Bending Analysis of Composite Sandwich Plates with Laminated Face Sheets: New Finite Element Formulation

M. O. Belarbi*, A. Tati
Laboratoire de Génie Energétique et Matériaux, LGEM. Université de Biskra, B.P. 145, R.P. 07000, Biskra, Algeria

Received 12 February 2016; accepted 27 March 2016

ABSTRACT
The bending behavior of composites sandwich plates with multi-layered laminated face sheets has been investigated, using a new four-noded rectangular finite element formulation based on a layer-wise theory. Both, first order and higher-order shear deformation; theories are used in order to model the face sheets and the core, respectively. Unlike any other layer-wise theory, the number of degrees of freedom in this present model is independent of the number of layers. The compatibility conditions as well as the displacement continuity at the interface ‘face sheets–core’ are satisfied. In the proposed model, the three translation components are common for the all sandwich layers, and are located at the mid-plane of the sandwich plate. The obtained results show that the developed model is able to give accurate transverse shear stresses directly from the constitutive equations. Moreover, a parametric study was also conducted to investigate the effect of certain characteristic parameters (core thickness to total thickness ratio, side-to-thickness ratio, boundary conditions, plate aspect ratio, core-to-face sheet anisotropy ratio, core shear modulus to the flexural modulus ratio and degree of orthotropy of the face sheet) on the transverse displacement variation. The numerical results obtained by our model are compared favorably with those obtained via analytical solution and numerical/experimental, results obtained by other models. The results obtained from this investigation will be useful for a more comprehensive understanding of the behavior of sandwich laminates.

Keywords: Layer-wise; Finite element; Sandwich plates; Bending.

1 INTRODUCTION
Currently, composite sandwich structures gained considerable attention and became increasingly important in different engineering fields (e.g. civil, aerospace, marine), due to their rigidity-and-resistance to weight ratios. However, there are still questions on the complexity of the behavior of these structures. The effect of shear deformation is quite significant which may lead to failure and becomes more complex in case of sandwich construction, as the material property variation is very large between the core and face layers [1]. Moreover, an accurate estimation of stress components, specifically the transverse shear stresses, plays an important role in reducing these failures [2].

In the literature, several two-dimensional theories have been proposed to study the behavior of composite sandwich structures. Starting by the simple classical laminated plate theory (CLPT), based on the Kirchhoff’s assumptions [3], which does not includes the effect of the transverse shear deformation [4-6], the first order shear
deformation theory (FSDT), where the effect of the transverse shear deformation is considered [7-10], but taken constant through the thickness, and the higher order shear deformation theories (HSDT) where a better representation of transverse shear effect can be obtained [11-16]. All these theories falls within the equivalent single layers (ESL) theories.

However, the ESL approach is unable to predict accurately the local behavior (e.g. interlaminar stresses) of sandwich structures. For that reason, many researchers developed more accurate theories such as the zig–zag theories (ZZT) [1, 17-22] and the global-local higher order shear deformation theories (GLHSDT) [23-27]. Recently, various works have adopted the layer-wise theories (LW) [28-37] to assume separate displacement field expansions within each material layer, thus providing a kinematically correct representation of the strain field in discrete laminated layer, and allowing accurate determination of ply level stresses [38]. For more details the reader may refer to [38-43].

In the finite elements development, many researchers have adopted the layer-wise theory for the sake of a good description of sandwich structures. On this topic, we can distinguish the work of Wu and Lin [35] where a two-dimensional mixed finite element based on higher order layer-wise model is presented for the analysis of thick sandwich plates. The displacement continuity at the interface is satisfied as well as the interlaminar stresses. These authors proposed for each layer, a cubic and quadratic polynomial functions for in-plane and transverse displacements, respectively. Afterwards, Lee and Fan [28] describe a new model using the first order shear deformation theory for the face sheets, whereas the displacement field at the core is expressed in terms of the two face sheets displacements. In this model, the transverse shear strain varies linearly while the transverse normal strain is constant through the thickness of the core. They used a nine-noded isoparametric finite element to study the bending and vibration of sandwich plates. On the other hand, a three-dimensional (3D), layer-wise finite element model was developed by Oskooei and Hansen [31] to analyze the sandwich plates with laminated face sheets. They used the first order shear deformation theory for the face sheets, whereas for the core a cubic and quadratic, functions for the in-plane and transverse, displacements, was adopted. In addition, an eighteen-node three-dimensional brick mixed finite element with six degrees of freedom (6 DOF) per node based on layer-wise theory has been developed by Ramtekkar et al. [44, 45] for an accurate evaluation of transverse stresses in laminated sandwich. In this model, the continuity of displacements as well as the transverse stresses is satisfied. In the same context, Linke et al. [29] developed a three-dimensional displacement finite element containing eleven DOF at each node (each face-sheet contains five DOF per node and only one DOF in the core) for static and stability analysis of sandwich plates. The formulation of this element is based on the layer-wise approach, where the face sheets are represented as elements of classical plate theory and the core is represented by the third order shear deformation theory (TSDT). The in and out-of, plane displacements of the core assume a cubic and quadratic variation, respectively. Later, a forty-five nodes triangular element having seven DOF per node was developed by Ramesh et al. [33] for accurate prediction of interlaminar stresses in laminated composite plates. The construction of this element is based on two theories; the third order shear deformation theory and the layer-wise plate theory of Reddy [34]. Recently, Mantari et al. [30] presented a new layer-wise model using a trigonometric displacement field for in-plane displacements and constant out-of plane displacements through the thickness. The authors used a C⁰ four-node isoparametric quadrilateral element to study the bending of thick sandwich panels.

In the present paper, the bending behavior of sandwich plates with laminated composites face sheets has been investigated by using a new four-noded rectangular finite element formulation based on a layer-wise theory. The face sheets are modeled based on the first order shear deformation theory, whereas the core is modeled using the third-order shear deformation plate theory. Unlike any other layer-wise theory, the number of degrees of freedom is independent of the number of layers. The performance of the proposed formulation is assessed by several study cases, considering different aspect ratios, loadings and boundary conditions. The obtained numerical results can be compared with the analytical solutions given by Pagano [46], the experimental results obtained by Kanematsu et al. [47] and the numerical results found by finite elements models [1, 17, 18, 22]. The second objective of this paper is to investigate the influence of material property parameters and plate geometry variables on the transverse displacement.

2 MATHEMATICAL MODEL

Sandwich plate is a structure composed of three principal layers, as shown in Fig.1, two face sheets (top-bottom) of thicknesses \((h_t), (h_b)\) respectively, and a central layer named core of thickness \((h_c)\) which is thicker than the
previous ones. Total thickness \( h \) of the plate is the sum of these thicknesses. The plane \((x, y)\) coordinate system coincides with mi-plane plate.

![Fig.1](image)

**Fig.1**
Geometry and notations of a sandwich plate.

### 2.1 Kinematic assumptions for the core

In the proposed model, the core is modeled using the third-order shear deformation theory (TSDT). The through-thickness variation of in-plane displacements \((u, v)\) and transverse displacement \((w)\) may be expressed, respectively as follows:

\[
\begin{align*}
    u_c &= u_0 + z \psi_x + z^2\eta_x + z^3 \zeta_x \\
    v_c &= v_0 + z \psi_y + z^2\eta_y + z^3 \zeta_y \\
    w_c &= w_0
\end{align*}
\]

where \(u_0, v_0\) and \(w_0\) are respectively, in-plane and transverse displacement components at the mid-plane of the sandwich plate. \(\psi_x, \psi_y\) represent normal rotations about the \(x\) and \(y\) axis respectively. The parameters \(\eta_x, \eta_y, \zeta_x\) and \(\zeta_y\) are higher order terms.

The strain-displacement relationships can be written in the following form

\[
\begin{align*}
    &\varepsilon_{xx} = \varepsilon_x^{(0)} + z \chi_x^{(1)} + z^2 \chi_x^{(2)} + z^3 \chi_x^{(3)} \\
    &\varepsilon_{yy} = \varepsilon_y^{(0)} + z \chi_y^{(1)} + z^2 \chi_y^{(2)} + z^3 \chi_y^{(3)} \\
    &\gamma_{xy} = \gamma_{xy}^{(0)} + z \chi_{xy}^{(1)} + z^2 \chi_{xy}^{(2)} + z^3 \chi_{xy}^{(3)} \\
    &\gamma_{yz} = \gamma_{yz}^{(0)} + z \chi_{yz}^{(1)} + z^2 \chi_{yz}^{(2)} + z^3 \chi_{yz}^{(3)} \\
    &\gamma_{xz} = \gamma_{xz}^{(0)} + z \chi_{xz}^{(1)} + z^2 \chi_{xz}^{(2)}
\end{align*}
\]

where,

\[
\begin{align*}
    \varepsilon^{(0)} &= \left(\varepsilon_x^{(0)}, \varepsilon_y^{(0)}, \gamma_{xy}^{(0)}\right) \\
    \chi^{(0)} &= \left(\chi_x^{(0)}, \chi_y^{(0)}, \chi_{xy}^{(0)}\right) \\
    \chi^{(1)} &= \frac{\partial \varepsilon_x}{\partial x} - \frac{\partial \varepsilon_y}{\partial y} \\
    \chi^{(2)} &= \frac{\partial \chi_x}{\partial x} - \frac{\partial \chi_y}{\partial y} \\
    \chi^{(3)} &= \frac{\partial \chi_x}{\partial x} - \frac{\partial \chi_y}{\partial y}
\end{align*}
\]

### 2.2 Kinematic assumptions for the face sheets

The face sheets are modeled using the first-order shear deformation theory (FSDT). The compatibility conditions as well as the displacement continuity at the interface (top face-sheet, core, bottom face-sheet), leads to the following improved displacement fields (Fig. 2)
a. Top face-sheet

\[ u_i = u_i \left( \frac{h^c}{2} + \left( z - \frac{h^c}{2} \right) \psi^c_i \right) , \quad v_i = v_i \left( \frac{h^c}{2} + \left( z - \frac{h^c}{2} \right) \psi^c_i \right) , \quad w_i = w_0 \]  

(4)

where \( \psi^c_i \) and \( \psi^c_j \) are the rotations of the top face-sheet cross section about the \( x \) and \( y \) axis, respectively.

with,

\[ u_i \left( \frac{h^c}{2} \right) = u_o + \left( \frac{h^c}{2} \right) \psi^c_i + \left( \frac{h^c}{4} \right) \eta^c_i + \left( \frac{h^c}{8} \right) \zeta^c_i \]  

, \[ v_i = v_o + \left( \frac{h^c}{2} \right) \psi^c_i + \left( \frac{h^c}{4} \right) \eta^c_i + \left( \frac{h^c}{8} \right) \zeta^c_i \]

(5)

Substitution of Eq. (5) into Eq. (4) leads to the following expression

\[ u_i = u_o + \left( \frac{h^c}{2} \right) \psi^c_i + \left( \frac{h^c}{4} \right) \eta^c_i + \left( \frac{h^c}{8} \right) \zeta^c_i + \left( z - \frac{h^c}{2} \right) \psi^c_i \]  

\[ v_i = v_o + \left( \frac{h^c}{2} \right) \psi^c_i + \left( \frac{h^c}{4} \right) \eta^c_i + \left( \frac{h^c}{8} \right) \zeta^c_i + \left( z - \frac{h^c}{2} \right) \psi^c_i \]  

\[ w_i = w_0 \]  

(6)

b. Bottom face-sheet

According to Fig. 2, the displacement field of the bottom face sheet can be written as:

\[ u_b = u_i \left( -\frac{h^c}{2} + \left( z + \frac{h^c}{2} \right) \psi^b_i \right) , \quad v_b = v_i \left( -\frac{h^c}{2} + \left( z + \frac{h^c}{2} \right) \psi^b_i \right) , \quad w_b = w_0 \]  

(7)

where \( \psi^b_i \) and \( \psi^b_j \) are the rotations of the bottom face-sheet cross section about the \( y \) and \( x \) axis respectively

with,

\[ u_i \left( -\frac{h^c}{2} \right) = u_o - \left( \frac{h^c}{2} \right) \psi^b_i - \left( \frac{h^c}{4} \right) \eta^b_i - \left( \frac{h^c}{8} \right) \zeta^b_i \]  

, \[ v_i = v_o - \left( \frac{h^c}{2} \right) \psi^b_i - \left( \frac{h^c}{4} \right) \eta^b_i - \left( \frac{h^c}{8} \right) \zeta^b_i \]

(8)

Substituting Eq. (8) into Eq. (7), leads to the following expression

\[ u_b = u_o - \left( \frac{h^c}{2} \right) \psi^b_i + \left( \frac{h^c}{4} \right) \eta^b_i - \left( \frac{h^c}{8} \right) \zeta^b_i + \left( z + \frac{h^c}{2} \right) \psi^b_i \]  

\[ v_b = v_o - \left( \frac{h^c}{2} \right) \psi^b_i + \left( \frac{h^c}{4} \right) \eta^b_i - \left( \frac{h^c}{8} \right) \zeta^b_i + \left( z + \frac{h^c}{2} \right) \psi^b_i \]  

\[ w_b = w_0 \]  

(9)

Fig. 2

Kinematics.
2.2.1 Strains

The strain-displacement relationships of the top face-sheet are given by

\[ \varepsilon_{xs} = \frac{\partial u}{\partial x} = \frac{\partial u_0}{\partial x} + \left( h_0 \frac{\partial \psi'_x}{\partial x} + \frac{k^2}{4} \frac{\partial \eta'_x}{\partial x} + \frac{k^1}{8} \frac{\partial \zeta'_x}{\partial x} + \frac{h}{2} \frac{\partial \psi'_x}{\partial x} \right) \]

\[ \varepsilon_{ys} = \frac{\partial v}{\partial y} = \frac{\partial v_0}{\partial y} + \left( h_0 \frac{\partial \psi'_x}{\partial y} + \frac{k^2}{4} \frac{\partial \eta'_x}{\partial y} + \frac{k^1}{8} \frac{\partial \zeta'_x}{\partial y} + \frac{h}{2} \frac{\partial \psi'_x}{\partial y} \right) \]

\[ \gamma_{ys} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{h}{2} \left( \frac{\partial \psi'_x}{\partial y} + \frac{\partial \psi'_y}{\partial x} \right) + \frac{k^2}{4} \left( \frac{\partial \eta'_x}{\partial y} + \frac{\partial \eta'_y}{\partial x} \right) + \frac{k^1}{8} \left( \frac{\partial \zeta'_x}{\partial y} + \frac{\partial \zeta'_y}{\partial x} \right) + \frac{h}{2} \left( \frac{\partial \psi'_x}{\partial y} + \frac{\partial \psi'_y}{\partial x} \right) \]

The strain-displacement relationships of the bottom face-sheet can be written in the following form

\[ \varepsilon_{bs} = \frac{\partial u}{\partial x} = \frac{\partial u_0}{\partial x} + \left( h_0 \frac{\partial \psi'_x}{\partial x} + \frac{k^2}{4} \frac{\partial \eta'_x}{\partial x} + \frac{k^1}{8} \frac{\partial \zeta'_x}{\partial x} + \frac{h}{2} \frac{\partial \psi'_x}{\partial x} \right) \]

\[ \varepsilon_{bs} = \frac{\partial v}{\partial y} = \frac{\partial v_0}{\partial y} + \left( h_0 \frac{\partial \psi'_x}{\partial y} + \frac{k^2}{4} \frac{\partial \eta'_x}{\partial y} + \frac{k^1}{8} \frac{\partial \zeta'_x}{\partial y} + \frac{h}{2} \frac{\partial \psi'_x}{\partial y} \right) \]

\[ \gamma_{bs} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{h}{2} \left( \frac{\partial \psi'_x}{\partial y} + \frac{\partial \psi'_y}{\partial x} \right) + \frac{k^2}{4} \left( \frac{\partial \eta'_x}{\partial y} + \frac{\partial \eta'_y}{\partial x} \right) + \frac{k^1}{8} \left( \frac{\partial \zeta'_x}{\partial y} + \frac{\partial \zeta'_y}{\partial x} \right) + \frac{h}{2} \left( \frac{\partial \psi'_x}{\partial y} + \frac{\partial \psi'_y}{\partial x} \right) \]

\[ \gamma_{bs} = \frac{\partial w}{\partial y} + \psi'_y + \gamma_{bs} = \frac{\partial w_b}{\partial x} + \psi'_y \]

2.3 Constitutive relationships

In this paper, the two face sheets (top and bottom) are considered as laminated composite, as shown in Fig. 3. So, the stress-strain relationship of the \( k \)th layer in the global coordinate system is given by

\[ \left[ \begin{array}{c} \sigma'_{x} \\ \sigma'_{y} \\ \tau'_{xs} \\ \tau'_{ys} \\ \tau'_{bk} \end{array} \right] = \frac{1}{Q_{ij}} \left[ \begin{array}{c} Q_{11} & Q_{21} & 0 & 0 & Q_{16} \\ Q_{21} & Q_{22} & 0 & 0 & Q_{26} \\ 0 & 0 & Q_{44} & Q_{45} & 0 \\ 0 & 0 & Q_{54} & Q_{55} & 0 \\ Q_{41} & Q_{51} & 0 & 0 & Q_{61} \end{array} \right] \left[ \begin{array}{c} \varepsilon'_{x} \\ \varepsilon'_{y} \\ \gamma'_{xs} \\ \gamma'_{ys} \\ \gamma'_{bk} \end{array} \right] \] \( f = t,b \)

(12)

The core is considered as an orthotropic composite material, and the stress-strain relationships given by

\[ \left[ \begin{array}{c} \sigma'_{x} \\ \sigma'_{y} \\ \tau'_{xs} \\ \tau'_{ys} \\ \tau'_{bk} \end{array} \right] = \frac{1}{Q_{ij}} \left[ \begin{array}{c} Q_{11} & Q_{21} & 0 & 0 & 0 \\ Q_{11} & Q_{22} & 0 & 0 & 0 \\ 0 & 0 & Q_{44} & 0 & 0 \\ 0 & 0 & 0 & Q_{55} & 0 \\ 0 & 0 & 0 & 0 & Q_{66} \end{array} \right] \left[ \begin{array}{c} \varepsilon'_{x} \\ \varepsilon'_{y} \\ \gamma'_{xs} \\ \gamma'_{ys} \end{array} \right] \]

(13)

The efforts resultants of the core are obtained by integration of the stresses through the thickness direction of laminated plate
\[
\begin{bmatrix}
N_x & M_{xy} & N_x & M_{xy} \\
N_y & M_{yx} & N_y & M_{yx} \\
N_{xx} & M_{xyy} & N_{xx} & M_{xyy} \\
N_{yy} & M_{yx} & N_{yy} & M_{yx}
\end{bmatrix} = \int \frac{1}{2} \begin{bmatrix}
\sigma_x \\
\sigma_y \\
\tau_{xy}
\end{bmatrix} \begin{bmatrix}
1, z, z^2, z^3
\end{bmatrix} dz \tag{14}
\]

\[
\begin{bmatrix}
V_x & S_x & R_x \\
V_y & S_y & R_y
\end{bmatrix} = \int \frac{1}{2} \begin{bmatrix}
\tau_{xx} \\
\tau_{yy}
\end{bmatrix} \begin{bmatrix}
1, z, z^2
\end{bmatrix} dz \tag{15}
\]

where \(N, M, \overline{N}, \overline{M}\) denote membrane, bending moment, higher order membrane and higher order moment resultants respectively. \(V\) is the shear resultant; \(S\) and \(R\) are the higher order shear resultants.

By introducing the constitutive equation in the expressions of the resultant stress (14) and (15), the generalized constitutive equations become

\[
\begin{bmatrix}
\mathbf{N} \\
\mathbf{M} \\
\mathbf{\overline{N}} \\
\mathbf{\overline{M}}
\end{bmatrix} = \begin{bmatrix}
\mathbf{[A]} & \mathbf{[B]} & \mathbf{[D]} & \mathbf{[E]} \\
\mathbf{[B]} & \mathbf{[D]} & \mathbf{[E]} & \mathbf{[F]} \\
\mathbf{[D]} & \mathbf{[E]} & \mathbf{[F]} & \mathbf{[G]} \\
\mathbf{[E]} & \mathbf{[F]} & \mathbf{[G]} & \mathbf{[H]}
\end{bmatrix} \begin{bmatrix}
\varepsilon^{(0)} \\
\chi^{(0)} \\
\chi^{(1)} \\
\chi^{(2)}
\end{bmatrix} \tag{16}
\]

\[
\begin{bmatrix}
\mathbf{V} \\
\mathbf{S} \\
\mathbf{R}
\end{bmatrix} = \begin{bmatrix}
\mathbf{[A'] [B'] [D']} \\
\mathbf{[B'] [D'] [E']} \\
\mathbf{[D'] [E'] [F']}
\end{bmatrix} \begin{bmatrix}
\chi^{(0)} \\
\chi^{(1)} \\
\chi^{(2)}
\end{bmatrix} \tag{17}
\]

where \(\mathbf{N} = (N_x, N_y, N_{xx}, N_{yy})^T, \mathbf{M} = (M_{xy}, M_{yx}, M_{xyy}, M_{yx})^T, \overline{\mathbf{N}} = (\overline{N_x}, \overline{N_y}, \overline{N_{xx}}, \overline{N_{yy}})^T, \overline{\mathbf{M}} = (\overline{M_{xy}}, \overline{M_{yx}}, \overline{M_{xyy}}, \overline{M_{yx}})^T, \mathbf{V} = (V_x, V_y)^T, \mathbf{S} = (S_x, S_y)^T, \mathbf{R} = (R_x, R_y)^T\).

The elements of the reduced stiffness matrices of the core ([\(A\)], [\(B\)], etc.) are defined by

\[
(\mathbf{A}_o, \mathbf{B}_o, \mathbf{D}_o, \mathbf{E}_o, \mathbf{F}_o, \mathbf{G}_o, \mathbf{H}_o) = \int \frac{1}{2} \begin{bmatrix}
\Omega_\phi \\
\Omega_\theta
\end{bmatrix} \begin{bmatrix}
1, z, z^2, z^3, z^4, z^5 \end{bmatrix} dz \quad (i, j = 1, 2, 6) \tag{18}
\]

and,

\[
(\mathbf{A}_o', \mathbf{B}_o', \mathbf{D}_o', \mathbf{E}_o', \mathbf{F}_o') = \int \frac{1}{2} \begin{bmatrix}
\Omega_\phi' \\
\Omega_\theta'
\end{bmatrix} \begin{bmatrix}
1, z, z^2, z^4 \end{bmatrix} dz \quad (i, j = 4, 5) \tag{19}
\]

According to the first order shear deformation theory, the elements of reduced stiffness matrices of the face-sheets are given by

**Top face-sheet**

\[
(\mathbf{A}_o, \mathbf{R}_o, \mathbf{D}_o') = \int \frac{1}{2} \begin{bmatrix}
\Omega_\phi \\
\Omega_\theta
\end{bmatrix} \begin{bmatrix}
1, z, z^2 \end{bmatrix} dz = \sum_{k=1}^{n_{layer}} \int \frac{1}{2} \begin{bmatrix}
\Omega_\phi^{(k)} \\
\Omega_\theta^{(k)}
\end{bmatrix} \begin{bmatrix}
1, z, z^2 \end{bmatrix} dz \quad (i, j = 1, 2, 6) \tag{20}
\]

© 2016 IAU, Arak Branch
Bending Analysis of Composite Sandwich Plates with Laminated Faces

\[
\left( \overline{A}_{ij} \right) = \int_{Z_{1}}^{Z_{2}} \overline{Q}_{ij} \, dz = \sum_{k=1}^{n_{layer}} \int_{Z_{1}}^{Z_{2}} \overline{Q}_{ij}^{(k)} \, dz \quad \text{ (i, j = 4, 5)}
\]  \hspace{1cm} (21)

Bottom face-sheet

\[
\left( A_{ij}^{b}, B_{ij}^{b}, D_{ij}^{b} \right) = \int_{\frac{b}{2}}^{Z_{2}} \overline{Q}_{ij}^{(k)} \left( 1, z, z^{2} \right) \, dz = \sum_{k=1}^{n_{layer}} \int_{\frac{b}{2}}^{Z_{2}} \overline{Q}_{ij}^{(k)} \left( 1, z, z^{2} \right) \, dz \quad \text{ (i, j = 1, 2, 6)}
\]  \hspace{1cm} (22)

\[
\left( \overline{A}_{ij}^{b} \right) = \int_{\frac{b}{2}}^{0} \overline{Q}_{ij}^{(k)} \, dz = \sum_{k=1}^{n_{layer}} \int_{\frac{b}{2}}^{0} \overline{Q}_{ij}^{(k)} \, dz \quad \text{ (i, j = 4, 5)}
\]  \hspace{1cm} (23)

Fig.3 Coordinate locations of a sandwich plate having laminated composite faces.

3 Finite Element Formulation

The present element, named RSFT52, is a \( C^{6} \) continuous four-noded Rectangular Sandwich plate element with thirteen DOF per node; based on the combination of the first order shear deformation theory and the third-order shear deformation plate theory. Each face-sheet has only two rotational DOF per node and the core has nine DOF per node: six rotational degrees and three translation components which are common for the all sandwich layers (Fig.4(a)).

The nodal displacement vector of the present element is defined by

\[
\delta_{i} = \begin{bmatrix} u_{i} \quad v_{i} \quad w_{i} \quad \psi_{x i} \quad \psi_{y i} \quad \eta_{i} \quad \xi_{i} \quad \zeta_{i} \quad \psi_{x i}^{b} \quad \psi_{y i}^{b} \end{bmatrix}^{T}
\]

The field variables may be expressed as follows

a. At mid-plate

\[
u_{b}(x, y) = \sum_{i=1}^{4} N_{i}(x, y) \, u_{b_{i}} ; \quad v_{b}(x, y) = \sum_{i=1}^{4} N_{i}(x, y) \, v_{b_{i}} ; \quad w(x, y) = \sum_{i=1}^{4} N_{i}(x, y) \, w_{b_{i}}
\]  \hspace{1cm} (24)

b. Core

\[
\begin{align*}
\psi_{x}(x, y) & = \sum_{i=1}^{4} N_{i}(x, y) \, \psi_{x_{i}}^{c} , & \psi_{y}(x, y) & = \sum_{i=1}^{4} N_{i}(x, y) \, \psi_{y_{i}}^{c} , & \eta_{i}(x, y) & = \sum_{i=1}^{4} N_{i}(x, y) \, \eta_{i}^{c} \\
\xi_{i}(x, y) & = \sum_{i=1}^{4} N_{i}(x, y) \, \xi_{i}^{c} , & \zeta_{i}(x, y) & = \sum_{i=1}^{4} N_{i}(x, y) \, \zeta_{i}^{c} , & \psi_{x i}^{b c} & = \sum_{i=1}^{4} N_{i}(x, y) \, \psi_{x i}^{b c} \\
\psi_{y i}^{b c} & = \sum_{i=1}^{4} N_{i}(x, y) \, \psi_{y i}^{b c}
\end{align*}
\]  \hspace{1cm} (25)
c. Top face-sheet

\[
\psi'(x, y) = \sum_{i=1}^{4} N_i(x, y) \psi_{u_i}^t, \quad \psi'(x, y) = \sum_{i=1}^{4} N_i(x, y) \psi_{v_i}^t
\]

(26)

d. Bottom face-sheet

\[
\psi'(x, y) = \sum_{i=1}^{4} N_i(x, y) \psi_{u_i}^b, \quad \psi'(x, y) = \sum_{i=1}^{4} N_i(x, y) \psi_{v_i}^b
\]

(27)

where \( N_i \) are the interpolation functions [48] associated with the node \( i (i = 1, 2, 3, 4) \).

For the core, the generalized strain vector of the Eq. (2) at any point of coordinates \((x, y)\) can be expressed in terms of nodal displacements as follows:

\[
\{\epsilon^{(t)}\} = [B_{x}]^{T}(\{\delta\}), \quad \{\epsilon^{(b)}\} = [B_{x}]^{T}(\{\delta\}), \quad \{\epsilon^{(l)}\} = [B_{x}]^{T}(\{\delta\}), \quad \{\epsilon^{(r)}\} = [B_{x}]^{T}(\{\delta\})
\]

(28)

where the matrices \([B_{x}]^{(0)}, [B_{x}]^{(1)}, [B_{x}]^{(2)}\) and \([B_{x}]^{(3)}\), are related the strains to nodal displacements.

For the top face-sheet, the generalized strain–displacement matrices given by

\[
\{\epsilon^t\} = [B_{x}]^{(i)} (\{\delta\}), \quad \{\epsilon^b\} = [B_{x}]^{(i)} (\{\delta\}), \quad \{\epsilon^l\} = [B_{x}]^{(i)} (\{\delta\}), \quad \{\epsilon^r\} = [B_{x}]^{(i)} (\{\delta\})
\]

(29)

In the same way, the generalized strain–displacement matrices for the bottom face-sheet are

\[
\{\epsilon^t\} = [B_{x}]^{(i)} (\{\delta\}), \quad \{\epsilon^b\} = [B_{x}]^{(i)} (\{\delta\}), \quad \{\epsilon^l\} = [B_{x}]^{(i)} (\{\delta\}), \quad \{\epsilon^r\} = [B_{x}]^{(i)} (\{\delta\})
\]

(30)

Fig.4

Geometry and corresponding degrees of freedom of the RSFT52 element.

3.1 Stiffness matrix

To establish the relationship between the forces and displacements, the principle of virtual work is used.

\[
\delta V = U - \partial W = 0
\]

(31)

Herein \( U \) indicates the total strain energy in the sandwich and \( W \) represents the work done by the external forces. \( \delta \) denote the variation operator.
The total virtual strain energy of the three layers (top, core and bottom) may be written in the general form

$$\delta U = \delta U_1 + \delta U_2 + \delta U_3$$

where

$$\delta U = \int_A \left( \sigma^{(1)} \right)^T \delta \epsilon^{(1)} dv + \int_B \left( \sigma^{(2)} \right)^T \delta \epsilon^{(2)} dv + \int_C \left( \sigma^{(3)} \right)^T \delta \epsilon^{(3)} dv$$

(33)

The thickness of the sandwich plate is assumed to remain unchanged and, accordingly, $$\varepsilon_z = 0$$. The expression for the strain energy (Eq. 33) simplifies to

$$\delta U = \int_A \left( \sigma^{(1)}_{xx} \delta \epsilon^{(1)}_{xx} + \sigma^{(1)}_{yy} \delta \epsilon^{(1)}_{yy} + \sigma^{(1)}_{xy} \delta \epsilon^{(1)}_{xy} + \sigma^{(1)}_{yz} \delta \epsilon^{(1)}_{yz} + \sigma^{(1)}_{xz} \delta \epsilon^{(1)}_{xz} \right) dz \ dA_x$$

$$+ \int_B \left( \sigma^{(2)}_{xx} \delta \epsilon^{(2)}_{xx} + \sigma^{(2)}_{yy} \delta \epsilon^{(2)}_{yy} + \sigma^{(2)}_{xy} \delta \epsilon^{(2)}_{xy} + \sigma^{(2)}_{yz} \delta \epsilon^{(2)}_{yz} + \sigma^{(2)}_{xz} \delta \epsilon^{(2)}_{xz} \right) dz \ dA_t$$

$$+ \int_C \left( \sigma^{(3)}_{xx} \delta \epsilon^{(3)}_{xx} + \sigma^{(3)}_{yy} \delta \epsilon^{(3)}_{yy} + \sigma^{(3)}_{xy} \delta \epsilon^{(3)}_{xy} + \sigma^{(3)}_{yz} \delta \epsilon^{(3)}_{yz} + \sigma^{(3)}_{xz} \delta \epsilon^{(3)}_{xz} \right) dz \ dA_b$$

(34)

The virtual work done by the external forces given by

$$\delta W = \int_{-b}^{b} \int_{-a}^{a} f(x,y) \delta w \ dx \ dy$$

(35)

In which $$f(x,y)$$ is the transverse static load.

a. Stiffness matrix of the core

By substituting the expressions of the stress resultants (Eqs. (16) and (17)) into the strain energy expression, the principle of virtual work for the core becomes

$$\Pi = \int_{-b}^{b} \int_{-a}^{a} \left[ \left( \delta \epsilon^{(0)} \right)^T [A] [\epsilon^{(0)}] + \left( \delta \epsilon^{(0)} \right)^T [B] [\chi^{(0)}] + \left( \delta \epsilon^{(0)} \right)^T [D] [\chi^{(1)}] + \left( \delta \epsilon^{(0)} \right)^T [E] [\chi^{(2)}] \right] dx \ dy$$

$$+ \left( \delta \chi^{(0)} \right)^T [B] [\epsilon^{(0)}] + \left( \delta \chi^{(0)} \right)^T [D] [\chi^{(0)}] + \left( \delta \chi^{(0)} \right)^T [E] [\chi^{(2)}] + \left( \delta \chi^{(0)} \right)^T [F] [\chi^{(0)}]$$

$$+ \left( \delta \chi^{(2)} \right)^T [D] [\epsilon^{(0)}] + \left( \delta \chi^{(2)} \right)^T [E] [\chi^{(0)}] + \left( \delta \chi^{(2)} \right)^T [F] [\chi^{(0)}] + \left( \delta \chi^{(2)} \right)^T [G] [\chi^{(0)}]$$

$$+ \left( \delta \chi^{(2)} \right)^T [E] [\epsilon^{(0)}] + \left( \delta \chi^{(2)} \right)^T [F] [\chi^{(0)}] + \left( \delta \chi^{(2)} \right)^T [G] [\chi^{(0)}] + \left( \delta \chi^{(2)} \right)^T [H] [\chi^{(0)}]$$

$$+ \left( \delta \chi^{(0)} \right)^T [A^+] [\gamma^{(0)}] + \left( \delta \chi^{(0)} \right)^T [B^+] [\chi^{(0)}] + \left( \delta \chi^{(0)} \right)^T [D^+] [\chi^{(0)}] + \left( \delta \chi^{(0)} \right)^T [B^+] [\gamma^{(0)}]$$

$$+ \left( \delta \chi^{(2)} \right)^T [D^+] [\chi^{(0)}] + \left( \delta \chi^{(2)} \right)^T [E^+] [\chi^{(0)}] + \left( \delta \chi^{(2)} \right)^T [D^+] [\gamma^{(0)}] + \left( \delta \chi^{(2)} \right)^T [E^+] [\gamma^{(0)}]$$

(36)

$$+ \left( \delta \chi^{(2)} \right)^T [F^+] [\chi^{(0)}] \right) dx \ dy - \int_{-b}^{b} \int_{-a}^{a} f(x,y) \ dxdy = 0$$

with Eq. (28), the equilibrium equation can be expressed as follows
where \( \{ f_{e}^{(c)} \} \) is the load vector for the core, \( [K_{e}^{(c)}] \) is the element stiffness matrix of the core which may be defined by

\[
[K_{e}^{(c)}] = \sum_{e=1}^{n} \int_{b=b}^{a=a} \left( [B_{e}^{(0)}]^T [A] [B_{e}^{(0)}] + [B_{e}^{(0)}]^T [B] [B_{e}^{(1)}]^T + [B_{e}^{(0)}]^T [D] [B_{e}^{(2)}]^T \\
+ [B_{e}^{(0)}]^T [E] [B_{e}^{(1)}] + [B_{e}^{(1)}]^T [B] [B_{e}^{(0)}] + [B_{e}^{(1)}]^T [D] [B_{e}^{(1)}] + [B_{e}^{(1)}]^T [E] [B_{e}^{(0)}] \\
+ [B_{e}^{(1)}]^T [F] [B_{e}^{(3)}] + [B_{e}^{(3)}]^T [D] [B_{e}^{(0)}] + [B_{e}^{(3)}]^T [E] [B_{e}^{(1)}] + [B_{e}^{(3)}]^T [F] [B_{e}^{(1)}] \\
+ [B_{e}^{(2)}]^T [G] [B_{e}^{(1)}] + [B_{e}^{(1)}]^T [E] [B_{e}^{(0)}] + [B_{e}^{(1)}]^T [F] [B_{e}^{(3)}] + [B_{e}^{(1)}]^T [G] [B_{e}^{(3)}] \\
+ [B_{e}^{(3)}]^T [H] [B_{e}^{(3)}] + [B_{e}^{(0)}]^T [A^T] [B_{e}^{(0)}] + [B_{e}^{(0)}]^T [B^T] [B_{e}^{(1)}]^T + [B_{e}^{(0)}]^T [D^T] [B_{e}^{(2)}]^T \\
+ [B_{e}^{(1)}]^T [B^T] [B_{e}^{(0)}] + [B_{e}^{(0)}]^T [D^T] [B_{e}^{(1)}]^T + [B_{e}^{(0)}]^T [E^T] [B_{e}^{(2)}]^T \\
+ [B_{e}^{(2)}]^T [D^T] [B_{e}^{(0)}] + [B_{e}^{(2)}]^T [E^T] [B_{e}^{(1)}]^T + [B_{e}^{(2)}]^T [F^T] [B_{e}^{(2)}]^T \right) \, dxdy
\]

where \( \{ f_{e}^{(c)} \} \) is the load vector for the core, \( [K_{e}^{(c)}] \) is the element stiffness matrix of the core which may be defined by

b. Stiffness matrix of the face sheets

The same steps are followed to elaborate the stiffness matrix of the two face sheets, therefore:

**Top face-sheet**

\[
[K_{e}^{(t)}] = \sum_{e=1}^{n} \int_{b=b}^{a=a} \left( [B_{e}^{(0)}]^T [A] [B_{e}^{(0)}] + [B_{e}^{(0)}]^T [B] [B_{e}^{(1)}]^T + [B_{e}^{(0)}]^T [D] [B_{e}^{(2)}]^T \\
+ [B_{e}^{(0)}]^T [E] [B_{e}^{(1)}] + [B_{e}^{(1)}]^T [B] [B_{e}^{(0)}] + [B_{e}^{(1)}]^T [D] [B_{e}^{(1)}] + [B_{e}^{(1)}]^T [E] [B_{e}^{(0)}] \\
+ [B_{e}^{(1)}]^T [F] [B_{e}^{(3)}] + [B_{e}^{(3)}]^T [D] [B_{e}^{(0)}] + [B_{e}^{(3)}]^T [E] [B_{e}^{(1)}] + [B_{e}^{(3)}]^T [F] [B_{e}^{(1)}] \\
+ [B_{e}^{(2)}]^T [G] [B_{e}^{(1)}] + [B_{e}^{(1)}]^T [E] [B_{e}^{(0)}] + [B_{e}^{(1)}]^T [F] [B_{e}^{(3)}] + [B_{e}^{(1)}]^T [G] [B_{e}^{(3)}] \\
+ [B_{e}^{(3)}]^T [H] [B_{e}^{(3)}] + [B_{e}^{(0)}]^T [A^T] [B_{e}^{(0)}] + [B_{e}^{(0)}]^T [B^T] [B_{e}^{(1)}]^T + [B_{e}^{(0)}]^T [D^T] [B_{e}^{(2)}]^T \\
+ [B_{e}^{(1)}]^T [B^T] [B_{e}^{(0)}] + [B_{e}^{(0)}]^T [D^T] [B_{e}^{(1)}]^T + [B_{e}^{(0)}]^T [E^T] [B_{e}^{(2)}]^T \\
+ [B_{e}^{(2)}]^T [D^T] [B_{e}^{(0)}] + [B_{e}^{(2)}]^T [E^T] [B_{e}^{(1)}]^T + [B_{e}^{(2)}]^T [F^T] [B_{e}^{(2)}]^T \right) \, dxdy
\]

**Bottom-face sheet**

\[
[K_{e}^{(b)}] = \sum_{e=1}^{n} \int_{b=b}^{a=a} \left( [B_{e}^{(0)}]^T [A] [B_{e}^{(0)}] + [B_{e}^{(0)}]^T [B] [B_{e}^{(1)}]^T + [B_{e}^{(0)}]^T [D] [B_{e}^{(2)}]^T \\
+ [B_{e}^{(0)}]^T [E] [B_{e}^{(1)}] + [B_{e}^{(1)}]^T [B] [B_{e}^{(0)}] + [B_{e}^{(1)}]^T [D] [B_{e}^{(1)}] + [B_{e}^{(1)}]^T [E] [B_{e}^{(0)}] \\
+ [B_{e}^{(1)}]^T [F] [B_{e}^{(3)}] + [B_{e}^{(3)}]^T [D] [B_{e}^{(0)}] + [B_{e}^{(3)}]^T [E] [B_{e}^{(1)}] + [B_{e}^{(3)}]^T [F] [B_{e}^{(1)}] \\
+ [B_{e}^{(2)}]^T [G] [B_{e}^{(1)}] + [B_{e}^{(1)}]^T [E] [B_{e}^{(0)}] + [B_{e}^{(1)}]^T [F] [B_{e}^{(3)}] + [B_{e}^{(1)}]^T [G] [B_{e}^{(3)}] \\
+ [B_{e}^{(3)}]^T [H] [B_{e}^{(3)}] + [B_{e}^{(0)}]^T [A^T] [B_{e}^{(0)}] + [B_{e}^{(0)}]^T [B^T] [B_{e}^{(1)}]^T + [B_{e}^{(0)}]^T [D^T] [B_{e}^{(2)}]^T \\
+ [B_{e}^{(1)}]^T [B^T] [B_{e}^{(0)}] + [B_{e}^{(0)}]^T [D^T] [B_{e}^{(1)}]^T + [B_{e}^{(0)}]^T [E^T] [B_{e}^{(2)}]^T \\
+ [B_{e}^{(2)}]^T [D^T] [B_{e}^{(0)}] + [B_{e}^{(2)}]^T [E^T] [B_{e}^{(1)}]^T + [B_{e}^{(2)}]^T [F^T] [B_{e}^{(2)}]^T \right) \, dxdy
\]

Finally, the total stiffness matrix \( [K_T] \) of the element is given by

\[
[K_T] = [K^{(t)}] + [K^{(c)}] + [K^{(b)}]
\]
4 NUMERICAL RESULTS AND DISCUSSIONS

In this section, some problem of sandwich plates with laminated face sheets will be analyzed to assess the performance and applicability of the present finite element model. The obtained numerical results are compared with the analytical solutions given by Pagano [46], the experimental results obtained by Kanematsu et al. [47] and others finite elements numerical results found in literature. Table 1. shows the boundary conditions, for which the numerical results have been obtained, where CCCC, SSSS, CSCS and CFCF respectively indicate: clamped, simply supported, clamped-simply supported and clamped-free boundary conditions.

The following non-dimensional quantities used in the present analysis are defined as:

Non-dimensional transverse shear stresses

$$\left( \bar{\sigma}_{xz}, \bar{\sigma}_{yz} \right) = \frac{h}{q_0 a} \left( \sigma_{xz}, \sigma_{yz} \right)$$  \hspace{1cm} (42)

Non-dimensional transverse displacement

$$\bar{w} = \left( \frac{100 E_h h \bar{w}}{a^2 q_0} \right)$$ \hspace{1cm} (43)

<table>
<thead>
<tr>
<th>Boundary conditions</th>
<th>Abbreviations</th>
<th>Restrained edges</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simply supported</td>
<td>SSSS</td>
<td>( w_o = \psi_x = \eta_x = \zeta_x = 0 ) at ( x = \pm \frac{a}{2} ) \hspace{1cm}  ( \psi_y = \eta_y = \zeta_y = 0 ) at ( y = \pm \frac{b}{2} )</td>
</tr>
<tr>
<td>Clamped</td>
<td>CCCC</td>
<td>( w_o = \psi_x = \psi_y = \zeta_x = 0 ) \hspace{1cm} ( \eta_x = \eta_y = \zeta_y = 0 )</td>
</tr>
<tr>
<td>Clamped-Simply supported</td>
<td>CSCS</td>
<td>Clamped at ( x = \pm \frac{a}{2} ) \hspace{1cm} Simply supported at ( y = \pm \frac{b}{2} )</td>
</tr>
<tr>
<td>Clamped-Free</td>
<td>CFCF</td>
<td>Clamped at ( x = \pm \frac{a}{2} ) \hspace{1cm} Free at ( y = \pm \frac{b}{2} )</td>
</tr>
</tbody>
</table>

4.1 Square sandwich plate \( (0/90/C / 0/90) \) having two-ply laminated stiff sheets at the faces with different boundary conditions

In this example, an unsymmetrical square laminated sandwich plate, subjected to sinusoidal loading is studied. The thickness of each laminate layer is 0.05\( h \), whereas the thickness of the core is 0.8\( h \). The mechanical properties of materials used are listed in Table 2. Different thickness ratios \( (h/a = 0.01, 0.05, 0.1, 0.2, 0.25 \text{ and } 0.5) \) and three types of boundary conditions (CCCC, SCSC and SSSS,) are taken into account. The values of non-dimensional transverse displacement and transverse shear stresses obtained in the present analysis are presented with those obtained from the literature \([1, 18, 22, 49]\) in Table 3. The obtained results present a very good performance and also confirm the robustness of the RSPT52 element in terms of stability, rapidity of convergence and accuracy for both thin and thick plates.

Moreover, we also note when \( h/a = 0.5 \), the values of the transverse displacement are higher than those found with lower thickness ratios for both boundary conditions (CCCC and SCSC). This may be from the effect of
transverse flexibility of the core and shear deformation effects. Also, the thickness of the core (0.8h) which has an important role in the sandwich plates because, it considerably affects on the flexional rigidity.

Table 2
Material properties (normalized) used for laminated sandwiches plates.

<table>
<thead>
<tr>
<th>Materials</th>
<th>Elastic properties</th>
<th>$E_{ij}$</th>
<th>$E_{ij}$</th>
<th>$G_{ij}$</th>
<th>$G_{ij}$</th>
<th>$G_{ij}$</th>
<th>$\nu_{ij}$</th>
<th>$\nu_{ij}$ = $\nu_{ij}$ = $\nu_{ij}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Core</td>
<td>0.04E</td>
<td>0.04E</td>
<td>0.016E</td>
<td>0.06E</td>
<td>0.06E</td>
<td>0.25</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Face sheets</td>
<td>25E</td>
<td>E</td>
<td>0.5E</td>
<td>0.5E</td>
<td>0.2E</td>
<td>0.25</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3
Normalized maximum deflection ($w$) and stresses ($\tau_{xz}, \tau_{yz}$) at the important points of square sandwich plate with laminated facings (0/90/C/0/90) under distributed load of sinusoidal variation with different boundary conditions.

<table>
<thead>
<tr>
<th>$h/a$</th>
<th>References</th>
<th>FE Models</th>
<th>$\tau_{xz}$</th>
<th>$\tau_{yz}$</th>
<th>$w$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>$0, b, 0$</td>
<td>$a, 0, 0$</td>
<td></td>
</tr>
</tbody>
</table>

Boundary condition: CCCC

0.01
- Present (8×8) | RSFT52 | 0.1877 | 0.1877 | 0.1075 |
- Present (12×12) | RSFT52 | 0.1981 | 0.2008 | 0.1178 |
- Present (16×16) | RSFT52 | 0.2008 | 0.2190 | 0.2279 |
- Khandelwal et al. [1] | FEM-Q9-HZZT | 0.2198 | 0.2189 | 0.2286 |
- Pandit et al. [18] | FEM-Q9-HZZT | 0.2189 | 0.2189 | 0.2286 |
- Singh et al. [49] | FEM-Q8-HZZT | 0.2348 | - | 0.2260 |
- Chalak et al. [22] | FEM-Q9-HZZT | 0.2171 | 0.2327 | 0.2267 |

0.05
- Present (8×8) | RSFT52 | 0.1639 | 0.1639 | 0.4166 |
- Present (12×12) | RSFT52 | 0.1694 | 0.1694 | 0.4226 |
- Present (16×16) | RSFT52 | 0.1682 | 0.1682 | 0.4245 |
- Khandelwal et al. [1] | FEM-Q9-HZZT | 0.1661 | 0.1661 | 0.4299 |
- Pandit et al. [18] | FEM-Q9-HZZT | 0.1828 | 0.1828 | 0.4296 |
- Singh et al. [49] | FEM-Q8-HZZT | 0.2004 | - | 0.4462 |
- Chalak et al. [22] | FEM-Q9-HZZT | 0.1568 | 0.2319 | 0.4283 |

0.1
- Present (8×8) | RSFT52 | 0.1518 | 0.1518 | 1.0538 |
- Present (12×12) | RSFT52 | 0.1513 | 0.1513 | 1.0451 |
- Present (16×16) | RSFT52 | 0.1474 | 0.1474 | 1.0443 |
- Khandelwal et al. [1] | FEM-Q9-HZZT | 0.1380 | 0.1383 | 1.0513 |
- Pandit et al. [18] | FEM-Q9-HZZT | 0.1587 | 0.1586 | 1.0489 |
- Singh et al. [49] | FEM-Q8-HZZT | 0.1651 | - | 1.0213 |
- Chalak et al. [22] | FEM-Q9-HZZT | 0.1308 | 0.2175 | 1.0484 |

0.2
- Present (8×8) | RSFT52 | 0.1377 | 0.1377 | 3.4824 |
- Present (12×12) | RSFT52 | 0.1336 | 0.1336 | 3.4560 |
- Present (16×16) | RSFT52 | 0.1283 | 0.1283 | 3.4501 |
- Khandelwal et al. [1] | FEM-Q9-HZZT | 0.1240 | 0.1242 | 3.4741 |
- Pandit et al. [18] | FEM-Q9-HZZT | 0.1396 | 0.1394 | 3.4521 |
- Singh et al. [49] | FEM-Q9-HZZT | 0.1422 | - | 3.3421 |

0.25
- Present (8×8) | RSFT52 | 0.1325 | 0.1325 | 5.2499 |
- Present (12×12) | RSFT52 | 0.1279 | 0.1279 | 5.2517 |
- Present (16×16) | RSFT52 | 0.1227 | 0.1227 | 5.2046 |
- Chalak et al. [22] | FEM-Q9-HZZT | 0.1228 | 0.1231 | 5.2470 |

0.5
- Present (8×8) | RSFT52 | 0.1147 | 0.1147 | 18.9206 |
- Present (12×12) | RSFT52 | 0.1107 | 0.1107 | 18.7881 |
- Present (16×16) | RSFT52 | 0.1068 | 0.1068 | 18.7406 |
- Pandit et al. [18] | FEM-Q9-HZZT | 0.1227 | 0.1217 | 18.3454 |
- Khandelwal et al. [1] | FEM-Q9-HZZT | 0.1264 | 0.1265 | 19.1560 |
- Singh et al. [49] | FEM-Q8-HZZT | 0.1325 | - | 18.3450 |
- Chalak et al. [22] | FEM-Q9-HZZT | 0.1182 | 0.1427 | 19.0444 |

* Nine-node quadratic finite element solution based on higher order zigzag plate theory.
Table 3
Continued.
\[ \frac{h}{a} \quad \text{References} \quad \text{FE Models} \quad \bar{r}_{yz} \left( a, b, 0 \right) \quad \bar{r}_{xz} \left( a, b, 0 \right) \quad \bar{w} \left( a, b, 0 \right) \]

<table>
<thead>
<tr>
<th>Boundary condition: SCSC</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>Present (8×8)</td>
<td>RSFT52</td>
<td>0.0674</td>
</tr>
<tr>
<td></td>
<td>Present (12×12)</td>
<td>RSFT52</td>
<td>0.0703</td>
</tr>
<tr>
<td></td>
<td>Present (16×16)</td>
<td>RSFT52</td>
<td>0.0714</td>
</tr>
<tr>
<td></td>
<td>Khandelwal et al. [1]</td>
<td>FEM-Q9-HZZT</td>
<td>0.0782</td>
</tr>
<tr>
<td></td>
<td>Pandit et al. [18]</td>
<td>FEM-Q9-IHZZT</td>
<td>0.0778</td>
</tr>
<tr>
<td></td>
<td>Singh et al. [49]</td>
<td>FEM-Q8-HZZT</td>
<td>0.0944</td>
</tr>
<tr>
<td></td>
<td>Chalak et al. [22]</td>
<td>FEM-Q9-HZZT</td>
<td>0.0775</td>
</tr>
<tr>
<td>0.05</td>
<td>Present (8×8)</td>
<td>RSFT52</td>
<td>0.0928</td>
</tr>
<tr>
<td></td>
<td>Present (12×12)</td>
<td>RSFT52</td>
<td>0.0962</td>
</tr>
<tr>
<td></td>
<td>Present (16×16)</td>
<td>RSFT52</td>
<td>0.0974</td>
</tr>
<tr>
<td></td>
<td>Khandelwal et al. [1]</td>
<td>FEM-Q9-HZZT</td>
<td>0.1075</td>
</tr>
<tr>
<td></td>
<td>Pandit et al. [18]</td>
<td>FEM-Q9-IHZZT</td>
<td>0.1061</td>
</tr>
<tr>
<td></td>
<td>Singh et al. [49]</td>
<td>FEM-Q8-HZZT</td>
<td>0.1542</td>
</tr>
<tr>
<td></td>
<td>Chalak et al. [22]</td>
<td>FEM-Q9-HZZT</td>
<td>0.1012</td>
</tr>
<tr>
<td>0.1</td>
<td>Present (8×8)</td>
<td>RSFT52</td>
<td>0.1248</td>
</tr>
<tr>
<td></td>
<td>Present (12×12)</td>
<td>RSFT52</td>
<td>0.1288</td>
</tr>
<tr>
<td></td>
<td>Present (16×16)</td>
<td>RSFT52</td>
<td>0.1302</td>
</tr>
<tr>
<td></td>
<td>Khandelwal et al. [1]</td>
<td>FEM-Q9-HZZT</td>
<td>0.1437</td>
</tr>
<tr>
<td></td>
<td>Pandit et al. [18]</td>
<td>FEM-Q9-IHZZT</td>
<td>0.1418</td>
</tr>
<tr>
<td></td>
<td>Singh et al. [49]</td>
<td>FEM-Q8-HZZT</td>
<td>0.1523</td>
</tr>
<tr>
<td></td>
<td>Chalak et al. [22]</td>
<td>FEM-Q9-HZZT</td>
<td>0.1366</td>
</tr>
<tr>
<td>0.2</td>
<td>Present (8×8)</td>
<td>RSFT52</td>
<td>0.1483</td>
</tr>
<tr>
<td></td>
<td>Present (12×12)</td>
<td>RSFT52</td>
<td>0.1530</td>
</tr>
<tr>
<td></td>
<td>Present (16×16)</td>
<td>RSFT52</td>
<td>0.1546</td>
</tr>
<tr>
<td></td>
<td>Khandelwal et al. [1]</td>
<td>FEM-Q9-HZZT</td>
<td>0.1708</td>
</tr>
<tr>
<td></td>
<td>Pandit et al. [18]</td>
<td>FEM-Q9-IHZZT</td>
<td>0.1683</td>
</tr>
<tr>
<td></td>
<td>Singh et al. [49]</td>
<td>FEM-Q8-HZZT</td>
<td>0.1620</td>
</tr>
<tr>
<td>0.25</td>
<td>Present (8×8)</td>
<td>RSFT52</td>
<td>0.1515</td>
</tr>
<tr>
<td></td>
<td>Present (12×12)</td>
<td>RSFT52</td>
<td>0.1563</td>
</tr>
<tr>
<td></td>
<td>Present (16×16)</td>
<td>RSFT52</td>
<td>0.1579</td>
</tr>
<tr>
<td></td>
<td>Khandelwal et al. [1]</td>
<td>FEM-Q9-HZZT</td>
<td>0.1747</td>
</tr>
<tr>
<td></td>
<td>Chalak et al. [22]</td>
<td>FEM-Q9-HZZT</td>
<td>0.1667</td>
</tr>
<tr>
<td>0.5</td>
<td>Present (8×8)</td>
<td>RSFT52</td>
<td>0.1490</td>
</tr>
<tr>
<td></td>
<td>Present (12×12)</td>
<td>RSFT52</td>
<td>0.1538</td>
</tr>
<tr>
<td></td>
<td>Present (16×16)</td>
<td>RSFT52</td>
<td>0.1555</td>
</tr>
<tr>
<td></td>
<td>Pandit et al. [18]</td>
<td>FEM-Q9-IHZZT</td>
<td>0.1691</td>
</tr>
<tr>
<td></td>
<td>Khandelwal et al. [1]</td>
<td>FEM-Q9-HZZT</td>
<td>0.1744</td>
</tr>
<tr>
<td></td>
<td>Singh et al. [49]</td>
<td>FEM-Q8-HZZT</td>
<td>0.1721</td>
</tr>
<tr>
<td></td>
<td>Chalak et al. [22]</td>
<td>FEM-Q9-HZZT</td>
<td>0.1659</td>
</tr>
</tbody>
</table>

\( ^{\text{c}} \) Nine-node quadratic finite element solution based on improved higher order zigzag plate theory.
\( ^{\text{d}} \) Eight-nodes quadratic finite element solution based on higher order zigzag plate theory.

Table 3
Continued.
\[ \frac{h}{a} \quad \text{References} \quad \text{FE Models} \quad \bar{r}_{xz} \left( \frac{a}{2}, 0, 0 \right) \quad \bar{r}_{yz} \left( \frac{a}{2}, b, 0 \right) \quad \bar{w} \left( \frac{a}{2}, \frac{b}{2}, 0 \right) \]

<table>
<thead>
<tr>
<th>Boundary condition: SSSS</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>Present (8×8)</td>
<td>RSFT52</td>
<td>0.1787</td>
</tr>
<tr>
<td></td>
<td>Present (12×12)</td>
<td>RSFT52</td>
<td>0.1797</td>
</tr>
<tr>
<td></td>
<td>Present (16×16)</td>
<td>RSFT52</td>
<td>0.1791</td>
</tr>
<tr>
<td></td>
<td>Pagano [46]</td>
<td>Elasticity solution</td>
<td>0.1773</td>
</tr>
<tr>
<td></td>
<td>Khandelwal et al. [1]</td>
<td>FEM-Q9-HZZT</td>
<td>0.1878</td>
</tr>
</tbody>
</table>
The previous example has been adopted. The plate has a stratification of angle-ply laminated stiff sheets at the center of the bottom edge. In this example, three orientation angles on the face sheets (0°, 30° and 45°) are considered. The non-dimensional results of transverse displacement at the center of the plate \(x = a/2, y = b/2 \text{ et } z = 0\), the transverse shear stresses \(\bar{\tau}_{xz}\) at the center of the left edge \((x = 0, y = a/2 \text{ et } z = \pm 0.4h)\) and the transverse shear stresses \(\bar{\tau}_{yz}\) at the center of the bottom edge \((x = a/2, y = 0 \text{ et } z = \pm 0.4h)\) of the plate, obtained in the present analysis, for three orientation angles on the face sheets \(0°, 30° \text{ and } 45°\), are tabulated in Table 4. The results obtained by the present element RSFT52 are in excellent agreement with those obtained by Chakrabarti and Sheikh [17] et Khandelwal et al. [1].

### Table 4

<table>
<thead>
<tr>
<th>(\frac{h}{a})</th>
<th>References</th>
<th>FE Models</th>
<th>(\bar{\tau}_{xz})</th>
<th>(\bar{\tau}_{yz})</th>
<th>(\bar{w})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\theta = 0°)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.2</td>
<td>Present ((8x8))</td>
<td>RSFT52</td>
<td>0.3033</td>
<td>0.3033</td>
<td>6.3228</td>
</tr>
<tr>
<td></td>
<td>Present ((12x12))</td>
<td>RSFT52</td>
<td>0.3242</td>
<td>0.3242</td>
<td>6.2940</td>
</tr>
<tr>
<td></td>
<td>Present ((16x16))</td>
<td>RSFT52</td>
<td>0.3343</td>
<td>0.3343</td>
<td>6.2909</td>
</tr>
<tr>
<td></td>
<td>Pagano [46]</td>
<td>Elasticity solution</td>
<td>0.3436</td>
<td>0.3413</td>
<td>6.2981</td>
</tr>
<tr>
<td></td>
<td>Khandelwal et al. [1]</td>
<td>FEM-Q9-HZZT (Const)*</td>
<td>0.3982</td>
<td>0.3993</td>
<td>6.3001</td>
</tr>
<tr>
<td></td>
<td>Khandelwal et al. [1]</td>
<td>FEM-Q9-HZZT (Equil)**</td>
<td>0.3455</td>
<td>0.3332</td>
<td>6.3001</td>
</tr>
<tr>
<td></td>
<td>Chakrabarti and Sheikh [17]</td>
<td>FEM-T6-HZZT</td>
<td>0.3482</td>
<td>-</td>
<td>6.3016</td>
</tr>
<tr>
<td>0.1</td>
<td>Present ((8x8))</td>
<td>RSFT52</td>
<td>0.3110</td>
<td>0.3110</td>
<td>2.6104</td>
</tr>
<tr>
<td></td>
<td>Present ((12x12))</td>
<td>RSFT52</td>
<td>0.3337</td>
<td>0.3337</td>
<td>2.6149</td>
</tr>
<tr>
<td></td>
<td>Present ((16x16))</td>
<td>RSFT52</td>
<td>0.3447</td>
<td>0.3447</td>
<td>2.6168</td>
</tr>
<tr>
<td></td>
<td>Pagano [46]</td>
<td>Elasticity</td>
<td>0.3496</td>
<td>0.3503</td>
<td>2.6168</td>
</tr>
<tr>
<td></td>
<td>Khandelwal et al. [1]</td>
<td>FEM-Q9-HZZT (Const)</td>
<td>0.4142</td>
<td>0.4366</td>
<td>2.6295</td>
</tr>
<tr>
<td></td>
<td>Khandelwal et al. [1]</td>
<td>FEM-Q9-HZZT (Equil)</td>
<td>0.3489</td>
<td>0.3397</td>
<td>2.6168</td>
</tr>
<tr>
<td></td>
<td>Chakrabarti and Sheikh [17]</td>
<td>FEM-T6-HZZT</td>
<td>0.3612</td>
<td>-</td>
<td>2.6296</td>
</tr>
</tbody>
</table>
### Table 4

Continued.

<table>
<thead>
<tr>
<th>( \theta = 30^\circ )</th>
<th>References</th>
<th>FE Models</th>
<th>( \bar{\tau}_{xz} )</th>
<th>( \bar{\tau}_{yz} )</th>
<th>( \bar{w} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Present (8\times8)</td>
<td>RSFT52</td>
<td>0.3185</td>
<td>0.3185</td>
<td>5.9480</td>
<td></td>
</tr>
<tr>
<td>Present (12\times12)</td>
<td>RSFT52</td>
<td>0.3382</td>
<td>0.3382</td>
<td>5.9326</td>
<td></td>
</tr>
<tr>
<td>Present (16\times16)</td>
<td>RSFT52</td>
<td>0.3472</td>
<td>0.3472</td>
<td>5.9308</td>
<td></td>
</tr>
<tr>
<td>Khandelwal et al. [1]</td>
<td>FEM-Q9-HZZT (Const)</td>
<td>0.4376</td>
<td>0.4321</td>
<td>5.9579</td>
<td></td>
</tr>
<tr>
<td>Chakrabarti and Sheikh [17]</td>
<td>FEM-T6-HZZT</td>
<td>0.3064</td>
<td>0.2951</td>
<td>5.9579</td>
<td></td>
</tr>
<tr>
<td>Present (8\times8)</td>
<td>RSFT52</td>
<td>0.3211</td>
<td>0.3211</td>
<td>2.2081</td>
<td></td>
</tr>
<tr>
<td>Present (12\times12)</td>
<td>RSFT52</td>
<td>0.3406</td>
<td>0.3406</td>
<td>2.2137</td>
<td></td>
</tr>
<tr>
<td>Present (16\times16)</td>
<td>RSFT52</td>
<td>0.3503</td>
<td>0.3503</td>
<td>2.2176</td>
<td></td>
</tr>
<tr>
<td>Khandelwal et al. [1]</td>
<td>FEM-Q9-HZZT (Const)</td>
<td>0.4280</td>
<td>0.4409</td>
<td>2.2322</td>
<td></td>
</tr>
<tr>
<td>Khandelwal et al. [1]</td>
<td>FEM-Q9-HZZT (Equil)</td>
<td>0.3275</td>
<td>0.3200</td>
<td>2.2322</td>
<td></td>
</tr>
<tr>
<td>Chakrabarti and Sheikh [17]</td>
<td>FEM-T6-HZZT</td>
<td>0.3659</td>
<td>-</td>
<td>2.2237</td>
<td></td>
</tr>
<tr>
<td>Present (8\times8)</td>
<td>RSFT52</td>
<td>0.3261</td>
<td>0.3261</td>
<td>1.2028</td>
<td></td>
</tr>
<tr>
<td>Present (12\times12)</td>
<td>RSFT52</td>
<td>0.3465</td>
<td>0.3465</td>
<td>1.2199</td>
<td></td>
</tr>
<tr>
<td>Present (16\times16)</td>
<td>RSFT52</td>
<td>0.3570</td>
<td>0.3570</td>
<td>1.2270</td>
<td></td>
</tr>
<tr>
<td>Khandelwal et al. [1]</td>
<td>FEM-Q9-HZZT (Const)</td>
<td>0.4637</td>
<td>0.5097</td>
<td>1.2450</td>
<td></td>
</tr>
<tr>
<td>Khandelwal et al. [1]</td>
<td>FEM-Q9-HZZT (Equil)</td>
<td>0.3592</td>
<td>0.3480</td>
<td>1.2450</td>
<td></td>
</tr>
<tr>
<td>Chakrabarti and Sheikh [17]</td>
<td>FEM-T6-HZZT</td>
<td>0.3603</td>
<td>0.3603</td>
<td>1.2381</td>
<td></td>
</tr>
</tbody>
</table>

* Values of transverse shear stresses obtained from constitutive relation.
** Values of transverse shear stresses obtained from equilibrium equations.

<table>
<thead>
<tr>
<th>( \theta = 45^\circ )</th>
<th>References</th>
<th>FE Models</th>
<th>( \bar{\tau}_{xz} )</th>
<th>( \bar{\tau}_{yz} )</th>
<th>( \bar{w} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Present (8\times8)</td>
<td>RSFT52</td>
<td>0.2955</td>
<td>0.2955</td>
<td>5.6998</td>
<td></td>
</tr>
<tr>
<td>Present (12\times12)</td>
<td>RSFT52</td>
<td>0.3144</td>
<td>0.3144</td>
<td>5.6615</td>
<td></td>
</tr>
<tr>
<td>Present (16\times16)</td>
<td>RSFT52</td>
<td>0.3231</td>
<td>0.3231</td>
<td>5.6516</td>
<td></td>
</tr>
<tr>
<td>Khandelwal et al. [1]</td>
<td>FEM-Q9-HZZT (Const)</td>
<td>0.4612</td>
<td>0.4484</td>
<td>5.6329</td>
<td></td>
</tr>
<tr>
<td>Khandelwal et al. [1]</td>
<td>FEM-Q9-HZZT (Equil)</td>
<td>0.3399</td>
<td>0.3187</td>
<td>5.6329</td>
<td></td>
</tr>
<tr>
<td>Chakrabarti and Sheikh [17]</td>
<td>FEM-T6-HZZT</td>
<td>0.2197</td>
<td>-</td>
<td>5.6079</td>
<td></td>
</tr>
<tr>
<td>Present (8\times8)</td>
<td>RSFT52</td>
<td>0.3025</td>
<td>0.3025</td>
<td>2.0132</td>
<td></td>
</tr>
<tr>
<td>Present (12\times12)</td>
<td>RSFT52</td>
<td>0.3223</td>
<td>0.3223</td>
<td>2.0035</td>
<td></td>
</tr>
<tr>
<td>Present (16\times16)</td>
<td>RSFT52</td>
<td>0.3320</td>
<td>0.3320</td>
<td>2.0019</td>
<td></td>
</tr>
<tr>
<td>Khandelwal et al. [1]</td>
<td>FEM-Q9-HZZT (Const)</td>
<td>0.4445</td>
<td>0.4592</td>
<td>1.9950</td>
<td></td>
</tr>
<tr>
<td>Khandelwal et al. [1]</td>
<td>FEM-Q9-HZZT (Equil)</td>
<td>0.3375</td>
<td>0.3232</td>
<td>1.9950</td>
<td></td>
</tr>
<tr>
<td>Chakrabarti and Sheikh [17]</td>
<td>FEM-T6-HZZT</td>
<td>0.2203</td>
<td>-</td>
<td>1.9764</td>
<td></td>
</tr>
<tr>
<td>Present (8\times8)</td>
<td>RSFT52</td>
<td>0.3034</td>
<td>0.3034</td>
<td>1.0615</td>
<td></td>
</tr>
<tr>
<td>Present (12\times12)</td>
<td>RSFT52</td>
<td>0.3215</td>
<td>0.3215</td>
<td>1.0671</td>
<td></td>
</tr>
<tr>
<td>Present (16\times16)</td>
<td>RSFT52</td>
<td>0.3311</td>
<td>0.3311</td>
<td>1.0697</td>
<td></td>
</tr>
<tr>
<td>Khandelwal et al. [1]</td>
<td>FEM-Q9-HZZT (Const)</td>
<td>0.4621</td>
<td>0.5309</td>
<td>1.0773</td>
<td></td>
</tr>
<tr>
<td>Khandelwal et al. [1]</td>
<td>FEM-Q9-HZZT (Equil)</td>
<td>0.3419</td>
<td>0.3335</td>
<td>1.0773</td>
<td></td>
</tr>
<tr>
<td>Chakrabarti and Sheikh [17]</td>
<td>FEM-T6-HZZT</td>
<td>0.2197</td>
<td>-</td>
<td>1.0615</td>
<td></td>
</tr>
</tbody>
</table>

### 4.3 Comparison with experimental results CFRP-faced rectangular sandwich plates

The objective of this problem is to describe the practical application of the present element with the experimental results available in the literature. Kanematsu et al. [47] have performed out experiments on rectangular (450 mm \times 300 mm) sandwich plates composed of CFRP face sheets and aluminum honeycomb core, for clamped...
edge conditions. Each face sheet is symmetrically laminated with three carbon-epoxy layers, where the thickness of each layer is 0.125 mm. The mechanical properties of the plate are presented in Table 5. In this study, four different types (designated as SP1, SP2, SP3 and SP4) of stacking sequences, as shown in Fig. 5, for the faces of sandwich plate, are considered. The stacking sequences are: [30/30/30], [0/0/0], [30/−30/30] and [0/90/0], respectively, for SP1, SP2, SP3 et SP4. The thickness of the core is 10 mm for specimens SP1 and SP2, and 7 mm for specimens SP3 and SP4. The plate is subjected to a uniformly distributed load \( P = 1,010 \, \text{KPa} \). This load is controlled by a manometer and the transverse displacement of the plate was measured with a linearly variable differential transducer (LVDT). A holographic technique is used in order to visualize the transverse displacement modes.

Furthermore, the authors also have provided analytical solutions, based on the Rayleigh-Ritz method, for the same problem of plate, using two types of boundary conditions, simply supported (SSSS) and clamped (CCCC). The results of transverse displacement \( w \), obtained from the present element, are presented in Table 6, using a mesh size of \((16 \times 16)\). The comparison was made with the analytical solutions given by Kanematsu et al. [47], the solutions based on experimental works presented by the same authors [47], as well as with the finite element models of Lee et Fan [28] and Nayak et al. [13]. The results of the comparison show the effectiveness and reliability of the present element in the analysis of this type of structures.

### Table 5

<table>
<thead>
<tr>
<th>Materials</th>
<th>Elastic properties (GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carbon/epoxy</td>
<td>( E_1 )</td>
</tr>
<tr>
<td>Al/Honeycomb</td>
<td>105</td>
</tr>
</tbody>
</table>

### Table 6

<table>
<thead>
<tr>
<th>Reference</th>
<th>FE Models</th>
<th>Central displacement ( w \left( \frac{a}{2}, \frac{b}{2}, 0 \right) ) (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Present element</td>
<td>RSFT52</td>
<td>SP1</td>
</tr>
<tr>
<td>Nayak et al. [14]</td>
<td>FEM-Q9-HSDT(^d)</td>
<td>0.04906</td>
</tr>
<tr>
<td>Kanematsu et al. [47]</td>
<td>Analytical solution</td>
<td>0.05040</td>
</tr>
<tr>
<td>Kanematsu et al. [47]</td>
<td>Experimental solution</td>
<td>0.06900</td>
</tr>
<tr>
<td>Lee et Fan [28]</td>
<td>FEM-Q9-LWT(^e)</td>
<td>0.05190</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Boundary condition: SSSS</th>
<th></th>
<th>SP1</th>
<th>SP2</th>
<th>SP3</th>
<th>SP4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Present element</td>
<td>RSFT52</td>
<td>0.1160</td>
<td>0.1733</td>
<td>0.1695</td>
<td>0.2010</td>
</tr>
<tr>
<td>Nayak et al. [14]</td>
<td>FEM-Q9-HSDT</td>
<td>-</td>
<td>0.1754</td>
<td>-</td>
<td>0.2111</td>
</tr>
<tr>
<td>Kanematsu et al. [47]</td>
<td>Analytical solution</td>
<td>0.1173</td>
<td>0.1829</td>
<td>0.1794</td>
<td>0.2206</td>
</tr>
<tr>
<td>Lee et Fan [28]</td>
<td>FEM-Q9-LWT(^e)</td>
<td>0.1213</td>
<td>0.1774</td>
<td>0.1729</td>
<td>0.2138</td>
</tr>
</tbody>
</table>

\(^d\) Nine-node quadratic finite element solution based on higher order shear deformation plate theory

\(^e\) Nine-node quadratic finite element solution based on layer wise plate theory

![Fig. 5](image_url) Sandwich plate cross sections.
4.4 Influence of the material and geometric properties on the transverse displacement

After verifying the accuracy and efficiency of the element against the known cases in the literature, we present below a parametric study to further understand the bending behaviour of sandwich plates having laminated composites face sheets.

This study aims to demonstrate the influence and evolution of different parameters (core thickness, side-to-thickness ratio, boundary conditions, aspect ratio, core-face sheet anisotropy ratio and degree of orthotropy of the face sheet) on the transverse displacement. The analysis is carried out for a square sandwich plate (0/90/0/C/0/90/0) subjected to sinusoidal loading and a thickness ratio (a/h = 10), where h is the total thickness of the plate. The core is made of HEREX-C70.130 closed-cell PVC (polyvinyl chloride) foam and the face sheets are made of glass fiber in polyester resin. The mechanical properties of the face sheets and the core can be found in Table 7. [50]. The thickness of the core is 7 mm, while the face sheets are considered as symmetric cross-ply laminates [0/90/0], where the thickness of each layer is 0.125 mm.

<table>
<thead>
<tr>
<th>Materials</th>
<th>Elastic properties (GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>E₁</td>
</tr>
<tr>
<td>Glass / polyester</td>
<td>24.51</td>
</tr>
<tr>
<td>HEREX-C70.130 foam</td>
<td>0.1036</td>
</tr>
</tbody>
</table>

The curves of Fig.6 show the effect of the relative thickness of the core (hᵣ/h), represented by the ratio (hᵣ/h), on the non-dimensional transverse displacement (w), with different type of boundary conditions (SSSS, CCCC, CFCF and CSCS). It can be seen that the values of transverse displacement increases with increasing (hᵣ/h) ratio, whatever boundary conditions. This elevation is probably due to the decrease of the flexural rigidity.

The variation of the non-dimensional transverse displacement (w) with different thickness ratios (a/h), has been plotted as shown in Fig.7, for the two cases of boundary conditions; simply supported (SSSS) and clamped (CCCC). It was seen that, for both boundary conditions, the values of transverse displacement decreases with increased a/h ratio up to a/h = 20 and then varies constantly in all cases. This can be explained by the effects of shear deformation and transverse flexibility of the core, which are more significant when the thickness increases. Furthermore, the core occupies major portion (90%) of the plate thickness, so the core compressibility plays very important role for highly thick plates.

The influence of aspect ratio (a/b) on the non-dimensional transverse displacement of a simply supported sandwich plate, is presented in Fig. 8. It is found that the variation of the transverse displacement decrease with increase in aspect ratio.

The next part concerns the study of the effect of the core-to-face sheet anisotropy ratio (E₁/Eₓ), of one part, and the ratio of the core shear modulus to the flexural modulus (Gₓ/Eₓ), of the other part, on the non-dimensional transverse displacement of a simply supported sandwich plate. In this study, the Young's modulus of the core (Eₓ) has been varied for each test, while the Young's modulus of the face sheet (E₁) is fixed. As shown in Fig. 9, the curves demonstrated that the transverse displacement is affected considerably by the variation of the modular ratio (E₁/Eₓ). It observed that, the values of the transverse displacement increase when the values of the anisotropy ratio increases, whatever the (Gₓ/Eₓ) ratio. This increase is probably due to the decrease in flexural stiffness caused by excess material of the core.

Fig.10 shows the effect of the degree of orthotropy of the faces (E₁/E₂) and the ratio h_c/h, on the non-dimensional transverse displacement (w), of a simply supported sandwich plate. From Fig. 10, it can be seen that, for different core thickness (h_c/h), the non-dimensional central deflection decrease with increase of the degree of orthotropy of the faces. It is a well-known fact, from the literature, that an increase in the stiffness of a sandwich plate leads to a decrease in its transverse displacement. A stiffness increase could be due to a decrease in the relative thickness of the core or to an increase in the degree of orthotropy of the faces.

© 2016 IAU, Arak Branch
Fig. 6
Effect of core thickness on the non-dimensional transverse displacement, of a sandwich plate (0/90/0/C/0/90/0) with different boundary conditions.

Fig. 7
Effect of plate side-to-thickness ratio, $a/h$, on the non-dimensional transverse displacement, of a sandwich plate (0/90/0/C/0/90/0) with two types of boundary conditions (SSSS and CCCC).

Fig. 8
Effect of plate aspect ratio, $a/b$, on the non-dimensional transverse displacement, of a simply supported sandwich plate (0/90/0/C/0/90/0).

Fig. 9
Effect of the modulus ratio, $E_1/E_2$, and ratio $G_c/E_c$, on the non-dimensional transverse displacement, of a simply supported sandwich plate (0/90/0/C/0/90/0).

Fig. 10
Effect of the degree of orthotropy of the face sheet, $E_1/E_2$, and ratio $h_c/h$, on the non-dimensional transverse displacement, of a simply supported sandwich plate (0/90/0/C/0/90/0).
5 CONCLUSIONS

In this paper, a study of bending behaviour of composites sandwich plates with laminated face sheets has been undertaken, using a new four-noded rectangular finite element formulation based on a layer-wise theory. The model is based on a proper combination of higher order and first order, shear deformation theories. Compatibility conditions as well as continuity of displacements, at the interface have been explicitly satisfied in the present formulation. Although the model is a layer-wise one, the number of unknowns is independent of the number of layers. Thus, the plate theory enjoys the advantage of a single-layer plate theory, even though it is based on the concept of a layer-wise plate approach. The results obtained for the transverse displacement and the transvers shear stresses, of a composite sandwich plate show very good performance of the present formulation. Hence, the potential of the element can be used to generate a number of new results, which are expected to be useful to the future research in this field.

In addition, the parametric effects of core thickness, side-to-thickness ratio, boundary conditions, plate aspect ratio, core-to-face sheet anisotropy ratio, core shear modulus to the flexural modulus ratio and degree of orthotropy of the face sheet, on the transverse displacement are discussed. The results reaffirm that these effects play an important role in the transverse displacement of composite sandwich plates with laminated face sheets.

REFERENCES