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Effect of Surface Energy on the Vibration Analysis of Rotating Nanobeam

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ABSTRACT

In this study, the free vibration behavior of rotating nanobeam is studied. Surface effects on the vibration frequencies of nanobeam are considered. To incorporate surface effects, Gurtin–Murdoch model is proposed to satisfy the surface balance equations of the continuum surface elasticity. Differential quadrature method is employed and in order to establish the accuracy and applicability of the proposed model, the numerical results are presented to be compared with those available in the literature. The effects of angular velocity, boundary conditions and surface elastic constants on the vibration characteristics are presented. Numerical results show that the softer boundary conditions cause an increase in the influence of the angular velocity on the nanobeam vibration frequencies.

Keywords : Vibration; Rotating nanobeam ; Differential quadrature method ; Surface energy.

1 INTRODUCTION

NANOTECHNOLOGY, encompassing nanoscience, engineering and technology, is a culmination of many facets of developments in the nanorealm. It involves imaging measuring, modeling, and manipulating matter in the nanoscale regime. Nanotechnology involves making useful devices with nanometer control in at least one dimension. Nanometer control infers that nanotechnology involves fabrication from molecular components. Therefore, in nanotechnology different molecular components must be integrated into a functional device. As a result, the benefit has been tangible in information technology, aerospace and defense industries, energy storage, environmental remediation and restoration, and human health, etc. Our quality of life has been changed enormously by all the current achievements.

In recent years, there has been significant interest in developing nanomechanical and nanoelectromechanical systems (NEMS), which are essentially sensors, actuators, machines, and electronics at the nanoscale [1, 2]. These devices can be used to measure extremely small displacements and forces that can contribute to novel technology developments with applications in medicine, computers, communications, etc.

Due to recent wide application of nanobeams in nanoelectromechanical systems (NEMS) and micro electromechanical systems (MEMS) technologies many attempts have been made for prediction of their mechanical properties. As controlled experiments in nanoscale are difficult and molecular dynamic simulations are highly computationally expensive, the theoretical modeling of nanomaterials has received the great deal of attention of

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scientific community. Thus, the continuum models have been proven to be important and efficient tools in the study of the nanostructures. Considerable sensitivity of static and dynamic behavior of nanosized materials to their mechanical properties exist which are not normally of concern in classic mechanics. For example, not taking surface effects or size effect into consideration, uncertainty in the prediction of the physical behaviors of nanomaterials will increase.

Classical continuum theories do not account for such size effects due to lack of material length scale parameters. Thus, size-dependent continuum theories such as classical couple stress theory [3, 4], modified couple stress theory [5, 6], nonlocal elasticity theory [7- 14], and strain gradient theory [15- 18] have been developed. In order to study the mechanical behaviors of nanostructures, the surface energy is one of the important fields which are investigated by researchers. To account for the effect of surfaces/interfaces on mechanical deformation, the surface elasticity theory is presented by modeling the surface as a two dimensional membrane adhering to the underlying bulk material without slipping [19, 20]. There are many studies related to the wave propagation, static, buckling and vibration analysis of nanobeams and carbon nanotubes (CNTs) based on different beam theories [21-31]. For example, Wang et al. [22] studied the surface buckling of a beam using the surface elasticity theory. The corresponding buckling wave number was analytically obtained in their work. They also reported that surfaces with positive surface elastic modulus may buckle under compression, while surfaces with negative surface elastic modulus are possible to wrinkle irrespective of the sign of surface strain. The nonlocal continuum mechanics is suitable for modeling sub micro-sized or nano-sized structures because it avoids enormous computational efforts when compared with discrete atomistic or molecular dynamics simulations. Alizada and Sofiyev [25] studied deformation and buckling of the beam with a nanocoating. In their paper, the behavior of the beam is modeled by the two-layered beam. The coating is carried out taking into account discreteness of the material. A stress analysis of a substrate coated by nanomaterials with a vacancy under uniform extension load is investigated by Alizada et al. [26]. In that paper, the extension of the coating is carried out by stretching of the first layer, which lies on the substrate and by the cohesive forces between the layers. Alizada and Sofiyev [27] obtained the Young's moduli in both directions for the two dimensional single crystal by using the continuum approach of continuum mechanics and taking into account only the interaction of neighboring atoms.

It is reported in literature that the surface effects include the surface elasticity, the surface stress, and the surface density. In addition, some literatures introduce relations for satisfying the balance condition between the nanostructure bulk and its surfaces. Most of the studies examine only the surface elasticity and stress effects and there is no work focusing on the influences of the surface density and satisfying the balance condition as well as the surface elasticity and stress effects. For example, Gheshlaghi and Hasheminejad [28] considered the surface elasticity and stress on the nonlinear free vibration of simply supported Euler-Bernoulli nanobeams without satisfying the balance condition. The similar situation can be found in Ref. [32] that the nonlinear free vibration of non-uniform nanobeams in the presence of the nonlocal effect as well as the surface elasticity and stress effects is studied using differential quadrature method (DQM). The other similar work is the one done by Mahmood et al. [33]. In this work, the static behavior of nonlocal Euler-Bernoulli nanobeams is considered using Galerkin finite element technique. Eltaher et al. [34] analyzed vibration analysis of nanobeam with consideration surface energy and nonlocal effect. They also reported that the surface properties have significant effect on the fundamental frequency in nano and micro regime. However, they did not investigate the effect of angular velocity, the effect of boundary conditions and material property effect on the frequency vibration of nanobeam. Pradhan and Murmu [35] studied vibration of rotating nanobeam. In that paper, the governing equation is derived by nonlocal continuum theory but they did not consider surface energy, the effect of different boundary condition, different cross section geometry and material properties.

From the literature survey, to date, no report has been found in the literature on the influence surface effect on the rotating nanobeam. On the other hand, the previous studies showed that this phenomenon have significant effects on the vibrational behaviors of nanobeams. Consequently, to perform an accurate vibration analysis, the formulation should include tense effect. Motivated by this idea, we aim to study the surface effects on the vibration response of the rotating nanobeam. Surface effects (surface elasticity, surface stress and surface density) on the vibration frequencies of nanobeam are considered. The influence of the angular velocity on the vibration frequency of the nanobeam is investigated. Differential quadrature method (DQM) is being used for the numerical solutions of the associated governing differential equations. Four different boundary conditions (i. e. Simply-Simply (SS), Clamp-Simply (CS), Clamp-Clamp (CC) and Clamp-Free (CF)) are investigated. The effects of (a) angular velocity, (b) surface elastic constants, (c) different cross section geometry, (d) different boundary conditions, (e) hub radius and (f) material properties on the vibration frequency of nanobeam are examined. The present work would be helpful while designing NEMS/MEMS devices using nanobeams.

2 MATHEMATICAL MODELING

At the micro/nanoscale, the fraction of energy stored in the surfaces becomes comparable with that in the bulk, because of the relatively high ratio of surface area to volume of nanoscale structures; therefore the surface and the induced surface forces cannot be ignored. The constitutive relations of the surface layers S^+ and S^- are given by Gurtin and Murdoch slipping [19, 20]. To take the surface stress effects into account, Gurtin and Murdoch model developed a theoretical framework based on the continuum mechanics, which has an excellent capability to incorporate the surface stress effects into the mechanical response of nanostrustures. This model has been widely used in different studies [36-40]. They proposed the following general and simple expression for surface stress–strain relation as:

$$\tau_{\alpha\beta}^{\pm}=\tau_0^{\pm}\delta_{\alpha\beta}+\left(\mu_0^{\pm}-\tau_0^{\pm}\right)\left(u_{\alpha,\beta}^{\pm}+u_{\beta,\alpha}^{\pm}\right)+\left(\lambda_0^{\pm}+\tau_0^{\pm}\right)u_{\gamma,\gamma}^{\pm}\delta_{\alpha\beta}+\tau_0^{\pm}u_{\alpha,\beta}^{\pm},\qquad \tau_{3\alpha}^{\pm}=\tau_0^{\pm}u_{3,\alpha}^{\pm} \tag{1}$$

where, α, β, γ=1, 2, $\tau_0^{\pm}$, are residual surface tensions under unconstrained conditions, $\lambda_0^{\pm}$ and $\mu_0^{\pm}$ are the surface Lame constants on the surfaces S^+ and S^- which can be determined from atomistic calculations [41], $\delta_{\alpha\beta}$ the Kronecker delta and $u_{\alpha}^{\pm}$ are the displacement components of the surfaces S^+ and S^-. In addition, the surface and bulk stresses satisfy the following balance equations of the surface layers in terms of the surface and bulk stress components as [19]

$$\tau_{xx,x}+T_x=\rho_0 \mathrm{u}_{,tt}^{s}\qquad\qquad \tau_{zx,x}+T_z=\rho_0 \mathrm{w}_{,tt}^{s} \tag{2}$$

where τ_{ij} are the components of bulk stresses at the boundary surface, T_x and T_z are the contact tractions on the contact surface between the bulk material and the surface layer. Moreover, u^s and w^s denotes the displacements at the boundary surface and ρ_0 is the density of the surface layer. In classical beam theory, the stress component σ_{zz} is neglected. However, σ_{zz} must be considered to satisfy the surface equilibrium equations of the Gurtin–Murdoch model. Following Lu et al. [42], it is assumed that the bulk stress σ_{zz} varies linearly through the nanobeam thickness. Therefore,

$$\sigma_{zz}=\frac{1}{2}\left(\sigma_{zz}^{+}+\sigma_{zz}^{-}\right)+\frac{z}{H}\left(\sigma_{zz}^{+}-\sigma_{zz}^{-}\right)=\frac{1}{2}\left(\tau_{zx,x}^{+}-\tau_{zx,x}^{-}\right)+\frac{z}{H}\left(\tau_{zx,x}^{+}+\tau_{zx,x}^{-}\right) \tag{3}$$

where σ_{zz}^{+} and σ_{zz}^{-} are stresses at the top and bottom fibers, respectively. Assuming the displacement is continuous with no slipping between the surface layer and the bulk. So, the displacement field at a point of the Euler–Bernoulli beam can be expressed as,

$$u(x,z,t)=u_0(x,t)-z\frac{\partial w_0(x,t)}{\partial x},\qquad w(x,z,t)=w_0(x,t) \tag{4}$$

where u_0 and w_0 are the axial and the transverse displacement of any point on the mid-plane, and t denote time. Since, the axial displacement at the mid-plane u_0 has a very small effect, we neglect it in the current analysis. The geometrical fit condition, strain–displacement relation, in case of small deformation can be described by:

$$\varepsilon_{xx}=\frac{\partial u}{\partial x}=-z\frac{\partial^2 w(x,t)}{\partial x^2} \tag{5}$$

By inserting Eq. (2) into Eq. (3) we have

$$\sigma_{zz}=\frac{1}{2}\left(\tau_0 w_{xx}^{+}-\tau_0 w_{xx}^{-}-\rho_0\ddot{w}^{+}+\rho_0\ddot{w}^{-}\right)+\frac{z}{H}\left(\tau_0 w_{xx}^{+}+\tau_0 w_{xx}^{-}-\rho_0\ddot{w}^{+}-\rho_0\ddot{w}^{-}\right) \tag{6}$$

where w^+ and w^- are vertical displacements of the top and bottom fibers, respectively. Based on the Euler–Bernoulli beam theory, the vertical displacements of the top and bottom fibers are identical because they are not function of thickness. So Eq. (6) can be reduced to

$$\sigma_{zz} = \frac{2z}{H}(\tau_0 \frac{\partial^2 w}{\partial x^2} - \rho_0 \frac{\partial^2 w}{\partial t^2}) \tag{7}$$

As a result of the interaction between the surface layer and bulk material, the contact tractions T_x and T_z exist on the contact surface between the bulk material and the surface layer.

$$\frac{\partial^2 M^l}{\partial x^2} + \frac{\partial}{\partial x}(\int_s T_x z ds) + \int_s T_z ds + q(x,t) - \frac{\partial}{\partial x}(N \frac{\partial w}{\partial x}) + \int_A \rho \frac{\partial^2 w}{\partial t^2} dA = 0 \tag{8}$$

Here, the moment stress resultant relation of the cantilever is expressed as:

$$M = \int_A z \sigma_{xx} dA \tag{9}$$

where N is the centrifugal tension force due to the rotation of the cantilever. The force $N(x)$ at a distance x from the origin is given by

$$N(x) = \int_x^l (\rho A)\bar{\omega}^2 (r+x) dx + \underbrace{\int_x^l (\rho_0 b)\bar{\omega}^2 (r+x) dx}_{\text{Upper Layer of nanobeam}} + \underbrace{\int_x^l (\rho_0 b)\bar{\omega}^2 (r+x) dx}_{\textit{Lower}\text{ layer of nanobeam}} \tag{10}$$

Here $\bar{\omega}$ denotes the angular velocity of the cantilever r is known as the hub radius and is the distance from the origin of rotation to inner end of the cantilever (Fig. 1). The second and the third terms in Eq. (10) denote the effect of angular velocity on the upper and lower layers of nanobeam, respectively. By inserting Eq. (2) into Eq. (8) ,the following equilibrium equation is obtained

$$\begin{aligned} &\frac{\partial^2 M^l}{\partial x^2} - \frac{\partial}{\partial x}(\int_s \frac{\partial \tau_{xx}}{\partial x} z ds) - \int_s \frac{\partial \tau_{zx}}{\partial x} ds + q(x,t) - \frac{\partial}{\partial x}(N \frac{\partial w}{\partial x}) \\ &+ \int_A \rho \frac{\partial^2 w}{\partial t^2} dA + \frac{\partial}{\partial x}(\int_s \rho_0 \frac{\partial^2 u^s}{\partial t^2} z ds) + \int_s \rho_0 \frac{\partial^2 w^s}{\partial t^2} ds = 0 \end{aligned} \tag{11}$$

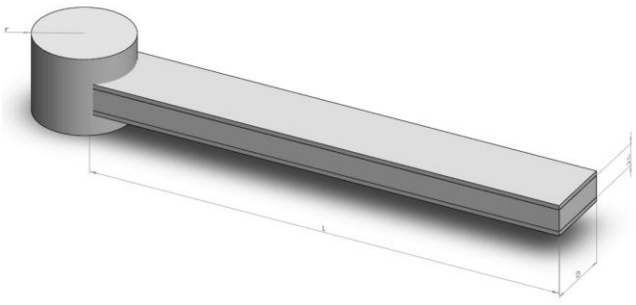

Fig.1
Configuration of a rotating nanobeam with clamp-free boundary condition.

Assuming a homogenous isotropic material and neglecting any residual stress in the bulk material due to surface tension, the relevant bulk stress–strain relation of the beam, in the context of local elasticity, can be expressed as:

$$\sigma_{xx} = E\,\varepsilon_{xx} + \upsilon\sigma_{zz} \tag{12}$$

Here, E is the Young modulus of material and υ is the Poisson's ratio of bulk material. By substituting Eq. (5) and (7) into Eq. (12), then into Eq. (9), the following equations can be obtained

$$M = -EI\frac{\partial^2 w}{\partial x^2} + \frac{2\upsilon I}{H}\left(\tau_0\frac{\partial^2 w}{\partial x^2} - \rho_0\frac{\partial^2 w}{\partial t^2}\right) \tag{13}$$

If the top and bottom layers have the same material properties, the stress–strain relations of the surface layers, i.e. Eq.(1), can be reduced following relation for nanobeams

$$\tau_{xx} = \tau_0 - (2\mu_0 + \lambda_0)z\frac{\partial^2 w}{\partial x^2}, \qquad \tau_{zx} = \tau_0\frac{\partial w}{\partial x} \tag{14}$$

Substituting Eq. (10), Eq. (13) and Eq. (14) into Eq. (11) yields the following governing equation of the local Euler–Bernoulli nanobeam, including the surface effect, in terms of the deflection

$$\begin{aligned}&\left(EI + (2\mu_0 + \lambda_0)I^*\right)\frac{\partial^4 w}{\partial x^4} - \frac{2\upsilon I}{H}\left(\tau_0\frac{\partial^4 w}{\partial x^4} - \rho_0\frac{\partial^4 w}{\partial x^2\partial t^2}\right) - \tau_0 s^*\frac{\partial^2 w}{\partial x^2} + q(x,t)\\ &+\left(\rho A + \rho_0 s^*\right)\frac{\partial^2 w}{\partial t^2} - \left(\rho A + \rho_0 s^*\right)\bar{\omega}^2\left(-(r+x)\frac{\partial w}{\partial x} + \begin{pmatrix} rl + \dfrac{l^2}{2} \\ -rx - \dfrac{x^2}{2}\end{pmatrix}\frac{\partial^2 w}{\partial x^2}\right) = 0\end{aligned} \tag{15}$$

where I is the moment of inertia of the beam cross section, I^* is the perimeter moment of inertia. In the case of beams with a rectangular cross section of height 2h and width *b* and *a* circular cross section of diameter *D*, we have

$$I = \begin{cases} 2bh^3/3 \\ \pi D^4/64\end{cases} \qquad I^* = \begin{cases} 2bh^2 + 4h^3/3 \\ \pi D^3/8\end{cases} \qquad S^* = \begin{cases} 2b \\ \pi D/2\end{cases} \qquad H = \begin{cases} 2h \\ D\end{cases} \tag{16}$$

Note that if σ_{zz} is neglected in the formulation, the second terms on the left-hand sides of Eq. (15) vanish. If the surface stress effect is completely neglected (μ_0, λ_0, ρ_0 and τ_0 are zero), then Eq. (15) reduce to the governing equations of classical Euler–Bernoulli beam theory [43]. It should be noted that if there is no surface residual stress (i.e., only τ_0 is zero) then the governing equations are identical to the classical governing equations with bending stiffness modified due to the contributions from the surface elastic constants and the beam inertia terms modified due to the contributions from surface layer density. If the contributions of σ_{zz}, τ_{zx} surface density and surface stresses on the vertical sides of a rectangular beam and angular velocity are neglected, then Eq. (15) reduces to the following governing equation, which is identical to the governing equation of Gurtin et al . [44]

$$\left(EI + (2\mu_0 + \lambda_0)I^{**}\right)\frac{\partial^4 w}{\partial x^4} + q(x,t) + \rho A\frac{\partial^2 w}{\partial t^2} = 0 \tag{17}$$

Here $I^{**} = 2bh^2$.

The vertical force equilibrium equation including the surface effects can be expressed as follow [34]

$$Q = \frac{dM}{dx} + \int_s T_x z ds - N \frac{\partial w}{\partial x} \tag{18}$$

where Q is the stress resultants defined as follow

$$\begin{aligned} Q_x &= -EI \frac{\partial^3 w}{\partial x^3} + \frac{2\upsilon I}{H}\left(\tau_0 \frac{\partial^3 w}{\partial x^3} - \rho_0 \frac{\partial^3 w}{\partial x \partial t^2}\right) + \left((2\mu_0 + \lambda_0) I^*\right) \frac{\partial^3 w}{\partial x^3} \\ &-\rho A \bar{\omega}^2 \left((rl + \frac{l^2}{2} - rx - \frac{x^2}{2}) \frac{\partial w}{\partial x} \right) \end{aligned} \tag{19}$$

It is obviously clear that if the angular velocity is neglected, ($\bar{\omega}$ is set to zero), Eq. (25), Eq. (27) and Eq. (29) are reduced to that of Euler-Bernoulli beam in presence of the surface effects [34]. For the nanobeam, the boundary conditions of SS, CS, CF and CC are given, respectively, by

$$\begin{aligned} &W(0) = M_{xx}(0) = 0;\ W(l) = M_{xx}(l) = 0 \\ &W(0) = \frac{\partial w}{\partial x}(0) = 0;\ W(l) = M_{xx}(l) = 0 \\ &w(0) = \frac{\partial w}{\partial x}(0) = 0;\ M_{xx}(l) = Q_{xx}(l) = 0 \\ &W(0) = \frac{\partial w}{\partial x}(0) = 0;\ W(l) = \frac{\partial w}{\partial x}(l) = 0 \end{aligned} \tag{20}$$

3 SOLUTION BY DIFFERENTIAL QUADRATURE METHOD

In addition to this, in a large number of practical applications where only reasonably accurate solutions at few specified physical coordinates are of interest, conventional numerical methods such as finite element or finite difference method require a large number of grid points and a large computer capacity. Among a variety of numerical methods, the finite element method is by far the most effectively and widely used method. Furthermore, the finite element method is still an effective method especially for systems with complex geometry and load conditions or applications with non-linear behavior and it has many successful applications. In seeking a more efficient numerical method that requires fewer grid points yet achieves acceptable accuracy, the method of differential quadrature (DQ) was developed by Shu [50]. Since then, applications of the differential quadrature method to various engineering problems have been investigated and their success has shown the potential of the method as an attractive numerical analysis technique [51-64]. The stability analysis of beams may be either in closed form or approximate. The closed form solutions consist of techniques for seeking direct solutions to the governing differential equation of beams. A closed form or rigorous solution of beams can be obtained for only a limited number of cases. For the majority of practical problems, a closed form namely analytical solution either cannot be obtained or is of such a complicated nature that it can be applied only with great difficulty in a practical computation. For many situations, numerical solutions are the only method that can be employed. The idea of the differential quadrature method is to quickly compute the derivative of a function at any grid point within its bounded domain by estimating a weighted linear sum of values of the function at a small set of points related to the domain. Exact solutions for beam problems are rather difficult to obtain, except for a few simple cases. In many cases, one may have to resort to various approximate, namely numerical methods. Each method has its own advantages and disadvantages. Among the various methods proposed in recent times, one can cite the finite differences, finite element and boundary element methods as the most efficient and universal methods for solving variant type beam problems.

As mentioned, DQM has been found to be an efficient numerical technique for the solution of initial and boundary value problems [65]. Since DQ technique provides simple formulation and low computational cost, it has been widely used for the analysis of mechanical behaviors of the structural elements at large scale, such as dynamic and stability problems. In recent years, many researchers used DQ approach in solving the governing equations of

nanostructures. Mohammadi et al. [57] applied new version of differential quadrature method for vibration analysis of embedded single-layer circular nanoplate. Farajpour et al. [58] used DQM for the buckling of orthotropic micro/nanoscale plate under linearly varing in-plane load. Danesh et al. [45] used DQM for the vibration analysis of tapered nanorod with different boundary conditions.

The basic idea of the DQM is that the derivative of a function with respect to a space variable at a given sampling point is approximated as a weighted linear sum of the sampling points in the domain of that variable. In this method, at a given grid point x, the derivatives of a function can be approximated as:

$$\left.\frac{\partial^m u}{\partial x^m}\right|_{x=x_i} = \sum_{k=1}^{N_x} C_{ik}^{(m)} u(x_k, t) = \sum_{k=1}^{N_x} C_{ik}^{(m)} u_{kj}(t) \tag{21}$$

where N_x is the number of grid points along the x-direction.

The method developed by Shu and Richard [66] is claimed to be computationally more accurate than other methods [65]. According to Shu and Richard rule [66], the weighting coefficients of the first-order derivatives in ζ direction $(\zeta = x)$ are determined as:

$$C_{ij}^{(1)} = \begin{cases} \dfrac{M(\zeta_i)}{(\zeta_i - \zeta_j) M(\zeta_j)} & \text{for } i \neq j \\ -\sum\limits_{j=1, i\neq j}^{N_\zeta} C_{ij}^{(1)} & \text{for } i = j \end{cases} \qquad i,j=1,2,...,N_\zeta \tag{22}$$

where

$$M(\zeta_i) = \prod_{j=1, i\neq j}^{N_\zeta} (\zeta_i - \zeta_j) \tag{23}$$

In order to evaluate the weighting coefficients of higher-order derivatives, recurrence relations are derived as:

$$C_{ij}^{(r)} = r\left[C_{ij}^{(r-1)} C_{ij}^{(1)} - \frac{C_{ij}^{(r-1)}}{(\zeta_i - \zeta_j)}\right] \text{ For } i, j = 1,2,...,N_\zeta, \ i \neq j \text{ and } \ 2 \le r \le N_\zeta - 1 \tag{24}$$

$$C_{ii}^{(r)} = -\sum_{j=1,\ i\neq j}^{N_\zeta} C_{ij}^{(r)} \text{ For } i = 1,2,...,N_\zeta \text{ and } \ 1 \le r \le N_\zeta - 1 \tag{25}$$

The natural and simplest choice of the grid points is equally spaced points in the direction of the coordinate axes of the computational domain. It was demonstrated that non-uniform grid points gives a better results with the same number of equally spaced grid points. In this paper, we choose these set of grid points in terms of natural coordinate directions ξ as:

$$\xi_i = \frac{1}{2}\left(1 - \cos\left(\frac{(i-1)\pi}{(N_\xi - 1)}\right)\right) \text{For } i = 1,2,...,N_\xi \tag{26}$$

A nanobeam is considered. The geometric properties of the nanobeam are denoted by length *l,* width *b,* thickness 2*h*. For convenience and generality, we introduce the following non-dimensional parameters

$$W=\frac{w}{a},\ \xi=\frac{x}{l},\ \delta=\frac{r}{l},\ \kappa=\frac{(2\mu_0+\lambda_0)I^*}{EI},\ \hbar=\frac{\rho_0 s^*}{\rho A},\ \not\prec=\frac{\tau_0 s^* l^2}{EI},$$
$$\hbar=\frac{\upsilon\tau_0}{EH},\ \chi=\frac{\upsilon\rho_0 I}{\rho A l^2 H},\ \Omega^2=\frac{\rho h\omega^2 l^4}{EI},\ \bar{\Omega}^2=\frac{\rho h\bar{\omega}^2 l^4}{EI}. \tag{27}$$

In the present study, it is assumed that the nanobeam is free from any transverse loadings $(q=0)$. The normal vibrations is harmonic. Therefore, the deflection in normal vibrations of nanobeam can be expressed as $w(x,y,t)=W(x,y)e^{i\omega_n t}$ where ω_n is the natural frequency and $i^2=-1$. Substitution of the above expressions and Eq. (27) into Eq. (17) yields a four order partial differential equation involving natural mode $W(x,y)$

$$\begin{aligned}&(1+\kappa)\frac{\partial^4 W}{\partial\xi^4}-2\hbar\frac{\partial^4 W}{\partial\xi^4}-2\chi\Omega^2\frac{\partial^2 W}{\partial\xi^2}-\not\prec\frac{\partial^2 W}{\partial\xi^2}-(1+\hbar)\Omega^2 W\\&-(1+\hbar)\bar{\Omega}^2\left(-(\delta+\xi)\frac{\partial W}{\partial\xi}+(\delta+0.5-\xi\delta-0.5\xi^2)\frac{\partial^2 W}{\partial\xi^2}\right)=0\end{aligned} \tag{28}$$

Eq. (28) can be solved by DQM approach for various boundary conditions. The computational domain of the nanobeam is $0\le\xi\le 1$. Making use of Eq. (21) and incorporating the boundary conditions by modified weighting coefficient method [50] we write Eq. (28) in non-dimensional form

$$\begin{aligned}&(1+\kappa)\sum_{k=1}^{N_\xi}C_{ik}^{(4)}W_{k,j}-2\hbar\sum_{k=1}^{N_\xi}C_{ik}^{(4)}W_{k,j}-2\chi\Omega^2\sum_{k=1}^{N_\xi}C_{ik}^{(2)}W_{k,j}-\not\prec\sum_{k=1}^{N_\xi}C_{ik}^{(2)}W_{k,j}\\&-(1+\hbar)\Omega^2 W_{i,j}-(1+\hbar)\bar{\Omega}^2\begin{pmatrix}-(\delta+\xi)\sum_{k=1}^{N_\xi}C_{ik}^{(1)}W_{k,j}\\+(\delta+0.5-\xi\delta-0.5\xi^2)\sum_{k=1}^{N_\xi}C_{ik}^{(2)}W_{k,j}\end{pmatrix}=0\end{aligned} \tag{29}$$

C is the weighting coefficient matrix in the x directions, respectively. After implementation of the boundary conditions, Eq. (29) can be written in matrix form as:

$$[K_{total}-\Omega^2 M_{total}]\{W\}=0, \tag{30}$$

The Eq. (30) can be solved by a standard eigenvalue solver. From this solution, the natural frequencies of the rotating nanobeam are obtained.

4 RESULTS AND DISCUSSION

In this section, selected numerical results are presented to demonstrate the salient features of the mechanical behavior of nanoscale beams with a rectangular cross section, as well as the effects of surface energy and beam boundary conditions. The bulk and surface elastic constants of selected materials have been obtained by Miller and Shenoy [67] and Shenoy [41] by using the embedded atom method proposed by Daw and Baskes [68]. Beams made of aluminum (Al) and silicon (Si) are considered in the numerical study. The relevant bulk material properties are E= 90 GPa, ν = 0.23 for Al and E= 107 GPa, ν = 0.33 for Si [67]. The surface material properties are $\mu_0=-5.4251\ N/m$, $\lambda_0=3.4939\ N/m$, $\tau_0=0.5689\ N/m$ and $\rho_0=5.46\times10^{-7}\ kg/m^2$ for Al; and $\mu_0=-2.7779\ N/m$, $\lambda_0=-4.4939\ N/m$, $\tau_0=0.6056\ N/m$ and $\rho_0=3.1688\times10^{-7}\ kg/m^2$ for Si [67]. In the absence of any previous solution for the present class of problems, the numerical solutions obtained from the current formulation for negligible values of angular velocity were compared with the surface beam solutions to ensure that appropriate limiting solutions were obtained.

A computer code is developed in MATLAB based on Eq. (30). As DQ results are sensitive to lower grid points, a convergence test is performed to determine the minimum number of grid points required to obtain stable and accurate results for Eq. (30). The analysis is carried out by length of nanobeam 10 nm, the non-dimensional angular velocity 5, the hub radius 1nm and the width of Si nanobeam 3nm. According to Fig. 3, present solution is convergent. It is clearly seen from the figure that tenth number of grid points ($N_\xi = 10$) are sufficient to obtain the accurate solutions for the present analysis.

To confirm the reliability of the present formulation and results, comparison studies are conducted for the natural frequencies of the nanobeams without considering rotating effect.

Here, the validity of the natural frequencies of nanobeam is investigated considering the surface effects and a comparative study for evaluation of the first four natural frequencies between the present solution and the results given by Liu and Rajapakse [31] and Hosseini-Hashemiet al. [69] is carried out in Table 1. for Al and Si nanobeams with SS, CC and CF boundary conditions.

In order to compare the non-dimensional angular velocity on the vibration response of the nanobeam, the percentage difference in the frequency vibration is defined as follows:

$$Percentage\ \text{difference}=\left|\frac{\text{Vibration}\ frequency_{\ \bar{\Omega}\neq 0}-\text{Vibration}\ frequency_{\bar{\Omega}=0}}{\text{Vibration}\ frequency_{\bar{\Omega}=0}}\right|$$

To investigate the influence of residual surface energy on the higher angular velocity of nanobeam, the variation of difference percent of frequency versus the non-dimensional angular velocity for various values of residual surface elastic modulus are shown in Fig. 2. Fig. 2 shows the solutions for Si [69] and geometrical parameters of nanobeam are assumed to be L=50 nm, b=3 nm, r=1 nm, h=b. In this figure, four cases of boundary conditions are considered. It is clearly seen that size effects increase by increasing non-dimensional angular velocity from 0 to 5 [35]. Also, it is interesting to note that as the surface residual stress increases, the percent difference decreases. In addition, Fig. 3 shows that the influences of the non-dimensional angular velocity on the natural frequency of Si nanobeam follows the order CC<CS<SS<CF, implying that using the softer boundary conditions cause an increase in the influence of the non-dimensional angular velocity. A final point to note is that the residual surface stress effect increases with increasing softness of boundary condition. It is seen from these results that the difference between the difference percent curves for nanobeam with CF boundary condition is higher than that for CC case and this observation means that the difference between any two residual surface stress curves decreases with increasing the stiff of boundary conditions.

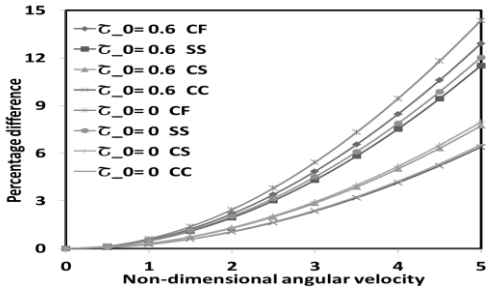

Fig.2
Change of percentage difference frequency of nanobeam with non-dimensional angular velocity for various values of residual surface elastic modulus.

Table 1
Comparison of the forth natural frequency of nanobeams obtained from the present model with some known results available in the literature for different boundary conditions.

Material	Boundary conditions	1st (GHz)			2nd (GHz)			3rd (GHz)			4th (GHz)		
		Ref. [28]	Ref. [69]	Present	Ref. [28]	Ref. [69]	Present	Ref.[28]	Ref. [69]	Present	Ref.[28]	Ref. [69]	Present
Al	CF	0.75	0.75	0.75	2.90	2.90	2.90	6.82	6.82	6.82	12.69	12.69	12.69
	SS	1.45	1.45	1.45	4.47	4.47	4.47	9.39	9.39	9.39	16.27	16.27	16.27
	CC	2.52	2.52	2.52	6.37	6.37	6.37	12.49	12.49	12.49	20.35	20.35	20.35
Si	CF	0.86	0.86	0.86	3.34	3.34	3.34	7.93	7.93	7.93	14.81	14.81	14.81
	SS	1.66	1.66	1.66	5.19	5.19	5.19	10.96	10.96	10.96	19.02	19.02	19.02
	CC	2.94	2.94	2.94	7.64	7.64	7.64	14.59	14.59	14.59	23.83	23.83	23.83

The variation of percent frequency of the nanobeam with the hub radius for different values of angular velocity is studied. The abovementioned variation is depicted in Figs. 3 and 4. Five values of angular velocity are considered 1, 2, 3, 4 and 5. The boundary condition in Figs. 3 and 4 are considered CC and CF, respectively. It is observed from these figures that as the non-dimensional angular velocity increases the fundamental frequency increases. In addition, it is observed that for any angular velocity, with increase in hub radius the frequency fraction increases. This is because of the fact that the nanobeam becomes stiffer with increase in hub radius. Further, the rate of increase in percent frequency for angular velocity 5 is more than that of the angular velocity 1. The present observation is attributed to the increasing coupling effect of hub radius and the angular velocity. A final point to consider is that the effect of the hub radius for CC boundary condition is more significant than other boundary conditions in every angular velocity.

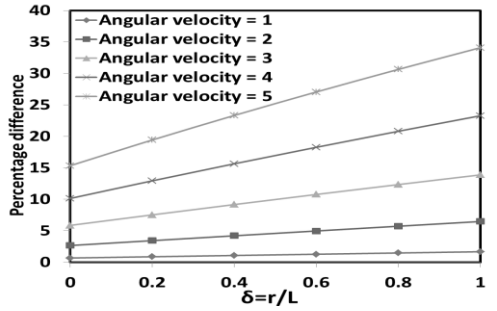

Fig.3
Change of percentage difference frequency of nanobeam with non-dimensional hub radius (δ) for various angular velocities and clamp-clamp boundary condition.

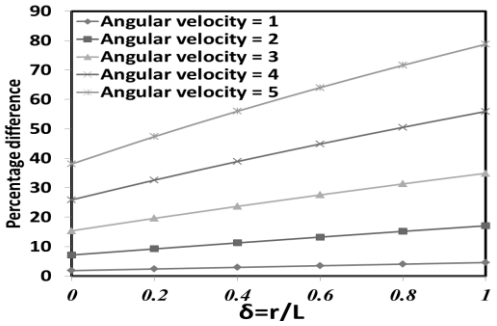

Fig.4
Change of percentage difference frequency of nanobeam with non-dimensional hub radius (δ) for various angular velocities and simply-simply boundary condition.

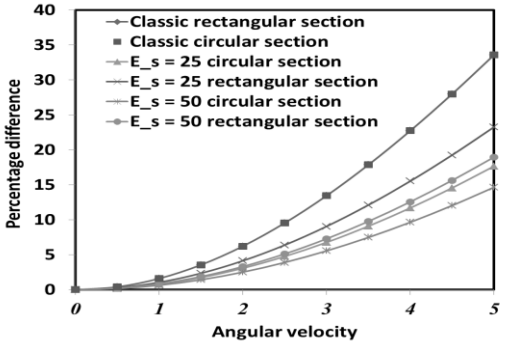

Fig.5
Change of percentage difference frequency of nanobeam with non-dimensional angular velocity for various cross section geometry and surface elastic modulus.

To investigate the influence of surface energy and geometry of cross section on the vibration frequency of nanobeam, the variation of percent frequency versus the angular velocity for various values of surface elastic modulus ($E_s = 2\lambda_0 + \mu_0$) and two shape of cross section (circular and rectangular) are shown in Fig. 5. The geometrical parameters of Si nanobeam are assumed to be L=50 nm, D=4 nm, h= b=3 nm and r=1 nm. It is found that as the surface elastic modulus increases from 0 to 50 N/m, the percent frequency decreases. This means that the surface effects are more significant in lower angular velocity of nanobeam. Moreover, the gap between the curves of circular and rectangular cross section widens with increasing the angular velocity in all of the surface constants. This observation means that the effect of angular velocity for circular cross section nanobeam is less important than the

rectangular cross section nanobeam. A final point to consider is that it is no difference between the nanobeam with circular cross section and rectangular cross section in classic theory.

5 CONCLUSIONS

The vibration analysis of rotating nanobeam is investigated by using surface energy. The surface effects are included by Gurtin–Murdoch model to satisfy the surface balance equations. Equation of motion based on surface energy has been derived. Numerical solutions are obtained for the non-dimensional frequency of nanobeam by employing the differential quadrature method. The effects of the angular velocity, surface elastic constants, boundary conditions and cross section geometry of nanobeam on the vibration frequency were investigated for four cases boundary conditions. It is found that the influence of the surface effects are quite significant in the vibration analysis of rotating nanobeam and cannot be neglected. From the results of the present work, the following conclusions are noticeable:

- The surface-to-bulk ratio of nanobeams is an important parameter in determining the influence of the surface effects so that it can be said that if an increase in the nanobeam length leads to decreasing the surfaceto-bulk ratio, the influence of the surface effects on frequency ratios will be diminished. Otherwise, the influence of the surface effects on frequency ratios will increase as the nanobeam length increases.
- The influences of the surface effects depend on the types of boundary conditions.
- The softer boundary conditions cause an increase in the influence of the non-dimensional angular velocity.
- It is observed that for any angular velocity, with increase in hub radius the frequency fraction increases.
- The effect of angular velocity for circular cross section nanobeam is less important than the rectangular cross section nanobeam.

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