Journal of Solid Mechanics Vol. 8, No. 4 (2016) pp. 773-780

Analyzing Frequency of Conical (Δ shaped) Tanks

F. Saljughi *

National Iranian Oil Company (NIOC), Iran

Received 18 July 2016; accepted 21 September 2016

ABSTRACT

A finite element analysis is presented for sloshing and impulsive motion of liquid-filled conical tanks during lateral anti-symmetric excitation. The performed analyses led to the development of a number of charts which can be used to identify the natural frequency, the mode shapes of conical tanks for both fundamental and the cos(θ)-modes of vibration. Conical tank geometry was described with several parameters namely, bottom radius(R_b) total height of liquid (h), angle of inclination of the tanks(θ_i), as variables. Numerical result of the free vibration was obtained for the cases of conical tanks with θ_i=0 and compared with existing experiments and other predicated results, showing a good agreement between the experiment and numerical results.

Keywords : Conical shell; Modal characteristic; Finite element method; Apex angle; Natural frequency.

1 INTRODUCTION

THE problem of liquid motion in axisymmetric tank has been a subject of interest in aerospace and civil engineering fields as exemplified by being applied to fuel sloshing, particularly the seismic analysis. The earlier study relevant to this topic was performed by Jacobson and Ayre [1], Graham and Rodriguez [2], and Abramson [3] among others. Linear liquid sloshing in conical tanks was reported by Feschenko et al. [4], Dokuchaev [5], and H. N. Abramson [6] , to analyze the hydrodynamic loads in rocket tanks. Mikishev and Dorozhkin [7] and Bauer [8] reported their experimental measurements of the lowest natural sloshing frequency. Linear liquid sloshing dynamics in v-shaped conical tanks was evaluated as a test problem for debugging the Computational Fluid Dynamics (CFD) methods (e.g. Bauer and Eidel [9], Lukovsky and Bilyk [10], Schiffner [11]. Yamaki and Gupta [12-13], developed an analytical method for free vibration of a clamped cylindrical shell filled with an ideal fluid using the Galerkin procedure. An experimental study was carried out and verified with FEM by Mazuch et al. [14]. Han and Liu [15], considered tanks with axial non -uniformity of the thickness when studying the same problem. Jeong and Kim[16], also developed a theoretical method for the free vibration of a circular cylindrical shell filled with a compressible bounded fluid using Fourier series expansion method. Jeong [17], investigated the effects of compressibility of fluid and fluid density on the coupled natural frequencies of a cylindrical tank filled with compressible fluid. It is important to study the effect of hydrodynamic loads caused by liquid motions in a full tank on the supporting tower. Such consideration could be modeled by a different theory. Since seismic events are always possible, the effect of hydrodynamic loads caused by liquid motions in a water tank on the supporting tower must be calculated. To model these events, equivalent mechanical models can be used, which relate liquid dynamics to oscillations of mass- spring systems [18-20]. The eigen frequencies of those mechanical systems should coincide with the lower natural sloshing frequencies. The reason is that the lower natural sloshing and impulsive mode are characterized by a relatively low

*Corresponding author. Tel.: +98 9161144650..
E-mail address: saljughi.f@nisoc.ir (F. Saljughi).

damping and, therefore, give a dominating contribution to the hydrodynamic moment and force applied to the tank wall. It is absolutely necessary to predict these natural frequencies and modes accurately (Dammaty et al. [21]). Housner [22], developed a theoretical lumped mass model of ground supported liquid storage tank with two degrees of freedom associated with impulsive mass and sloshing mass to investigate the seismic response. The problem of determination of the hydrodynamic pressure relies on the concept of fundamental natural period of the tank-liquid system.

The motion of fluid coupled with the shell is resolved to by means of the velocity potential flow theory. In order to compute the normalized natural frequencies that illustrate the fluid effect on a fluid-coupled system, finite element analyses of fluid-filled conical tanks are carried out. Also, the effect of angle of inclination on the frequencies is investigated. However, few theoretical studies on free vibration of fluid-filled conical tanks were taken into consideration. This study attempts to suggest an analytical approach which can calculate the natural frequencies of a conical (Δ -shaped) tank. A schematic view of conical (Δ -shaped) tanks is depicted in Fig .1.

Fig.1
Picture of conical tanks (Δshape) located in Iran.

2 THEORY

2.1 Governing equations

The response of the liquid is governed by the system of differential equations,

$$\frac{\partial^2 \phi}{\partial r^2}+\frac{1}{r}\frac{\partial \phi}{\partial r}+\frac{1}{r^2}\frac{\partial^2 \phi}{\partial \theta^2}-\frac{\partial^2 d}{\partial z \partial t}=0 \tag{1}$$

And

$$\frac{\partial}{\partial z}\left(\rho \frac{\partial \phi}{\partial t}\right)+\rho \frac{\partial^2 d}{\partial t^2}-\frac{\partial \rho}{\partial z} g d=0 \tag{2}$$

In which $d=d(r,z,\theta,t)$ is the vertical sloshing displacement of the liquid at arbitrary point and time, and $\phi=\phi(r,z,\theta,t)$ is a velocity potential function which is related to the hydrodynamic pressure, $p=p(r,z,\theta,t)$, by

$$p=\rho \frac{\partial \phi}{\partial t} \tag{3}$$

And to radial and tangential components of liquid velocity, υ_r and υ_θ , by

$$\upsilon_r=-\frac{\partial \phi}{\partial r} \qquad \upsilon\theta=-\frac{1}{r}\frac{\partial \phi}{\partial \theta} \tag{4}$$

Eqs. (1) and (2) are deduced from more general expressions presented by Yih [24] by expressing the latter in cylindrical coordinates and specializing them to the incompressible liquid considered herein. For a homogeneous liquid with ρ =constant,

$$\frac{\partial d}{\partial t}=-\frac{\partial \phi}{\partial z} \tag{5}$$

And Eq. (1) is reduced to the well-known Laplace's equation $\nabla^2\phi=0.$ The solutions of Eqs. (1) and (2) must satisfy the continuity of radial velocities at the tank-wall, defined by

$$(\frac{\partial \phi}{\partial r})_{r=R}=-\dot{x}_g(t)\cos\theta \tag{6}$$

The condition of no vertical motion at the tank-base is defined by

$$\left(d\right)_{z=0}=0 \tag{7}$$

And the linearized pressure condition at the free liquid surface is defined by

$$\left(\frac{\partial \phi}{\partial t}-gd\right)_{z=h}=0 \tag{8}$$

where, 'g' is the acceleration of gravity.

2.2 Sloshing component

The equation governing the liquid motion under arbitrary horizontal base excitation can be expressed in terms of the relative Cartesian co-ordinates (*x, y*) and the relative velocity potential ϕ_s as follows:

$$\nabla^2\phi_s=0 \tag{9}$$

And the operator ∇^2 is defined by

$$\nabla^2=\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}+\frac{\partial^2}{\partial z^2} \tag{10}$$

In cylindrical coordinate system. Also, the corresponding hydrodynamic pressure is shown in

$$P_s=\rho\frac{\partial \phi_s}{\partial t} \tag{11}$$

where:

$$\phi_s=\sum_{j=1}^{\infty}\left[P_j(t)\cosh\lambda_j\eta\ \right]J_1(\lambda_j\zeta)\cos\theta \tag{12}$$

where $\zeta=\frac{r}{R},\eta=\frac{z}{R}$ and $P_j(t)$ are time-dependent coefficients which must be determined from the conditions at the lower and upper boundaries of the fluid; J_1 is the Bessel function of the first kind order and λ_j is the *m-th* zero of the first derivative of J_1, i.e., the *m-th* root of $J_1^{'}=0$; the value λ_j for different values of the angle of inclination (θ_i) and dimensionless parameter (h/R_b) presented in Fig. 2.

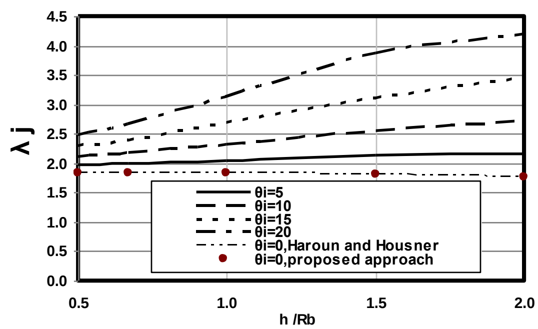

Fig.2
values of λ_j for different values of the angle of inclination (θ_i) and dimensionless parameter (h/R_b).

2.2.1 Sloshing frequency

The circular natural frequency corresponding to the *cos(θ)*-sloshing mode (n=1) of vibration may be conveniently expressed in a generalized Eq. (5) as:

$$\omega_j = c_j \sqrt{\frac{\lambda_j * g}{R_b}} \tag{13}$$

where

$$c_j = \sqrt{tagh(\frac{\lambda_j * h}{R_b})} \tag{14}$$

In which c_j is a dimensionless factor which depends on tank shape and h/R_b, the relative thickness and relative densities of the liquid in order of the frequency or mode to be considered. Fig. 2 presents the value of λ_j for different values of the angle of inclination (θ_i) and dimensionless parameter (h/R_b). As can be seen from Fig. 2, good agreement between the results of the proposed approach and the results is achieved by Haroun and Housner [23].

2.3 *Impulsive component*

The hydrodynamic pressure distribution over the tank circumference is well known, and it is a pure translation of (*cos (θ)*) type mode. The methodology for determining this hydrodynamic pressure includes obtaining the pressure distribution over the tank conical height, as this pressure will vary over the circumference as the cosine function of the angle (α) between the point to be considered and the excitation direction. Fig. 3 presents the mode shapes with the axial mode '1' and circumferential modes.

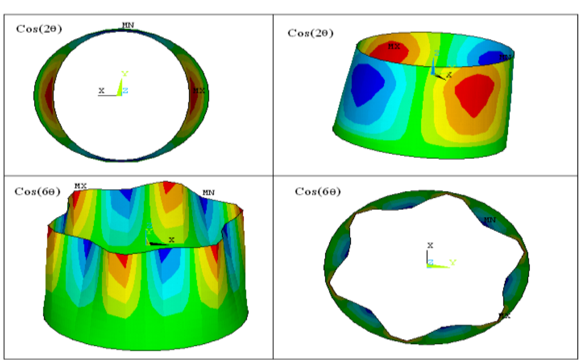

Fig.3
Circumferential mode shapes for the *1st* axial mode.

2.3.1 Impulsive frequency

The free vibration analyses indicate that the fundamental impulsive mode of the liquid-filled shells usually involves a cos ($n\theta$), n>1, circumferential variation. It is clear that this mode does not lead to shear force or overturning moment acting on the cross section of the tank. The natural frequency of the n-th vibration mode of the tank-liquid system in cycles per second, f_i , is conveniently expressed as:

$$f_i = \frac{1}{\beta_i . h} \sqrt{\frac{E_{sh} t_{sh}}{\rho_F R_b}} \tag{15}$$

where f_i is the first $\cos(\theta)$-mode, n=1, and β_i is a dimensionless coefficient which depends on $h / R_b, \theta_i$, and ρ_F , is the mass density of the contained-fluid and E_{sh} is Young's modulus of the tanks shell. The value β_i for different values of the angle of inclination (θ_i) and dimensionless parameter (h/R_b) are presented in Table 2. and Fig. 4. As Fig. 4 shows, a good agreement between the results of the proposed approach and the results is achieved by Haroun and Housner [23].

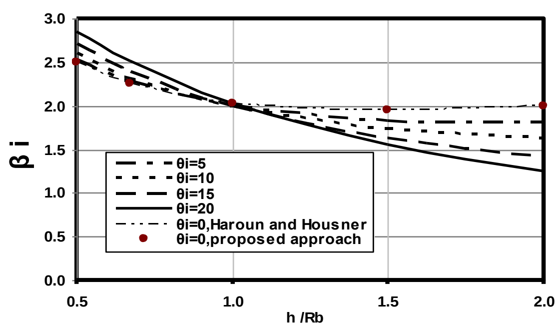

Fig.4
The value of β_i for different values of the angle of inclination (θ_i) and dimensionless parameter (h/R_b).

3 FINITE ELEMENT ANALYSIS

The present finite element modeling was done to model a tank structure using finite element of shell. A numerical study was conducted to determine the influence of the wall flexibility, fluid viscosity, filling level and base excitation characteristic on the sloshing response in conical liquid storage tanks. The finite element based software, ANSYS 10, is used. In this research, the Langrage-Lagrange method is used to model interaction between fluid and structure. Elements which are used for fluid and structure in ANSYS software are Fluid80 and Shell63 where Fluid80 element has eight nodes with three degrees of freedom in each node and Shell63 element has four nodes with six degrees of freedom in each node. Fig. 5, showed the geometry and coordinate system of a liquid-filled conical tank. In this figure R_b and h are the bottom radius and height of the tank, respectively. The angle between the tank generator (the *S*-axis) and the vertical direction (the *Z*-axis) is denoted as θ_i.

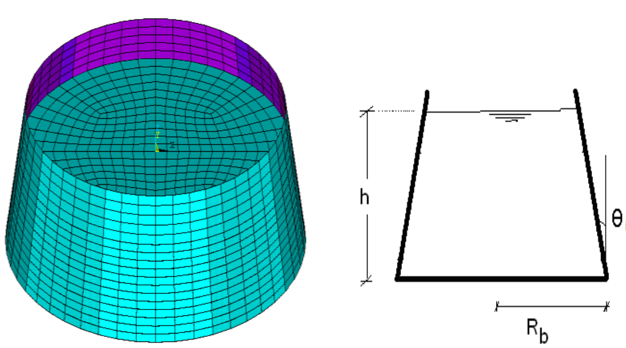

Fig.5
Finite element modeling and geometric coordinate system of tank.

4 COMPARISION

Two models with different aspect ratios are used for analysis: a tall tank and a broad tank. The dimensions and material properties of the analysis models are summarized in Table 1.

As Table 2. shows, a good agreement between the results of the proposed approach and the results is achieved by Haroun and Housner [23]. The differences relating to the tall tank are below 1.538%, and the differences relating to the broad tank are below 0.645%. Figs. 6 and 7 showed the plots of the sloshing and impulsive frequencies, respectively.

Table 1
Dimensions and material properties of a liquid storage tank.

Properties	Broad tank	Tall tank
Shell		
Thickness(mm)	18.3	12
Radius(m)	18.3	3.0
Density(kg /m^3)	7830	7830
Poison's Ratio	0.3	0.3
Young's Modulus(Gpa)	200	200
Water		
Density(kg /m^3)	1000	1000
Height(m)	12.2	6.0

Table 2
Comparison between the Parameters needed in the Present study with ($\theta i=0^o$) to Haroun and Housner mechanical model.

Parameter	Broad tank			Tall tank		
	Present study	Haroun and Housner	Differences (%)	Present study	Haroun and Housner	Differences (%)
f_s	0.145	0.145	0.000	0.384	0.390	1.538
f_i	6.133	6.173	0.645	23.52	23.67	0.633

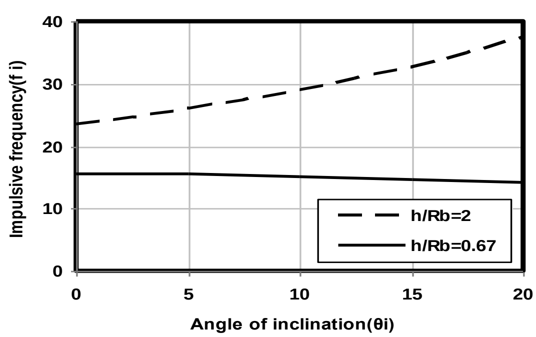

Fig.6
The value impulsive frequency for different values of (θ_i) and (h/R_b=0.67, 2.0).

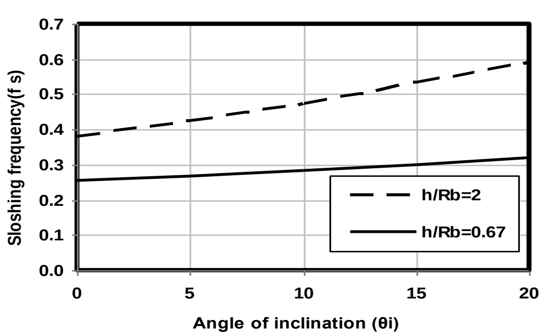

Fig.7
The value sloshing frequency for different values of (θ_i) and (h/R_b=0.67,2.0).

4 CONCLUSIONS

The natural frequencies of Cos (θ) modes for liquid-filled conical tanks are evaluated based on the validated finite element boundary Analysis. The study focuses on the fundamental mode shape which is found to have typically a cos ($n\theta$), n>1, variation along the circumference and the Cos (θ) mode shape in which the cross section of the tank remains circular. As a result, charts were suggested to present variations of such natural frequencies related to conical tanks geometry parameters. The proposed approximate methods accurately predict the sloshing and impulsive frequency of liquid-filled conical tanks (Δ shape) due to a dynamic excitation with significantly lower computational efforts. Result obtained from this study provides a basis for the development of such procedure to be implemented in the codes of design.

ACKNOWLEDGEMENTS

The financial support of this work by National Iranian South Oil Company under Grant No. 1346/115-23069 is gratefully acknowledged.

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