195.0	1421.6	1488.0	1549.3	1786.9	1878.5
0.06	0	1210.5	1271.8	1458.7	1888.7	2427.5	2993.1	3067.9	3088.0	3290.8	3378.8
	0.5	969.48	1006.3	1184.3	1514.0	1943.6	2406.5	2448.9	2587.6	2774.7	2788.4
	1	767.75	786.47	967.83	1219.9	1575.1	1943.8	1988.0	2096.6	2238.3	2440.9
	2	634.85	660.96	816.72	1046.9	1339.7	1577.5	1620.9	1770.4	1828.5	2052.6
0.08	0	1530.6	1607.5	1869.0	2400.0	3082.3	3088.5	3645.6	3728.2	3825.4	4132.8
	0.5	1154.4	1198.5	1421.8	1818.3	2328.3	2791.9	2836.6	2939.6	3132.6	3261.8
	1	889.35	924.58	1101.6	1419.8	1834.8	2242.7	2304.4	2453.7	2518.4	2558.8
	2	784.29	802.90	951.64	1211.8	1579.1	1915.3	1961.5	2091.5	2154.9	2176.5

Table 10
Variation of natural frequencies with the volume fraction index *n* for Al/Al_2O_3 FG rectangular plates with two circular holes (h/b=0.04)

		n			
		0	0.5	1	2
Boundary condition	Mode				
CCCC	1	947.42	820.79	656.11	667.44
	2	1225.6	1017.3	838.38	819.91
	3	1627.8	1352.4	1102.9	1066.8
	4	2258.9	1872.3	1529.8	1449.8
	5	2270.6	1909.3	1568.4	1521.0
CFCF	1	856.00	716.37	582.40	567.73
	2	915.63	742.00	607.83	582.81
	3	1063.9	919.51	720.84	743.11
	4	1378.6	1155.7	637.92	943.25
	5	1783.1	1490.2	1208.1	1195.0
CSCS	1	947.24	820.68	656.15	657.42
	2	1224.8	1016.9	838.38	814.09
	3	1626.3	1351.7	1102.7	1065.0
	4	2258.2	1871.3	1529.6	1446.6
	5	2268.2	1908.9	1568.3	1544.4

4.3 Trapezoidal plates

Trapezoidal plates with multiple cutouts is shown in Fig. 4.

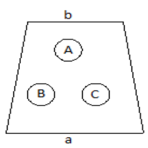

Fig. 4
Trapezoidal plates with multiple cutouts.

Table 11 shows the comparison of natural frequencies of FG trapezoidal plate whose side ratio is b/a=0.7 and height is 1. Thickness of the plate is h=0.05. The plate has two holes of radius ratio r/a=0.05 at location A(x/a=0.25, y=0.45) and B(x/a=0.70, y=0.45) respectively. Table 12 shows the comparison of natural frequencies of FG trapezoidal plate of same side ratio, height and thickness with three holes of radius ratio r/a=0.05, 0.1 and 0.15 at location A(x/a=0.25, y/a=0.35), B(x/a=0.65, y/a=0.35) and C(x/a=0.45, y/a=0.75) respectively.

Table 11
Comparison of natural frequencies of FG trapezoidal plate with two circular cutouts (fully clamped for external boundaries)

r/a	Mode 1	2	3	4	5	6	7	8	9	10
0.05	852.11	1449.5	1792.1	2368.8	2511.7	2971.5	3030.2	3089.8	3529.2	3686.3
[16]	852.84	1489.5	1867.0	2405.1	2536.4	3156.6	3542.1	3775.3	3945.0	-
0.1	842.86	1418.5	1835.8	2353.4	2413.6	2902.9	2985.2	3105.7	3615.6	3643.5
[16]	840.27	1469.6	1869.8	2367.4	2501.3	3245.1	3549.3	3710.7	3951.8	-
0.15	819.27	1390.5	1891.7	2276.5	2328.1	2826.8	3015.4	3191.8	3581.9	3614.7
[16]	817.18	1444.3	1897.7	2332.4	2463.6	3277.3	3648.8	3736.3	3982.1	-

Table 12
Comparison of natural frequencies of FG trapezoidal plate with three circular cutouts (fully clamped for external boundaries)

r/a	Mode 1	2	3	4	5	6	7	8	9	10
0.05	427.44	539.52	1007.6	1109.5	1295.3	1330.8	1861.4	1963.1	2173.8	2335.5
[16]	438.87	541.99	985.51	1166.8	1320.7	1829.2	1899.5	2237.9	2279.4	-
0.1	444.41	545.51	1010.2	1131.8	1233.0	1322.2	1882.8	2065.0	2201.5	2252.5
[16]	447.66	537.26	936.57	1167.4	1301.8	1769.7	1874.7	2132.0	2135.4	-
0.15	461.74	564.00	994.79	1111.0	1184.5	1337.5	1828.4	2068.7	2137.3	2165.8
[16]	462.49	534.93	881.02	1206.3	1301.6	1686.2	1805.1	1892.9	2122.0	-

Table 13
Variation of natural frequencies with the volume fraction index *n* for Al/Al_2O_3 FG trapezoidal plates with two circular holes (fully clamped for external boundaries)

h/a	*n*	Mode 1	2	3	4	5	6	7	8	9	10
0.04	0	2149.8	3907.4	4528.9	5656.4	6388.7	6965.2	7508.9	7806.4	8508.8	8796.2
	0.5	1591.6	2966.8	3412.9	4282.2	4854.7	5794.1	6281.3	6611.3	6843.6	7060.2
	1	1218.9	2300.2	2636.0	3362.1	3853.5	4589.1	5190.6	5473.8	5730.2	6025.4
	2	1040.9	1936.5	2196.0	2833.2	3163.7	3783.2	4390.5	4445.7	4826.3	5044.1
0.06	0	2742.2	4890.8	5695.5	6933.5	7047.8	7777.9	7894.2	9030.7	9206.0	10311.
	0.5	2110.7	3800.9	4471.6	5591.9	6239.1	6354.5	7114.3	7410.6	8113.1	8410.6
	1	1663.4	3033.1	3532.2	4431.5	4938.1	5748.5	5873.2	6442.2	6765.3	6944.4
	2	1358.7	2470.0	2824.9	3510.0	3968.3	4725.2	4819.2	5322.6	5457.9	5482.7
0.08	0	3274.1	5776.2	6701.4	7140.6	7988.8	8050.8	8887.1	9207.9	10702.	11519.
	0.5	2558.4	4480.1	5217.4	6386.9	6439.3	7144.4	7218.7	8191.9	8458.6	9740.6
	1	2083.6	3752.3	4281.4	5328.5	5803.7	6009.7	6480.4	7045.8	7398.6	7998.7
	2	1629.3	2911.3	3344.0	4074.5	4609.6	4848.6	5349.2	5438.3	6070.8	6163.4

The variation of natural frequencies with the volume fraction exponent for Al/Al_2O_3 FG trapezoidal plates whose side ratio is b/a=0.6 and height is 1. Thickness ratio of the plate is h/a=0.04, 0.06 and 0.08. The plate has two holes of radius ratio r/b=0.1 at locations A(x/a=0.25, y/a=0.45) and B(x/a=0.70, y/a=0.45), respectively are shown in Tables 13 and 14. The results for first ten modes are computed. For the FG plates with fully clamped external boundary condition, the frequencies in all ten modes decreases as the volume fraction index increases. This is expected, because a large volume fraction index means that a plate has a smaller ceramic component and thus its stiffness is reduced. Similar results are also observed for the clamped-free and clamped-simply supported external boundary conditions. From Tables 13 and 14 it is also observed that natural frequencies slowly improve as the thickness ratio h/b increases from 0.04 to 0.06. Table 15 shows that the variation of natural frequencies for first five modes of Al/Al_2O_3 FG trapezoidal plate (h/a=0.04) with two holes (r/a=0.1) for fully clamped, clamped-free and clamped-simply supported external boundary conditions are quite close to each other.

Table 14
Variation of natural frequencies with the volume fraction index *n* for Al/Al_2O_3 FG trapezoidal plates with two circular holes (clamped-free for external boundaries)

		Mode									
		1	2	3	4	5	6	7	8	9	10
h/a	*n*										
0.04	0	1349.8	1585.4	2392.9	3266.1	3522.2	3692.9	4537.1	4581.4	6048.0	6297.7
	0.5	998.16	1197.8	1803.8	2483.6	2639.1	3331.2	3405.0	3492.1	4572.4	4592.4
	1	764.14	912.76	1392.8	1923.2	2071.3	2626.5	2674.7	3035.7	3610.7	3808.0
	2	646.20	773.78	1175.8	1601.9	1713.1	2189.6	2251.4	2562.8	3018.4	3181.9
0.06	0	1772.9	2096.7	3094.1	3745.6	4122.0	4424.3	5738.9	5832.7	6374.2	6416.3
	0.5	1348.7	1617.1	2417.6	3194.9	3366.1	3473.3	4485.0	4517.0	5718.8	5764.2
	1	1042.5	1253.4	1883.2	2528.8	2743.9	3050.6	3493.2	3533.1	4639.2	4983.7
	2	861.09	1018.4	1536.1	2084.0	2232.6	2567.4	2851.1	2896.1	3796.2	4010.1
0.08	0	2159.3	2583.7	3785.2	3792.8	4917.9	5335.6	6541.5	6616.3	6824.2	6945.2
	0.5	1622.2	1916.9	2863.3	3398.1	3741.7	4005.0	5191.7	5314.9	5772.7	5830.6
	1	1329.5	1556.9	2332.5	3077.8	3153.1	3383.6	4321.7	4396.7	5204.9	5260.7
	2	1058.9	1246.2	1837.8	2459.8	2573.7	2629.4	3380.6	3425.7	4250.6	4324.3

Table 15
Variation of natural frequencies with the volume fraction index *n* for Al/Al_2O_3 FG trapezoidal plates with two circular holes (h/a=0.04)

Boundary condition	Mode	n			
		0	0.5	1	2
CCCC	1	2149.8	1591.6	1218.9	1040.9
	2	3907.4	2966.8	2300.2	1936.5
	3	4528.9	3412.9	2636.0	2196.0
	4	5656.4	4282.2	3362.1	2833.2
	5	6388.7	4854.7	3853.5	3163.7
CFCF	1	1349.8	998.16	764.14	646.20
	2	1585.4	1197.8	912.76	773.78
	3	2392.9	1803.8	1392.8	1175.8
	4	3266.1	2483.6	1923.2	1601.9
	5	3522.2	2639.1	2071.3	1713.1
CSCS	1	2137.5	1518.1	1213.9	1035.7
	2	3888.6	2956.3	2294.9	1931.1
	3	4489.3	3381.0	2616.8	2179.7
	4	5591.4	4249.1	3342.3	2813.5
	5	6346.1	4831.1	3835.7	3147.1

4.4 Circular Plates or Discs

Circular plates with different cutouts are shown in Fig. 5

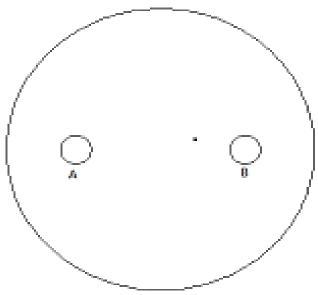

Fig. 5
Circular plates with two cutouts.

Table 16 shows the comparison of natural frequencies of FG circular disc of radius R=1 with circular holes of radius r=0.1 at locations A(x/R=-0.55, y/R=0) and B(x/R=0.55, y/R=0) respectively. Thickness of the plate is h=0.05. Table 17 shows the variation of natural frequencies of Al/Al_2O_3 FG circular plate of radius R=1 having two holes of radius r = 0.1 at locations A(x/R= -0.5, y/R=0) and B(X/r = 0.5, y/R=0) respectively. Thickness ratio of the plate is h/R=0.04, 0.06 and 0.08 respectively. The results for first ten modes are computed. For the FG plates with fully clamped external boundary condition, the frequencies in all ten modes decreases as the volume fraction index increases. This is expected, because a large volume fraction index means that a plate has a smaller ceramic component and thus its stiffness is reduced. The comparisons show good agreement with most of the differences being less than 10 % and the maximum difference being 4 %.

Table 16
Comparison of natural frequencies of FG circular disc with two circular cutouts (fully clamped for external boundaries)

	Mode									
r/R	1	2	3	4	5	6	7	8	9	10
0.1	202.04	413.77	421.03	668.48	684.35	782.04	954.79	973.43	1123.9	1248.7
[16]	206.01	420.34	427.10	684.40	694.61	782.63	991.48	1009.4	1173.5	-
0.15	209.07	430.69	446.52	695.70	741.85	808.34	961.35	1071.9	1131.8	1265.5
[16]	207.64	420.05	437.29	678.39	720.89	795.19	977.63	1051.3	1161.2	-
0.2	210.75	420.98	460.54	676.99	775.70	836.75	959.11	1105.6	1178.8	1254.4
[16]	204.84	411.34	444.64	659.38	727.09	823.53	946.89	1069.6	1176.0	-

Table 17
Variation of natural frequencies with the volume fraction index n for Al/Al_2O_3 FG circular plates with two circular holes (fully clamped for external boundaries)

		Mode									
h/*a*	*n*	1	2	3	4	5	6	7	8	9	10
0.04	0	464.14	946.79	959.26	1541.2	1566.2	1732.9	2239.6	2286.1	2554.0	2643.8
	0.5	410.78	832.36	870.92	1357.0	1377.4	1562.2	1911.7	1944.6	2269.0	2466.2
	1	336.07	685.58	692.46	1129.2	1154.5	1265.5	1600.2	1625.3	1848.2	1985.5
	2	291.81	568.65	582.18	936.01	992.84	1037.0	1345.5	1388.8	1513.8	1727.1
0.06	0	576.64	1144.5	1167.6	1837.6	1883.2	2079.2	2626.3	2677.9	3026.9	3125.2
	0.5	502.96	1000.4	1031.8	1656.6	1683.2	1850.3	2378.7	2419.1	2714.3	2895.6
	1	401.20	809.53	825.37	1343.2	1357.3	1522.4	1881.5	1937.5	2237.7	2357.0
	2	343.23	681.15	682.30	1105.3	1121.9	1239.0	1552.3	1594.4	1778.8	1902.7
0.08	0	757.34	1481.6	1507.7	2367.5	2384.6	2638.5	3244.0	3291.6	3307.3	3348.0
	0.5	584.96	1167.9	1195.9	1909.6	1924.6	2189.9	2673.0	2741.2	2939.1	2996.8
	1	468.48	913.55	941.98	1493.7	1512.4	1647.8	2123.0	2188.6	2394.1	2505.6
	2	392.02	764.37	792.53	1248.7	1267.0	1388.4	1766.3	1795.6	1990.1	2105.6

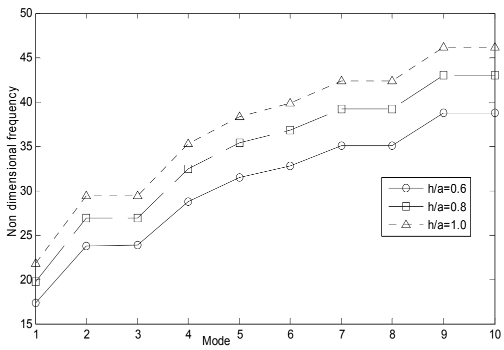

Fig. 6
Variation of non dimensional fundamental frequencies with different thickness ratio (h/a) of FG square plate with circular hole.

The variation of first ten natural frequencies with the thickness ratio of 0.6, 0.8 and 1.0 are shown in Fig. 6. This is observed from the figure that the variation in frequencies is almost constant as the thickness ratio increases from 0.6 to 0.8 and from 0.8 to 1.0. The variation of first ten natural frequencies with the different value of circular cut outs is shown in Fig. 7. From this figure, it is clear that the first frequency of vibration is only increasing as the size of cutouts increases and the rest of other frequencies are not showing the clear distinction in the variation of frequencies.

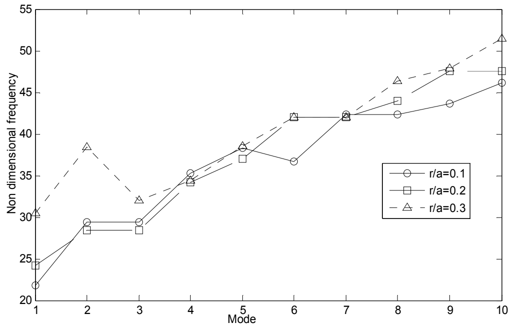

Fig. 7
Variation of non dimensional fundamental frequency with different circular cutouts in a square FG plate.

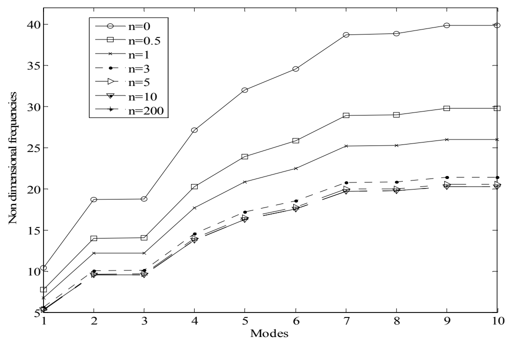

Fig. 8
Variation of non dimensional fundamental frequency with different volume fraction index for FG square plate with circular hole.

The first ten modes of frequencies with the different values of volume fraction index for FG square plate with circular cutouts are shown in Fig. 8. From this figure, it is observed that as the volume fraction index increases from 0 to 3, the frequencies continuously decreasing but as the volume fraction index increases from 5 to 200, the variation in the frequencies are negligible.

5 CONCLUSIONS

Free vibration problem of moderately thick functionally graded plates of different shapes with different cutouts has been solved using the finite element method. The methodology developed proved robust and efficient solutions for the vibration analysis of functionally graded plates for different boundary conditions. The effect of volume fraction exponent on natural frequencies have been carried out and found that a volume fraction exponent that ranges from 0 to 3 has a significant influence on the natural frequency of functionally graded plates. The natural frequency of moderately thick functionally graded plates decreases as the volume fraction index increases and increases as the thickness ratio increases. Extensive parametric studies with respect to the effect on natural frequencies have been carried out and the results have been plotted for future reference.

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