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Torsion of Poroelastic Shaft with Hollow Elliptical Section

M. Jabbari *, M.F. Khansanami

Department of Mechanical Engineering, Islamic Azad University, South Tehran Branch, Iran

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ABSTRACT

In this paper torsion of hollow Poroelastic shaft with Elliptical section is developed. Using the boundary equation scheme. It looks for a stress function where satisfied Poisson equation and vanishes on boundary. It also analyzed stress function and warping displacement for the hollow elliptical section in Poroelastic shaft. At the end, the result of elastic and poroelastic shaft in warping displacement and stress function is compared.

Keywords : Torsion; Stress function; Warping ; Poroelastic ; Inhomogeneous.

1 INTRODUCTION

TORSION is one of the interesting fields for researches. Prandtl [1] introduced the stress function of the Saint-Venant torsion and the method of membrane analogy [2]. In 1903, Prandtl presented a membrane analogy for torsional analysis and proved the accuracy and efficiency of his approximation. Baron [3] studied torsion of hollow tubes by multiplying the connected cross sections. He used an iterative method to satisfy the equilibrium and compatibility equations. A computational method for calculating torsional stiffness of multi-material bars with arbitrary shape was studied by Li et Al. [4]. In this work, they considered additional compatibility and equilibrium equations in common boundaries of different materials in their formulation and took good results. Mijak [5] considered a new method to design an optimum shape in beams with torsional loading. In his work, cost function was torsional rigidity of the domain and constraint was the constant area of the cross-section while shape parameters were co-ordinates of the finite element nodes along the variable boundary. The problem was solved directly by optimizing the cost function with respect to the shape parameters. He solved this problem using finite elements (FE method. A method based on finite elements for torsional analysis of prismatic bars by modeling only a small slice of the bar was published by Jiang et al. [6]. Another work related to the torsion in prismatic bars was introduced by Louis et al. [7], in which they presented a solution using a power fit model for the torsion problem of a rectangular prismatic bar. Recently, Doostfatemeh et al. [8] obtained a closed-form approximate formulation for torsional analysis of hollow tubes with straight and circular edges. In this work, the problem was formulated in terms of Prandtl's stress function. Also, accuracy of the formulas was verified by accurate finite element method solutions. In recent years, the composition of several different materials has been often used in structural components in order to optimize responses of the structures subjected to thermal and mechanical loads.

Since these pioneering works established the theory of torsion and solved many problems in engineering application, the torsion of a straight bar became a classical problem in the theory of elasticity, which was also presented as a numerical example in a seminal paper about the finite element method by Courant [9]. Some analytical solutions of the homogeneous section with various shapes are available in the literatures [10, 11]. The torsion of composite shafts has attracted many researchers' attention in the development and application of

*Corresponding author. Tel.: +98 912 2447595.
E-mail address: mohsen.jabbari@gmail.com (M. Jabbari).

composite materials. Muskhelishvilli [12] presented the governing equation and boundary condition of the torsion of composite bars and its solution in Fourier series for composite section with two sub-rectangles. This solution was extended later for multiple rectangular composite section by Booker and Kitipornchai [13]. Kuo and Conway [14–17] analyzed the torsion of the composite sections of various shapes. Packham and Shail [18] extended their work on two-phase fluid to the torsion of composite shafts. Ripton [19] investigated the torsional rigidity of composite section reinforced by fibers. Chen et al. [20] also analyzed exactly the torsion of composite bars. Apart from these analytical methods, numerical methods have also been employed to solve the torsion of the straight bars. Ely and Zienkiewicz [21] firstly solved the Poisson’s equation of the Prandtl’s stress function using finite difference method and investigated the rectangular section with and without holes. Herrmann [22] utilized the finite element method to calculate the warping function of the torsion of irregular sectional shapes. The boundary element method was applied to solve the boundary integral equation of the warping function of the torsion in Refs. [23–26].

Recently Poro's whose properties continuously vary with spatial coordinates, have been developed to overcome the problems associated with interfaces in traditional composite materials due to the abrupt change of the materials properties [27]. Although Ely and Zienkiewicz [21] and Plunkett [28] presented the governing equation of the torsion of inhomogeneous material before the introduction of the conception of Poros, it was paid little attention as there is no engineering significance at that time. Once the FGMs were fabricated and applied in engineering practice, Rooney and Ferrari [29,30] and Horgan and Chan [31] resumed the research on the torsion of FGM bars. More recently, Tarn and Chang [32] obtained the exact solution of the torsion of orthotropic inhomogeneous cylinders and also analyzed the end effect. In particular, the torsion problem for inhomogeneous isotropic elastic materials has been investigated recently in [33].

Poroelasticity is a theory that models the interaction of deformation and fluid flow in a fluid-saturated porous medium. The deformation of the medium influences the flow of the fluid and vice versa. The theory was proposed by Biot [34-36]. As a theoretical extension of soil consolidation models developed to calculate the settlement of structures placed on fluid-saturated porous soils. The historical development of the theory is sketched by de Boer (1996). The theory has been widely applied to geotechnical problems beyond soil consolidation, most notably problems in rock mechanics. there has been recently a growing interest in the context of non-homogeneous and/or anisotropic shaft. Arghavan and Hematiyan[37] analyzed the torsion of functionally graded hollow tubes. Batra [38], Horgan and Chan [39] work on Torsion of a functionally graded cylinder.; Rooney and Ferrari [40]; Udea et al [41] and Yausisi and Shigeyasu [42] analyzed the torsion and flexure of inhomogeneous elements. Sofiyev A.H worked on The torsional buckling analysis of cylindrical shells with material non-homogeneity in thickness direction under impulsive loading and torsional buckling of cross-ply laminated orthotropic composite cylindrical shells subject to dynamic loading [43,44].

In this paper, we analyze torsion in Poroelastic shaft and calculate mechanical effects on circles and ellipses section shaft that made up of inhomogeneous materials. Boundary condition includes unbalanced torsion load that is linear in elastic condition and shift in internal and external surfaces in X ,Y direction. The problem will be solved based on direct Euler equation and by separation of variables. Finally differential equation will be solved. At the end of the paper comparison made between Elastic and Poroelstic shaft by use of graphs and plots, is provided and will be achieved.

2 GOVERNING EQUATIONS [45]

2.1 Stress-stress function formulation

The stress formulation leads to the use of a stress function similar to the results of the plane problem discussed. Using the displacement form, the strain-displacement relations give the following strain field:

$$e_x = e_y = e_z = e_{xy} = 0$$
$$e_{xz} = \frac{1}{2}\left(\frac{\partial w}{\partial x} - \alpha y\right) \quad , \quad e_{yz} = \frac{1}{2}\left(\frac{\partial w}{\partial y} + \alpha x\right) \tag{1}$$

The corresponding stresses follow from Hooke’s law:

$$\sigma_x = \sigma_y = \sigma_z = \tau_{xy} = 0 \tag{2}$$

$$\tau_{xz} = \mu(\frac{\partial w}{\partial x} - \alpha y) \tag{3}$$

$$\tau_{yz} = \mu(\frac{\partial w}{\partial y} + \alpha x) \tag{4}$$

Note the strain and stress fields are functions only of *x* and *y*. For this case, with zero body forces, the quilibrium equations reduce to

$$\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} = 0 \tag{5}$$

Rather then using the general Beltrami-Michell compatibility equations, it is more direct to develop a special compatibility relation for this particular problem. This is easily done by simply differentiating (3) with respect to *y* and (4) with respect to *x* and subtracting the results to get

$$\frac{\partial \tau_{xz}}{\partial y} - \frac{\partial \tau_{yz}}{\partial x} = -2\mu\alpha \tag{6}$$

This represents an independent relation among the stresses developed under the continuity conditions of w(x,y). Relations (5) and (6) constitute the governing equations for the stress formulation. The coupled system pair can be reduced by introducing a stress function approach. For this case, the stresses are represented in terms of the Prandtl stress function $\phi = \phi(x, y)$ by

$$\tau_{xz} = \frac{\partial \phi}{\partial y} \tag{7}$$

$$\tau_{yz} = -\frac{\partial \phi}{\partial x} \tag{8}$$

The equilibrium equations are then identically satisfied and the compatibility relation gives

$$\nabla^2 \phi = \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = -2\mu\alpha \tag{9}$$

This single relation is then the governing equation for the problem and (9) is a Poisson equation that is amenable to several analytical solution techniques.

To complete the stress formulation we now must address the boundary conditions on the problem. As previously mentioned, the lateral surface of the cylinder S is to be free of tractions, and thus

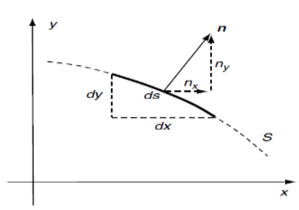

Fig.1
Differential surface element..

2.2 Multiply connected cross-sections

We now wish to develop some additional relations necessary to solve the torsion of hollow cylinders with multiply connected cross-sections. Fig.2 illustrates a typical section of this type with a single hole, and we shall establish theory capable of handling any number of holes. It is assumed that the original boundary conditions of zero tractions on all lateral surfaces applies to the external boundary so and all internal boundaries S1, . . . Therefore, the stress function is a constant and the displacement is specified on each boundary Si, i = 0, 1, . . .

$$\begin{aligned} &\phi = \phi_i \quad on \quad S_i \\ &\frac{dw}{dn} = \alpha\left(yn_x - xn_y\right) \quad on \quad S_i \end{aligned} \tag{10}$$

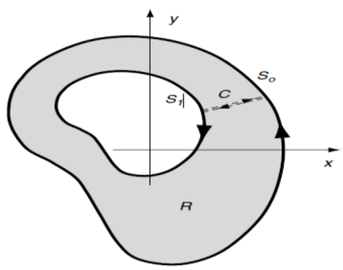

Fig.2
Multiply connected cross-section.

where Φ_i are constants. These conditions imply that the stress function and warping displacement can be determined up to an arbitrary constant on each boundary S_i. With regard to the stress function, the value of Φ_i may be arbitrarily chosen only on one boundary, and commonly this value is taken as zero on the outer boundary S_o similar to the simply connected case. For multiply connected sections, the constant values of the stress function on each of the interior boundaries are determined by requiring that the displacement w be single-valued. Considering the doubly connected example shown in Fig.2, the displacement will be single-valued if

$$\oint_{S_1} dw\,(x,y) = 0 \tag{11}$$

This integral can be written as:

$$\oint_{S_1} dw\,(x,y) = \oint_{S_1}\left(\frac{\partial w}{\partial x}dx + \frac{\partial w}{\partial y}dy\right) = \frac{1}{\mu}\oint_{S_1}(\tau_{xz}dx + \tau_{yz}dy) - \alpha\oint_{S_1}(xdy - ydx) \tag{12}$$

Now $\tau_{xz}dx + \tau_{yz}dy = \tau ds$, where τ_i is the resultant shear stress. Using Green's theorem.

$$\oint_{S_1}(xdy - ydx) = \iint_{A_1}\left(\frac{\partial x}{\partial x} + \frac{\partial y}{\partial y}\right)dxdy = 2\iint_{A_1} dxdy = 2A_1 \tag{13}$$

where A_1 is the area enclosed by S_1. Combining these results, the single-valued condition (11) implies that

$$\oint_{S_1} \tau ds = 2\mu\alpha A_1 \tag{14}$$

The value of Φ_1 on the inner boundary S_1 must therefore be chosen so that (14) is satisfied. If the cross-section has more than one hole, relation (14) must be satisfied for each; that is,

$$\oint_{S_1} \tau ds = 2\mu\alpha A_k \tag{15}$$

where k =1, 2, 3, . . . is the index corresponding to each of the interior holes. The resultant torque condition will give

$$T = 2\iint_R \phi dxdy + 2\phi_1 A_1 \tag{16}$$

For the case with *N* holes, this relation becomes

$$T = 2\iint_R \phi dxdy + \sum_{k=1}^{N} 2\phi_k A_k \tag{17}$$

Justifying these developments for multiply connected sections requires contour integration in a cut domain following the segments S_0, C, S_1, as shown in Fig.2.

3 SOLUTION

3.1 Consider the torsion of a bar with a hollow elliptical section as shown in Fig. 2. The inner boundary is simply a scaled ellipse similar to that of the outer boundary

The stress function solution for the hollow case is given by

$$\phi = \left(\frac{x^2}{a^2} + \frac{y^2}{b^2} - 1\right)\left(k_1 x^2 + k_2 y^2 + |k_3 xy| + |k_4 x| + |k_5 y| + k_6\right) \tag{18}$$

$$\phi = (\frac{k_1 x^4}{a^2} + \frac{k_2 x^2 y^2}{a^2} + \frac{k_3 x^3 y}{a^2} + \frac{k_4 x^3}{a^2} + \frac{k_5 x^2 y}{a^2} + \frac{k_6 x^2}{a^2} + \frac{k_1 x^2 y^2}{b^2} + \frac{k_2 y^4}{b^2} + \frac{k_3 xy^3}{b^2} + \frac{k_4 xy^2}{b^2} + \frac{k_5 y^3}{b^2} + \frac{k_6 y^z}{b^2} - k_1 x^2 - kzy^2 - k_3 yx - k_4 x - k_5 y - k_6) \tag{19}$$

and this form satisfies the governing equation, boundary conditions, and the multiply connected condition 25. The constant value of the stress function on the inner boundary is found to be using the boundary equation scheme, we look for a stress function of the form:

$$\phi_1 = -\frac{a^2 b^2 \mu_1 \alpha}{a^2 + b^2}(k^2 - 1) \tag{20}$$

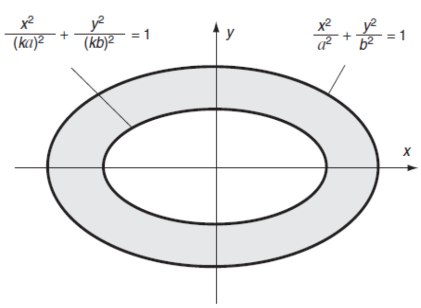

Fig.3
Hollow elliptical section.

We introduce μ function by the relation:

$$\mu = A_1 x^2 + A_2 y^2 + A_3 xy + A_4 x + A_5 y + A_6 \tag{21}$$

Substitution ϕ into Eq. (9) give

$$\nabla^2\Phi = (\frac{12k_1x^2}{a^2} + \frac{2k_2y^2}{a^2} + \frac{6k_3xy}{a^2} + \frac{6k_4x}{a^2} + \frac{2k_5Y}{a^2} + \frac{2k_6}{b^2} + \frac{2k_1x^2}{b^2} - 2k_1) + (\frac{2k_2x^2}{a^2} + \frac{2k_1x^2}{b^2} + \frac{12k_2y^2}{b^2} + \frac{6k_3xy}{b^2} + \frac{2k_4x}{b^2} + \frac{6k_5Y}{b^2} + \frac{2k_6}{b^2} - 2k_2) = -2\mu\alpha = -2(A_1X^2 + A_2Y^2 + A_3XY + A_4X + A_5Y + A_6)\alpha \tag{22}$$

Equating the coefficients of the identical powers yields

$$\begin{aligned}
&\frac{12k_1}{a^2} + \frac{2k_2}{a^2} + \frac{2k_1}{b^2} = -2A_1\alpha \\
&\frac{2k_1}{b^2} + \frac{2k_2}{a^2} + \frac{12k_2}{b^2} = -2A_2\alpha \\
&\frac{6k_3}{a^2} + \frac{6k_3}{b^2} = -2A_3\alpha \\
&\frac{6k_4}{a^2} + \frac{2k_4}{b^2} = -2A_4\alpha \\
&\frac{2k_5}{a^2} + \frac{6k_5}{b^2} = -2A_5\alpha \\
&\frac{2k_6}{a^2} + \frac{2k_6}{b^2} - 2k_1 - 2k_2 = -2A_6\alpha
\end{aligned} \tag{23}$$

Substituting Eqs. (19, 20) into Eq. (16) gives

$$\begin{aligned}
T &= 8(\pi K_1a^3b)/32 + (\pi K_2ab^3)/96 + (K_3a^2b^2)/24 + (2K_4a^2b)/15 + \\
&(K_5ab^2)/15 + (\pi K_6ab)/16 + (\pi K_1a^3b)/96 + (\pi K_2ab^3)/32 + (K_3a^2b^2)/24 + \\
&(K_4a^2b)/15 + (2K_5ab^2)/15 + (\pi K_6ab)/16 - (\pi K_1a^3b)/16 - (\pi K_2ab^3)/16 - \\
&(K_3a^2b^2)/8 - (K_4a^2b)/3 - (K_5ab^2)/3 - \pi(K_7a)^3(K_7b)^3\mu_1/(K_7a)^2 + (K_7b)^2
\end{aligned} \tag{24}$$

Eqs. (23) and (24) are a system of algebraic equations, where the solution is given by the Cramer's method as:

$$\begin{aligned}
&k_1 = (A_1a^2b^4 + 6A_1a^4b^2 - A_2a^2b^4)(6a^2 + 36a^2b^2 + 6b^4)\alpha \\
&k_2 = -(6A_2a^2b^4 - A_1a^4b^2 + A_2a^4b^2)(6a^4 + 36a^2b^2 + 6b^4)\alpha \\
&k_3 = (-A_3a^2b^2)(3a^2 + 3b^2)\alpha \\
&k_4 = (-A_4a^2b^2)(a^2 + 3b^2)\alpha \\
&k_5 = (-A_5a^2b^2)(3a^2 + b^2)\alpha \\
&k_6 = -((A_1a^4b^6 + 5A_1a^6b^4 + 5A_2a^4b^6 + A_2a^6b^4 + 6A_6a^2b^6 + 36A_6a^4b^4 + 6A_6a^6b^2)(6a^6 + 42a^4b^2 + \\
&42a^2b^4 + 6b^6)\alpha)
\end{aligned} \tag{25}$$

Substituting Eq.(25) back into (24) give

$$\begin{aligned}
T &= 8((A_3a^4b^4\alpha(3a^2 + 3b^2))/24 + (2A_4a^4b^3\alpha(a^2 + 3b^2))/15 + (2A_5a^3b^4\alpha(3a^2 + b^2))/15 - \\
&(\pi a^3b\alpha(36a^2b^2 + 6a^2 + 6b^4)(A_1a^2b^4 + 6A_1a^4b^2 - A_2a^2b^4))/48 + (\pi ab^3\alpha(6a^4 + 36a^2b^2 + 6b^4) \\
&(6A_2a^2b^4 - A_1a^4b^2 + A_2a^4b^2))/48 - (\pi ab\alpha(6a^6 + 42a^4b^2 + 42a^2b^4 + 6b^6)(A_1a^4b^6 + 5A_1a^6b^4 + \\
&5A_2a^4b^6 + A_2a^6b^4 + 6A_6a^2b^6 + 36A_6a^4b^4 + 6A_6a^6b^2))/8 - \pi(k_7a)^3(k_7b)^3\mu_1/(k_7a)^2 + (k_7b)^2
\end{aligned} \tag{26}$$

which can be cast in the form to determine the angle of twist in terms of the applied loading as:

$$\begin{aligned}\alpha =& T\ /8((A_3a^4b^4(3a^2+3b^2))/24+(2A_4a^4b^3(a^2+3b^2))/15+(2A_5a^3b^4(3a^2+b^2))/15-\\ &(\pi a^3b(36a^2b^2+6a^2+6b^4)(A_1a^2b^4+6A_1a^4b^2-A_2a^2b^4))/48+(\pi ab^3(6a^4+36a^2b^2+6b^4)\\ &(6A_2a^2b^4-A_1a^4b^2+A_2a^4b^2))/48-(\pi ab(6a^6+42a^4b^2+42a^2b^4+6b^6)(A_1a^4b^6+5A_1a^6b^4+\\ &5A_2a^4b^6+A_2a^6b^4+6A_6a^2b^6+36A_6a^4b^4+6A_6a^6b^2))/8-\pi(k_7a)^3(k_7b)^3\mu_1/(k_7a)^2+(k_7b)^2\end{aligned} \tag{27}$$

The shear stresses (7, 8) resulting are given by:

$$\sigma_{xz}=2k_2x^2\frac{y}{a^2}+\frac{k_3x^3}{a^2}+\frac{k_5x^2}{a^2}+\frac{2k_1x^2}{b^2}+\frac{4k_2y^3}{b^2}+\frac{3k_3xy^2}{b^2}+\frac{2k_4xy}{b^2}+\frac{3k_5y^2}{b^2}+\frac{2k_6y}{b^2}-2k_2y-k_3x-k_5 \tag{28}$$

$$\sigma_{yz}=-(\frac{4k_1x^3}{a^2}+\frac{2k_2xy^2}{a^2}+\frac{3k_3x^2y}{a^2}+\frac{3k_4x^2}{a^2}+\frac{2k_5xy}{a^2}+\frac{2k_6x}{a^2}+\frac{2k_1xy^2}{b^2}+\frac{k_3y^3}{b^2}+\frac{k_4y^2}{b^2}-2k_1x-k_3y-k_4) \tag{29}$$

Substituting (25) back into (28) yields:

$$\begin{aligned}\sigma_{xz}=&2\alpha y\,(6a^4+36a^2b^2+6b^4)(6A_2a^2b^4-A_1a^4b^2+A_2a^4b^2)-A_3b^2\alpha x^3(3a^2+3b^2)+\\ &(2\alpha x^2(36a^2b^2+6a^2+6b^4)(A_1a^2b^4+6A_1a^4b^2-A_2a^2b^4))/b^2-(4\alpha y^3(6a^4+36a^2b^2+6b^4)\\ &(6A_2a^2b^4-A_1a^4b^2+A_2a^4b^2))/b^2-(2\alpha y\,(6a^6+42a^4b^2+42a^2b^4+6b^6)(A_1a^4b^6+5A_1a^6b^4+5A_2a^4b^6+\\ &A_2a^6b^4+6A_6a^2b^6+36A_6a^4b^4+6A_6a^6b^2))/b^2+A_5a^2b^2\alpha(3a^2+b^2)-3A_5a^2y^2\alpha(3a^2+b^2)-\\ &A_5b^2x^2\alpha(3a^2+b^2)-2A_4a^2x\,\alpha y\,(a^2+3b^2)-(2\alpha x^2y\,(6a^4+36a^2b^2+6b^4)(6A_2a^2b^4-A_1a^4b^2+A_2a^4b^2))/a^2+\\ &A_3\alpha a^2b^2x\,(3a^2+3b^2)-3A_3\alpha a^2xy^2(3a^2+3b^2)\end{aligned} \tag{30}$$

Substituting (25) back into (29) give

$$\begin{aligned}\sigma_{yz}=&2\alpha x\,(36a^2b^2+6a^2+6b^4)(A_1a^2b^4+6A_1a^4b^2-A_2a^2b^4)+A_3a^2y^3\alpha(3a^2+3b^2)\\ &-(4x^3\alpha(36a^2b^2+6a^2+6b^4)(A_1a^2b^4+6A_1a^4b^2-A_2a^2b^4))/a^2\\ &+(2\alpha x\,(6a^6+42a^4b^2+42a^2b^4+6b^6)(A_1a^4b^6-5A_1a^6b^4+5A_2a^4b^6+A_2a^6b^4+6A_6a^2b^6\\ &+36A_6a^4b^4+6A_6a^6b^2))/a^2-A_4a^2b^2\alpha(a^2+3b^2)+A_4a^2y^2\alpha(a^2+3b^2)+3A_4b^2x^2\alpha(a^2+3b^2)\\ &+2A_5b^2xy\,(3a^2+b^2)+(2xy^2\alpha(6a^4+36a^2b^2+6b^4)(6A_2a^2b^4-A_1a^4b^2+A_2a^4b^2))/a^2-\\ &(2\alpha xy^2(36a^6b^2+6a^2+6b^4)(A_1a^2b^4+6A_1a^4b^2-A_2a^2b^4))/b^2-\\ &A_3a^2b^2y\,\alpha(3a^2+3b^2)+3A_3b^2x^2y\,\alpha(3a^2+3b^2)\end{aligned} \tag{31}$$

Intuition from strength of materials theory would suggest that the maximum stress should occur at the boundary point most removed from the section's center; that is, at x =+-a and y = 0 (assuming a > b). However, the membrane analogy would argue for a boundary point closest to the center of the section where the membrane slope would be the greatest shear stress becomes

$$\sigma=\sqrt{\sigma_{xz}{}^2+\sigma_{yz}{}^2} \tag{32}$$

Using the stress relations (32),it yields a system that can be integrated to determine the displacement field

$$W=\int(\frac{\sigma_{xz}}{\mu}+\alpha y\,)dx \tag{33}$$

4 RESULTS AND DISCUSSION

Consider a Poroelastic shaft with hollow elliptical section, where a=0.04 m and b=0.02 m are the semi major and minor axes in elliptic section. The modulus of elasticity is $\mu = A_1 x^2 + A_2 y^2 + A_3 xy + A_4 x + A_5 y + A_6$. For simplicity of analysis we consider $A_1 = A_2 = A_3 = A_6 = 0$. To examine the proposed solution method, one example problems are considered. As the example, consider a Poroelastic shaft with hollow elliptical section where the modulus of elasticity $A_4 = 10$ Gpa, $A_5 = 35$ Gpa and $\mu_1 = 80$ Gpa.The stress function and the warping displacement are substituted in appendix.

Contour lines of the stress function for poroelasic with hollow elliptical section shaft are shown in Figs.3 and 4 , and it is observed that the maximum slope of the stress function (membrane) occurs at $x = 0$ and $y = \pm b$ (on the top and bottom of the section). A three-dimensional plot of the warping displacement surface is also shown in Fig.5, illustrating the behavior of the w displacement**.** Contour lines of displacement field are represented by hyperbolas in the x , y-plane and are shown in Fig. 6 for the case of a positive counter clockwise torque applied to the section. Solid lines correspond to positive values of *w*, indicating that points move out of the section in the positive *z* direction, while dotted lines indicate negative values of displacement.

Fig.7 compare the change of the warping in Elastic and Poroelastic shaft, where minimum chang of warping for both Elastic and Poroelastic due to mechanical loads is in the center of the shaft and it is increase at the outer radius of shaft. It also shows that Increasing the major axis of shaft will enhance the quntity of change of warping in the shaft. The Fig.7 also illustrates that changes of the warping in Poroelastic shaft follows a trend similar to the Elastic shaft, except its quantity which is different. Considerable point is that for both Elastic and poroelastic shaft the maximum quantity is between 2 and 2.5. Along each of the coordinate axes the displacement is zero, and for the special case with a = b (circular section), the warping displacement vanishes everywhere. If the ends of the elliptical cylinder are restrained, normal stresses σ_{xz} are generated as a result of the torsion. The changes of the Stress Function in Elastic compare with Poroelastic shaft are shown in Fig.8, where illustrate a linear trend but looking to it in depth, shows it is like having its maximum near boundary point. As change of the warping, changes of the Stress Function (Fig.8) follow a trend similar to each other in both Elastic and Poroelastic shaft, but the quantity is not same, where the quantity of the stress function in elastic shaft is much more than the Poroelastic shaft.

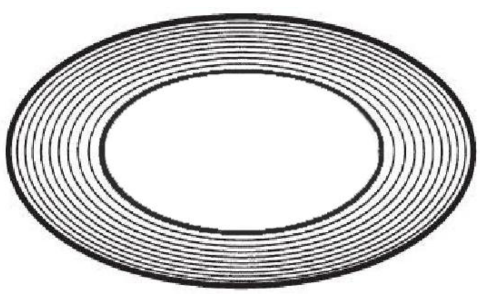

Fig.4
Stress function for the elliptical section (2D).

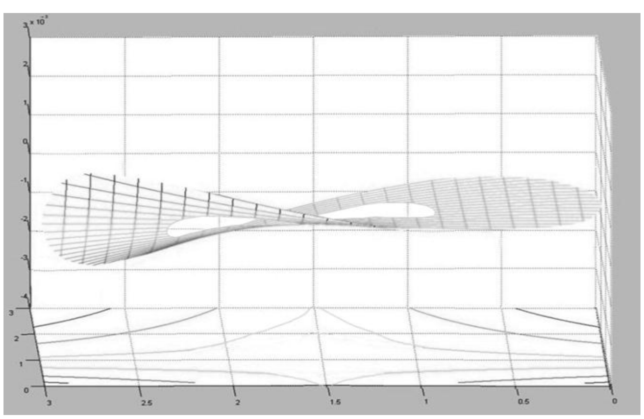

Fig.5
Warping displacement surface (3D).

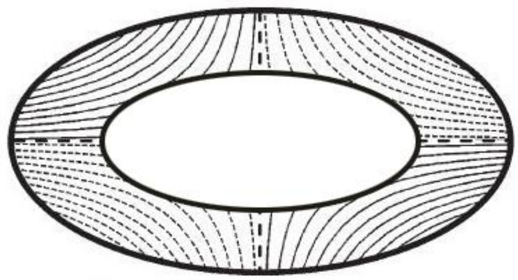

Fig.6
Displacement contours.

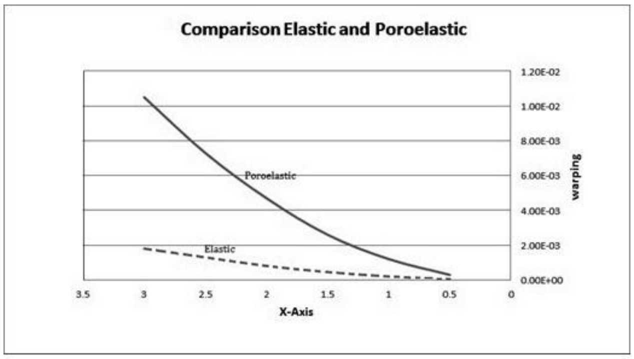

Fig.7
Compare elastic with poroelastic.

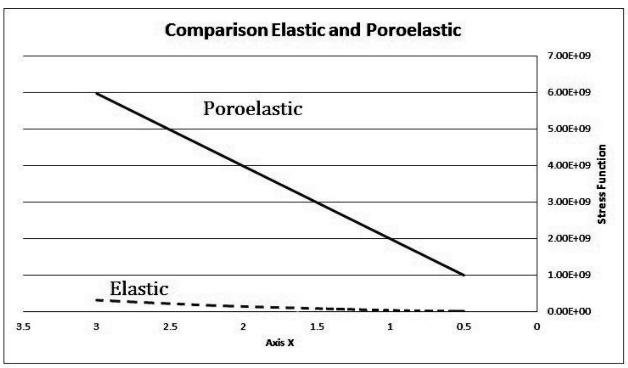

Fig.8
Compare elastic with poroelastic.

5 CONCLUSIONS

This paper presents the analytical solution for the torsion of Poroelastic shaft with hollow elliptical section. The method of solution is based on the direct method and use of stress function where satisfies Poisson equation and vanishes on boundary. Because the boundary is expressed by the relation $f(X, Y) = 0$, this paper suggests a possible simple solution which is a scheme of expressing the stress function in terms of the boundary equation $\phi = K_f(x, y)$. It is to be emphasized that the proposed method does not have the mathematical limitations to handle the general types of boundary conditions which are usually countered in the potential function method.

APPENDIX

Stress for elliptic section

$$\sigma_{xz} = (A_5 a^2 b^2 T\,((2A_4 a^4 b^3 (a^2+3b^2))/15+(2A_5 a^3 b^4 (3a^2+b^2))/15)(3a^2+b^2))/8-$$
$$(A_5 b^2 T x^2 ((2A_4 a^4 b^3 (a^2+3b^2))/15+(2A_5 a^3 b^4 (3a^2+b^2))/15)(3a^2+b^2))/8-$$
$$(3A_5 a^2 T y^2 ((2A_4 a^4 b^3 (a^2+3b^2))/15+(2A_5 a^3 b^4 (3a^2+b^2))/15)(3a^2+b^2))/8-$$
$$(A_4 a^2 Txy\,((2A_4 a^4 b^3 (a^2+3b^2))/15+(2A_5 a^3 b^4 (3a^2+b^2))/15)(a^2+3b^2))/4$$

$$W = (Ty^2 (b^2 k_7{}^2 + (2A_4 a^4 b^3 (a^2+3b^2))/15+(2A_5 a^3 b^4 (3a^2+b^2))/15-\pi a b^3 k_7{}^4 u_1))/16-$$
$$(x(15A_4 a^2 Ty((2A_4 a^4 b^3 (a^2+3b^2))/15+(2A_5 a^3 b^4 (3a^2+b^2))/15)(a^2+3b^2)-$$
$$(15A_5{}^2 b^2 Ty((2A_4 a^4 b^3 (a^2+3b^2))/15+(2A^5 a^3 b^4 (3a^2+b^2))/15)(3a^2+b^2))/(2A_4)))/(60A_4)+$$
$$(\log(A_4 x + A_5 y)(300TA_4{}^3 A_5 a^{10} b^5 - 7TA_4{}^3 A_5 a^{10} b^3 y^2 + 1000TA_4{}^3 A_5 a^8 b^7 - 18TA_4{}^3 A_5 a^8 b^5 y^2 +$$
$$300TA_4{}^3 A_5 a^6 b^9 + 9TA_4{}^3 A_5 a_6 b^7 y^2 + 9TA_4{}^2 A_5^2 a^9 b^6 - 21TA_4{}^2 A_5{}^2 a^9 b^4 y^2 +$$
$$6TA_4{}^2 A_5{}^2 a^7 b^8 + 2TA_4{}^2 A_5{}^2 a^7 b^6 y^2 + TA_4{}^2 A_5{}^2 a^5 b^{10} + 3TA_4{}^2 A_5{}^2 a^5 b^8 y^2 - 3TA_4 A_5{}^3 a^8 b^5 y^2$$
$$-10TA_4 A_5{}^3 a^6 b^7 y^2 - 3TA_4 A_5{}^3 a^4 b^9 y^2 - 9TA_5{}^4 a^7 b^6 y^2 - 6TA_5{}^4 a^5 b^8 y^2 - TA_5{}^4 a^3 b^{10} y^2))/(60A_4{}^3)$$
$$-(A_5 b^2 T x^2 ((2A_4 a^4 b^3 (a^2+3b^2)))/15+(2A_5 a^3 b^4 (3a^2+b^2))/15)(3a^2+b^2))/(16A_4)$$

REFERENCES

[1] Timoshenko S.P., Goodier J.N., 1970, *Theory of Elasticity*, New York:, McGraw-Hill.

[2] Timoshenko S.P., 1953, *History of Strength of Materials*, New York:, McGraw-Hill.

[3] Baron F. M ., 1942,Torsion of multi-connected thin-walled cylinders, *Journal of Applied Mechanics* **9**:72-74.

[4] Li Z.., Ko J. M., Ni Y. Q., 2000, Torsional rigidity of reinforced concrete bars with arbitrary sectional shape, *Finite Elements in Analysis and Design* **35**:349-361.

[5] Mejak G ., 2000, Optimization of cross-section of hollow prismatic bars in torsion, *Communications in Numerical Methods in Engineering* **16**:687-695.

[6] Jiang W. G ., Henshall J. L ., 2002 , A coupling cross-section finite element model for torsion analysis of prismatic bars, *European Journal of Mechanics A-solids* **21**: 513-522.

[7] Louis Angelo M ., Ryan M ., 2007 , Torsion of a rectangular prismatic bar: Solution using a power fit model, *Philippine Engineering Journal* **28**(1) : 77-98.

[8] Doostfatemeh A., Hematiyan M.. R ., Arghavan S., 2009 , Closed-form approximate formulations for torsional analyses of hollow tube with straight and circular edges, *Journal of Mechanics* **25**:401-409.

[9] Courant R., 1943 , Variational methods for the solution of problems of equilibrium and vibration, *Bulletin of the American Mathematical Society* **49**(1):1-23.

[10] Timoshenko S.P., 1956, *Strength of Materials*, Berkshire (England) ,Van Nostrand.

[11] Quinlan P.M., 1964, The torsion of an irregular polygon, *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Science* **282** :208-227.

[12] Muskhelishvilli N.I., 1953, *Some Basic Problems of the Mathematical Theory of Elasticity*, Groningen ,Holland.

[13] Booker J.R., Kitipornchai S., 1971, Torsion of multilayered rectangular section, *Journal of the Engineering Mechanics Division ASCE* **97**:1451-1468.

[14] Kuo Y.M., Conway H.D., 1973 ,The torsion of composite tubes and cylinders, *International Journal of Solids and Structures* **9**(12):1553-1565.

[15] Kuo Y.M., Conway H..D., 1974, Torsion of cylinders with multiple reinforcement, *Journal of the Engineering Mechanics Division ASCE* **100**:221-234.

[16] Kuo Y.M., Conway H.D., 1974, Torsion of composite rhombus cylinder, *Journal of Applied Mechanics* **41**(1):302-303.

[17] Kuo Y.M., Conway H.D., 1980, Torsion of reinforced square cylinder, *Journal of the Engineering Mechanics Division* **106** :1341-1347.

[18] Packham B.A., Shail R.., 1978 , St. venant torsion of composite cylinders, *Journal of Elasticity* **8**(4):393-407.

[19] Ripton R.,1998 , Optimal fiber configurations for maximum torsional rigidity, *Archive for Rational Mechanics and Analysis* **144**(1):79-106.

[20] Chen T., Benveniste Y., Chuang P.C , 2002 , Exact solutions in torsion of composite bars: thickly coated neutral inhomogeneities and composite cylinder assemblages, *Proceedings of the Royal Society A : Mathematical, Physical and Engineering Science* **458**(2023):1719-1759.

[21] Ely J.F., Zienkiewicz O.C., 1960, Torsion of compound bars a relaxation solution, *International Journal of Mechanical Sciences* **1**(4):356-365.

[22] Herrmann L.R., 1965 , Elastic torsional analysis of irregular shapes, *Journal of the Engineering Mechanics Division* **91**(6): 11-20.

[23] Jaswon M.A., Ponter A.R., 1963, An integral equation solution of the torsion problem, *Proceedings of the Royal Society A: Mathematical Physical and Engineering Science* **273**:237-246.

[24] Kasikadelis J.T., Sapountzakis E.J, 1986 , Torsion of composite bars by boundary element method, *Journal of Engineering Mechanics* **111**(9):1197-1210.

[25] Sapountzakis E.J., 2000 , Solution of non-uniform torsion of bars by an integral equation method, *Computer and Structures* **77**(6):659-667.

[26] Sapountzakis E.J., 2001 , Nonuniform torsion of multi-material composite bars by the boundary element method, *Computer and Structures* **79**(32):2805-2816.

[27] Koizumi M., 1993 ,The concept of FGM, *Ceram Trans Function Grad Mater* **34**(1):3-10.

[28] Plunkett R., 1965 , Torsion of inhomogeneous elastic prismatic bars, *Journal of Engineering for Industry* **87**:391-392.

[29] Rooney F.J., Ferrari M., 1995, Torsion and flexure of inhomogeneous elements, *Engineering of Composite* **5**(7):901-911.

[30] Rooney F.J., Ferrari M., 1999, On the St. venant problems for inhomogeneous circular bars, *Journal of Applied Mechanics* **66**(2):32-44.

[31] Horgan C.O., Chan A..M., 1999, Torsion of functionally graded isotropic linearly elastic bars, *Journal of Elasticity* **52**(2):181-199.

[32] Tarn J.G., 2008, Chang HH. Torsion of cylindrically orthotropic elastic circular bars with radial inhomogeneity: some exact solutions and end effects, *International Journal of Solids and Structures* **45**(1):303-319.

[33] Rooney F.J ., Ferrari M.,1995,Torsion and flexure of inhomogeneous elements, *Composites Engineering* **5**: 901-911.

[34] Biot M..A., 1962 , Generalized theory of acoustic propagation in porous dissipative media, *Journal of the Acoustical Society of America* **34**: 1254-1264.

[35] Biot M.A., 1972, Theory of finite deformation of porous solid, *Indiana University Mathematics Journal* **21**:597-620.

[36] Biot M..A., 1982, Generalized Lagrangian equations of non-linear reaction- diffusion, *Chemical Physics* **66**:11-26.

[37] Arghavan S., Hematiyan, M..R., 2009, Torsion of functionally graded hollow tubes, *European Journal Mechanics A/Solids* **28**(3): 551-559.

[38] Batra R..C, 2006, Torsion of a functionally graded cylinder, *The American Institute of Aeronautics and Astronautics* **44** (6):1363-1365.

[39] Horgan C.O, 2007, On the torsion of functionally graded anisotropic linearly elastic bars, *Journal of Applied Mathematics* **72** (5): 556-562.

[40] Rooney F.J, Ferrari M., 1995, Torsion and flexure of inhomogeneous elements, *Composites Engineering* **5** (7):901-911.

[41] Udea M., Nishimura T., Sakate T, 2002, Torsional analysis of functionally graded materials., *Advances in Mechanics of Structures and Materials, Proceedings of 17th Australian Conference (ACMS17)*, Tayor and Francis, Queensland, Australia.

[42] Yaususi T., Shigeyasu A., 2000, Torsional characteristics of hemp palm branch with triangular cross-section (2-composite bar), *The Japan Society of Mechanical Engineers* **66** (649): 1806-1811.

[43] Sofiyev A..H, 2005, The torsional buckling analysis of cylindrical shells with material non-homogeneity in thickness direction under impulsive loading, *Structural Engineering and Mechanics an International Journal* **19**(2):231-236.

[44] Sofiyev A.H., 2003, Torsional buckling of cross-ply laminated orthotropic composite cylindrical shells subject to dynamic loading, *European Journal of Mechanics A/Solids* **22**:943-951.

[45] Sadd M. H.,. 2009, *Elasticity Theory, Application, and Numerics*, Department of Mechanical Engineering and Applied Mechanics University of Rhode Island.