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Free Vibration and Buckling Analysis of Sandwich Panels with Flexible Cores Using an Improved Higher Order Theory

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ABSTRACT

In this paper, the behavior of free vibrations and buckling of the sandwich panel with a flexible core was investigated using a new improved high-order sandwich panel theory. In this theory, equations of motion were formulated based on shear stresses in the core. First-order shear deformation theory was applied for the procedures. In this theory, for the first time, incompatibility problem of velocity and acceleration field existing in Frostig's first theory was solved using a simple analytical method. The main advantage of this theory is its simplicity and less number of equations than the second method of Frostig's high-order theory. To extract dynamic equations of the core, three-dimensional elasticity theory was utilized. Also, to extract the dynamic equations governing the whole system, Hamilton's principle was used. In the analysis of free vibrations, the panel underwent primary pressure plate forces. Results demonstrated that, as plate pre-loads got closer to the critical buckling loads, the natural frequency of the panel tended zero. The results obtained from the present theory were in good correspondence with the results of the most recent papers.

Keywords : Free vibration; Buckling; Plate sandwich; Flexible core; Navier's methods.

1 INTRODUCTION

TODEY, the use of lightweight and durable structures with high stiffness to weight and strength to weight ratios has become prevalent in engineering applications. One of the most recent and common strong engineering structures are sandwich beam and sheets. A sandwich structure, including beam and sheet, is composed of two thin and strong layers, encompassing a soft, flexible and relatively thick core. Layers are typically manufactured from thin and strong metal sheets or multi-layered composite sheets. Cores are mainly produced from lightweight polymers, foams or honeycomb structures. Generally, composite shells and sheets theories include equivalent single-layer theories (multi-layer classical theory and multi-layer shear deformation theories) and 3-D elasticity theories (general theory of 3-D elasticity and layerwise theory). The equivalent single-layer theory assumes the multiple composite layers as a single equivalent layer and considers the kinematic displacement relations for it. This type of theory converts a 3-D problem into a 2-D one. On the contrary, there are 3-D theories in which each layer is considered as a separate 3-D object. The layerwise theory has an acceptable level of accuracy in calculating the natural frequencies since number of unknown functions depend on the laminate layers.

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Rao et al. [1] Developed an analytical method for calculating the natural frequencies of multi-layered composites and sandwich beams based on high order hybrid theory. Each layer of multiple layers was considered as a 2-D orthotropic material in the plane stress case. Hamilton's principle was used to derive the equilibrium equations. Kant and Swamnata [2] proposed a formulation and analytical solution for free vibration analysis of multi-layered composite and sandwich sheets based on the improved high order theory. In the presented theory, multi-layer deformations were calculated considering the effects of transverse shear deformation, transverse normal stresses and strains and non-linear variables for the in-plane displacement. Mionir and Shenwi [3] applied Reddy's improved shear deformation theory to investigate effects of damping on the dynamic response of sandwich panels. Nayak et al. [4] Proposed a new method based on Reddy's higher order theory and finite element method to calculate the natural frequencies of multi-layered composite sheets as well as sandwich composite layers. They studied the effects of different parameters such as geometry and material properties on the structure's natural frequencies. Frostig and Thomson [5] used the higher order theory of sandwich panels for the free vibration analysis of sandwich panels with flexible core. Malekzadeh et al. [6] Proposed an improved higher order theory for the sandwich sheets based on the theory by Frostig and Thompson [5] to conduct a free vibration analysis on sandwich panels. In this theory, contribution of in-plane forces in the top and lower layers of the sandwich sheet and the equivalent dissipation factor of the sandwich sheet were calculated; system's damping was also studied for the vibration analysis. Frostig and Barak [7] carried out a research based on the higher order theory considering the effects of out-of-plane flexibility and shear stiffness of the core for a sandwich beam. Frostig and Barak investigated buckling and vibrations in panel with a soft core and composite layers and asymmetric and unparalleled lamination [8-9]. Frostig and Thompson [10] investigated buckling of a sandwich panel with a soft core under non-linear mechanical-thermal loading. A higher order layerwise model was proposed by Defader et al. [11]. In this model, three displacements and three transverse stresses were assumed to exist at the intersection of the core and layer which was used in calculation of third order variations of displacement components in each layer. Since there is a large number of the unknown, a simplified model was proposed in which the proposed model was applied to a single equivalent layer which eliminates the accuracy of predicting the panel's buckling behavior. Pundit et al. [12] proposed an improved high order theory to investigate buckling in a multi-layered sandwich panel. Changes in the in-plane displacements for both layers and core along the direction of thickness and changes in the transverse displacement of the core were respectively considered to be of third and second orders. Setkovic and Vaksanoic [13] studied bending, free vibration and buckling of the sandwich panels using layerwise displacement model. In the proposed model, changes in the in-plane displacement components and changes in the transverse displacement along the sheet thickness were considered were considered linear and constant, respectively. Using the assumed displacement field, strain-displacement ratio, 3-D single-layer structural equations and motion equations were derived from Hamilton's law. Buckling and vibrations of multi-layered composite sheets with different fiber spacing were investigated by Yaoko et al. Using finite element analysis [14]. Position-dependent stiffness matrix formulation was extracted considering properties of inhomogeneous materials. Fiedler et al. [15] investigated the initiation of buckling phenomenon in multi-layered composite square sheets under uniaxial loading. They used the generalized higher order shear displacement theory which was able to include the effects of shear displacements in the sheet's transverse and thickness directions. Displacement field was modeled using Taylor series. They demonstrated that higher order polynomial terms are required to reach the solutions divergence for the critical buckling load coefficient. Shari'at [16] proposed a generalized higher order theory for sheets to study bending and buckling in sandwich panels under mechanical-thermal loads. In order to provide an exact and comprehensive theory which satisfies the continuity of all the transverse stress components, non-linear strain-displacement relations were used for sandwich panels under thermal-mechanical loading. Dariush and Sedighi [17], investigated behavior of a sandwich beam under static loading based on the non-linear higher order theory.

In this study, free vibration and buckling behavior of a sandwich panel with flexible core was investigated using an improved higher order theory. In this theory, equations of motion were formulated based on shear stresses in the core. The first order shear theory was used for analysis of the layers. In this theory, for the first time the problem of inconsistencies between acceleration and velocity fields in the Frostig's first order theory was resolved by a simple method. The main benefit of this theory is its simplicity and limited number of equations with respect to the second method of Frostig's higher order theory. To extract the dynamic equations for the core, 3-D elasticity theory was utilized.

2 STATEMENT OF PROBLEM

The sandwich sheet in this study is a rectangle with two composite layers and a middle core made of soft and flexible material. The thicknesses of the middle core, top layer and lower layer are respectively c, d_t and d_b. Fig. 1 also shows the coordinate axes. Length and width of the sheet are respectively a and b. The conducted analysis is within the linear elastic area and the layers and the middle core are completely attached to each other and strain functions at the layers' contact points are continuous.

2.1 Kinematic relations

For top and bottom layers, the first order shear theory was used by considering small displacement and rotations. Therefore, displacement components in top and bottom layers are:

$$\begin{aligned} u_j(x,y,z,t) &= u_{0j}(x,y,t) + z_j \,.\, \varphi_{xj}(x,y,t) \\ v_j(x,y,z,t) &= v_{0j}(x,y,t) + z_j \,.\, \varphi_{yj}(x,y,t) \\ w_j(x,y,z,t) &= w_{0j}(x,y,t) \qquad (j = t,b) \end{aligned} \tag{1}$$

φ_{xj} and φ_{yj} are rotational components of transverse sections of the sheet around respectively x and y axes in top and bottom layers. $u_{0j}(x,y,t), v_{0j}(x,y,t)$ and $w_{0j}(x,y,t)$ are displacement components of in the middle plane of the layers along *x, y* and z axes. Z_j is the vertical coordinate in each layer which is measured from the midpoint of each layer downwards (Fig. 1).

Kinematic relations for the core based on small displacement are as follows:

$$\varepsilon_{zz}^{c} = w_{c,z} \quad , \ \gamma_{xz}^{c} = u_{c,z} + w_{c,x} \ \ , \gamma_{yz}^{c} = v_{c,z} + w_{c,y} \tag{2}$$

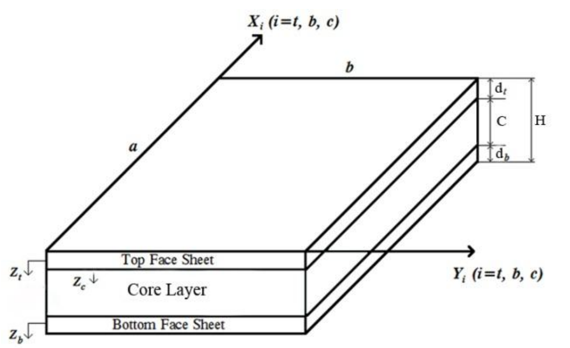


Fig.1
Geometry of the sandwich plate.

where u_c, v_c and w_c , are displacements along x and y axes and core creep, respectively. Consistency relation, with the assumption of complete contact of core between top and bottom layers is as follows:

$$\begin{aligned} u_c(z_c = 0) &= u_t\left(z_t = \frac{d_t}{2}\right), u_c(z_c = c) = u_b\left(z_t = -\frac{d_b}{2}\right) \\ v_c(z_c = 0) &= v_t\left(z_t = \frac{d_t}{2}\right), v_c(z_c = c) = v_b\left(z_t = -\frac{d_b}{2}\right) \\ w_c(z_c = 0) &= w_{0t}, w_c(z_c = c) = w_{0b} \end{aligned} \tag{3}$$

It is assumed that τ_{xz}^{c} and τ_{yz}^{c} are constant along z direction and are only functions of x and y. In fact, it is assumed that the core acts as a mediator that transfers the load to the layers, instead of bearing it [9].

Stress-strain relationship for a compressible orthotropic core is as follows:

$$\varepsilon_{zz}^{c}=\frac{\sigma_{zz}^{c}}{E_c},\gamma_{xz}^{c}=\frac{\tau_{xz}^{c}}{G_{cx}},\gamma_{yz}^{c}=\frac{\tau_{yz}^{c}}{G_{cy}} \tag{4}$$

where G_c and E_c are shear modulus in the vertical plane and elastic modulus, respectively.

To describe the relations of the core based on layers displacement fields, first core deformation field must be determined. Using the results obtained in reference [9], and relations (2), (3) and (4), normal stress and vertical displacement in the core are as follows:

$$\sigma_{zz}^{c}\left(x,y,z_c\right)=-\frac{\left(\tau_{xz,x}^{c}+\tau_{yz,y}^{c}\right)}{2}\left(2z_c-c\right)+\frac{w_{0b}-w_{0t}}{c}E_c \tag{5a}$$

$$w_c\left(x,y,z_c\right)=-\frac{\left(\tau_{xz,x}^{c}+\tau_{yz,y}^{c}\right)}{2E_c}\left(z_c^2-cz_c\right)+\frac{w_{0b}-w_{0t}}{c}z_c+w_{0t} \tag{5b}$$

Furthermore, using Eq.(3), the core displacement along x and y directions are as follows:

$$\begin{aligned}
u_c\left(x_c,y_c,z_c,t\right)&=\frac{z_c^2\left(2z_c-3c\right)}{12\mathrm{E_c}}\left(\tau_{xzc,xx}\left(x_c,y_c,t\right)+\tau_{yzc,xy}\left(x_c,y_c,t\right)\right)+\frac{z_c^2}{2c}\left(w_{t,x}\left(x_c,y_c,t\right)-w_{b,x}\left(x_c,y_c,t\right)\right)\\
&+\frac{z_c}{G_{\mathrm{cx}}}\tau_{xzc}\left(x_c,y_c,t\right)-z_c w_{t,x}\left(x_c,y_c,t\right)+u_{0t}\left(x_c,y_c,t\right)+\frac{1}{2}d_t\varphi_{xt}\left(x_c,y_c,t\right)\\
v_c\left(x_c,y_c,z_c,t\right)&=\frac{z_c^2\left(2z_c-3c\right)}{12E_c}\left(\tau_{xzc,xy}\left(x_c,y_c,t\right)+\tau_{yzc,yy}\left(x_c,y_c,t\right)\right)+\frac{z_c^2}{2c}\left(w_{t,y}\left(x_c,y_c,t\right)-w_{b,y}\left(x_c,y_c,t\right)\right)\\
&+\frac{z_c}{G_{cy}}\tau_{yzc}\left(x_c,y_c,t\right)-z_c w_{t,y}\left(x_c,y_c,t\right)+v_{0t}\left(x_c,y_c,t\right)+\frac{1}{2}d_t\varphi_{yt}\left(x_c,y_c,t\right)
\end{aligned} \tag{6}$$

2.2 Strain components in the layers

The strain-displacement relations for the face sheets $(j=t,b)$ can be defined as:

$$\begin{aligned}
&\varepsilon_{xx}^{j}=u_{0j,x}+z_j\varphi_{xj,x} \qquad \varepsilon_{yy}^{j}=v_{0j,y}+z_j\varphi_{yj,y} \qquad \varepsilon_{zz}^{j}=0\\
&\gamma_{xy}^{j}=u_{0j,y}+v_{0j,x}+z_j\left(\varphi_{xj,y}+\varphi_{yj,x}\right) \quad \gamma_{xz}^{j}=w_{0j,x}+\varphi_{xj} \quad \gamma_{yz}^{j}=w_{0j,y}+\varphi_{yj}
\end{aligned} \tag{7}$$

2.3 Stress-strain relationships

Layers are produced from multi-layered composite sheet. A single composite layer can be considered an orthotropic material. For this orthotropic single layer, the reduced stress-strain relation can be defined as follows:

$$\begin{Bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \tau_{xy}\end{Bmatrix}=\begin{bmatrix}\bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16}\\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26}\\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66}\end{bmatrix}\begin{Bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \gamma_{xy}\end{Bmatrix} \qquad \begin{Bmatrix}\tau_{yz}\\ \tau_{xz}\end{Bmatrix}=\begin{bmatrix}\bar{Q}_{44} & \bar{Q}_{45}\\ \bar{Q}_{45} & \bar{Q}_{55}\end{bmatrix}\begin{Bmatrix}\gamma_{yz}\\ \gamma_{xz}\end{Bmatrix} \tag{8}$$

In this equation, $\bar{Q}_{mn}(m,n=1,2,6)$ are the reduced in-plane spring constants and $\bar{Q}_{kl}(k,l=4,5)$ are the transverse shear constants for the assumed single layer.

2.4 Equation of motion

In this section, minimum potential energy principle is used to calculate the governing equations of motion for the problem. Based on this principle, we have:

$$\delta\int_{t_1}^{t_2}\left(\mathrm{U}+\mathrm{V}-\mathrm{T}\right)\mathrm{dt}=0 \tag{9}$$

In the above relation, *U* is the total strain energy of the sheet, *V* is the work done by the external loads on the sheet, *T* is the total kinetic energy of the sheet and δ is the variation operator. The general equation of the first order variation of strain energy for a sandwich panel has been introduced by relation (10).

$$\begin{aligned}\delta\mathrm{U}=&\int_{v_t}\left(\sigma_{xx}^{t}\delta\varepsilon_{xx}^{t}+\sigma_{yy}^{t}\delta\varepsilon_{yy}^{t}+\tau_{xy}^{t}\delta\gamma_{xy}^{t}+\tau_{xz}^{t}\delta\gamma_{xz}^{t}+\tau_{yz}^{t}\delta\gamma_{yz}^{t}\right)dv_t\\&+\int_{v_b}\left(\sigma_{xx}^{b}\delta\varepsilon_{xx}^{b}+\sigma_{yy}^{b}\delta\varepsilon_{yy}^{b}+\tau_{xy}^{b}\delta\gamma_{xy}^{b}+\tau_{xz}^{b}\delta\gamma_{xz}^{b}+\tau_{yz}^{b}\delta\gamma_{yz}^{b}\right)dv_b+\int_{v_c}\left(\tau_{xz}^{c}\delta\gamma_{xz}^{c}+\tau_{yz}^{c}\delta\gamma_{yz}^{c}+\sigma_{zz}^{c}\delta\varepsilon_{zz}^{c}\right)dv_c\end{aligned} \tag{10}$$

ε_{ii} and $\sigma_{ii}(i=x,y)$ are respectively strains and normal stresses in *x* and *y* direction and indices *t, b* and *c* indicate top layer, bottom layer and core. γ_{iz} and $\tau_{iz}(i=x,y)$ are respectively strains and normal stresses. v_t and v_b and v_c are respectively volumes of top layer, the bottom layer, and core.

The variation of the external energy equals:

$$\begin{aligned}\delta V=&-\sum_{i=t,b}\left[\int_A\left(q_t\delta w_0^t+q_b\delta w_0^b+\bar{n}_{xi}\delta u_0^i+\bar{n}_{yi}\delta v_0^i\right)dA\right]\\&-\sum_{j=1}^{2}\int_0^a\int_0^b\left(\bar{N}_{xxj}^{t}\delta u_0^t+\bar{N}_{xyj}^{t}\delta v_0^t+\bar{N}_{xxj}^{b}\delta u_0^b+\bar{N}_{xyj}^{b}\delta v_0^b\right)\delta_D\left(x-x_j\right)dxdy\\&-\sum_{j=1}^{2}\int_0^a\int_0^b\left(\bar{N}_{yyj}^{t}\delta v_0^t+\bar{N}_{xyj}^{t}\delta u_0^t+\bar{N}_{yyj}^{b}\delta v_0^b+\bar{N}_{xyj}^{b}\delta u_0^b\right)\delta_D\left(x-x_j\right)dxdy\end{aligned} \tag{11}$$

q_i are the normal dynamic loads distributed over the top and bottom layers of the sheet. $\bar{n}$ are the external plane loads which are stress in nature and are present on surfaces of top and bottom layers. $\bar{N}_{xxj}^{t}$ and $\bar{N}_{xxj}^{b}$ are external loads normal to the edge of per unit length along the *x* axis on edges $x_1=0,x_2=a$ and $\bar{N}_{yyj}^{t}$ and $\bar{N}_{yyj}^{b}$ are external loads normal on the edge per unit length along *y* axis on edges $x_1=0$ and $,y_2=b$ in top and bottom layers. In fact, these loads are forces that cause buckling of the panel. $\bar{N}_{xyj}^{t}$ and $\bar{N}_{xyj}^{b}$ are external shear in-plane loads on the edge per unit length of the top and bottom layers. $\delta_D(x-x_j)$ and $\delta_D(y-y_j)$ are dirac delta functions in loading coordinates $(j=1,2)$.

u_0,v_0 and w_0 are respectively displacements in *x, y* and *z* directions on the top and bottom layers and product of terms related forces to terms such as $z_j\varphi_{xj}$ were neglected since the layers were assumed to be thin. General equation of variation of first order kinetic energy for a panel sandwich is written in relation (12).

$$\delta T=-\int_{t_1}^{t_2}\left[\begin{array}{l}\int_0^a\int_0^b\int_{-\frac{h_t}{2}}^{\frac{h_t}{2}}\rho_t\left(\ddot{u}_t\delta u_t+\ddot{v}_t\delta v_t+\ddot{w}_t\delta w_t\right)dxdydz+\int_0^a\int_0^b\int_{-\frac{h_b}{2}}^{\frac{h_b}{2}}\rho_b\left(\ddot{u}_b\delta u_b+\ddot{v}_b\delta v_b+\ddot{w}_b\delta w_b\right)dxdydz\\+\int_0^a\int_0^b\int_0^c\rho_c\left(\ddot{u}_c\delta u_c+\ddot{v}_c\delta v_c+\ddot{w}_c\delta w_c\right)dxdydz\end{array}\right] \tag{12}$$

where Eq.(12) are $\rho_i (i=t,b,c)$ respectively densities of top, bottom and core and $\ddot{u}_i,\ddot{v}_i,\ddot{w}_i (i=t,b,c)$ are respectively acceleration components of top, bottom and core.

In Frostig's theory, velocity and acceleration fields are linear, while the displacement fields resulting from elasticity theory are non-linear. Thus, velocity and acceleration are not obtained from the first and second derivatives of movement fields, which is a contradiction. This problem was solved in this new theory. The distributions of the accelerations through the depth of the core are assumed to take the shape of the static displacement fields so, according to Eqs. (5b) and (6) we have:

$$\begin{aligned}
&\ddot{u}_c\left(x_c,y_c,z_c,t\right)=\frac{z_c^2\left(2z_c-3c\right)}{12E_c}\left(\ddot{\tau}_{xzc,xx}\left(x_c,y_c,t\right)+\ddot{\tau}_{yzc,xy}\left(x_c,y_c,t\right)\right)+\frac{z_c^2}{2c}\left(\ddot{w}_{t,x}\left(x_c,y_c,t\right)-\ddot{w}_{b,x}\left(x_c,y_c,t\right)\right)\\
&+\frac{z_c}{G_{cx}}\ddot{\tau}_{xzc}\left(x_c,y_c,t\right)-z_c\ddot{w}_{t,x}\left(x_c,y_c,t\right)+\ddot{u}_{0t}\left(x_c,y_c,t\right)+\frac{1}{2}d_t\,\ddot{\varphi}_{xt}\left(x_c,y_c,t\right)\\
&\ddot{v}_c\left(x_c,y_c,z_c,t\right)=\frac{z_c^2\left(2z_c-3c\right)}{12E_c}\left(\ddot{\tau}_{xzc,xy}\left(x_c,y_c,t\right)+\ddot{\tau}_{yzc,yy}\left(x_c,y_c,t\right)\right)+\frac{z_c^2}{2c}\left(\ddot{w}_{t,y}\left(x_c,y_c,t\right)-\ddot{w}_{b,y}\left(x_c,y_c,t\right)\right)\\
&+\frac{z_c}{G_{cy}}\ddot{\tau}_{yzc}\left(x_c,y_c,t\right)-z_c\ddot{w}_{t,y}\left(x_c,y_c,t\right)+\ddot{v}_{0t}\left(x_c,y_c,t\right)+\frac{1}{2}d_t\,\ddot{\varphi}_{yt}\left(x_c,y_c,t\right)\\
&\ddot{w}_c\left(x_c,y_c,z_c,t\right)=-\frac{z_c\left(z_c-c\right)}{2E_c}\left(\ddot{\tau}_{xzc,x}\left(x_c,y_c,t\right)+\ddot{\tau}_{yzc,y}\left(x_c,y_c,t\right)\right)-\frac{\ddot{w}_t\left(x_c,y_c,t\right)-\ddot{w}_b\left(x_c,y_c,t\right)}{c}z_c+\ddot{w}_t\left(x_c,y_c,t\right)
\end{aligned} \tag{13}$$

2.5 Resultant of stress on the layers

In order to extract the governing equations of the problem, it is necessary to replace the strain energies in the sheet. This can be done by defining a number of parameters with the general name of resultant of stress. Parameters of the resultant stress in the layers are as follows:

$$\begin{bmatrix}N_{xx}^{(j)} & N_{yy}^{(j)}\\ M_{XX}^{(j)} & M_{yy}^{(j)}\end{bmatrix}=\int_{-\frac{h_j}{2}}^{\frac{h_j}{2}}\begin{bmatrix}1\\ z_j\end{bmatrix}\begin{bmatrix}\sigma_{xx}^{(j)} & \sigma_{yy}^{(j)}\end{bmatrix}dz_j\qquad \begin{pmatrix}N_{xy}^{(j)}\\ M_{Xy}^{(j)}\end{pmatrix}=\int_{-\frac{h_j}{2}}^{\frac{h_j}{2}}\begin{pmatrix}1\\ z_j\end{pmatrix}\tau_{xy}^{(j)}dz_j\qquad \begin{bmatrix}Q_{xz}^{(j)}\\ Q_{yz}^{(j)}\end{bmatrix}=\int_{-\frac{h_j}{2}}^{\frac{h_j}{2}}\begin{bmatrix}\tau_{xz}^{(j)}\\ \tau_{yz}^{(j)}\end{bmatrix}dz_j\qquad (j=t,b) \tag{14}$$

Now, by substituting relations (10), (11), (12) in Eq. (9) and applying the variation operator and considering the resultants of stress, the governing equations can be extracted.

$$\begin{aligned}
&N_{xx,x}^{(t)}+N_{xy,y}^{(t)}+\tau_x=\left(\frac{13}{35}\rho_c c+I_{0t}\right)\ddot{u}_{0t}+\left(\frac{9}{70}\rho_c c\right)\ddot{u}_{0b}-\left(\frac{11}{210}\rho_c c^2\right)\ddot{w}_{t,x}+\left(\frac{13}{420}\rho_c c^2\right)\ddot{w}_{b,x}+\left(\frac{3\rho_c c^2}{140G_{cx}}\right)\ddot{\tau}_{xz}\\
&+\left(\frac{13}{70}\rho_c c d_t\right)\ddot{\varphi}_{xt}-\left(\frac{9}{140}\rho_c c d_b\right)\ddot{\varphi}_{xb}
\end{aligned} \tag{15a}$$

$$N_{xx,x}^{(b)}+N_{xy,y}^{(b)}-\tau_x=\left(\frac{13}{35}\rho_c c+I_{0b}\right)\ddot{u}_{0b}+\left(\frac{9}{70}\rho_c c\right)\ddot{u}_{0t}+\left(\frac{11}{210}\rho_c c^2\right)\ddot{w}_{b,x}-\left(\frac{13}{420}\rho_c c^2\right)\ddot{w}_{t,x}-\left(\frac{3\rho_c c^2}{140\mathrm{G_{cx}}}\right)\ddot{\tau}_{xz}$$
$$-\left(\frac{13}{70}\rho_c c d_b\right)\ddot{\varphi}_{xb}+\left(\frac{9}{140}\rho_c c d_t\right)\ddot{\varphi}_{xt} \qquad (15b)$$

$$N_{yy,y}^{(t)}+N_{xy,x}^{(t)}+\tau_y=\left(\frac{13}{35}\rho_c c+I_{0t}\right)\ddot{v}_{0t}+\left(\frac{9}{70}\rho_c c\right)\ddot{v}_{0b}-\left(\frac{11}{210}\rho_c c^2\right)\ddot{w}_{t,y}+\left(\frac{13}{420}\rho_c c^2\right)\ddot{w}_{b,y}+\left(\frac{3\rho_c c^2}{140\mathrm{G_{cy}}}\right)\ddot{\tau}_{yz}$$
$$+\left(\frac{13}{70}\rho_c c d_t\right)\ddot{\varphi}_{yt}-\left(\frac{9}{140}\rho_c c d_b\right)\ddot{\varphi}_{yb} \qquad (15c)$$

$$N_{yy,y}^{(b)}+N_{xy,x}^{(b)}-\tau_y=\left(\frac{13}{35}\rho_c c+I_{0b}\right)\ddot{v}_{0b}+\left(\frac{9}{70}\rho_c c\right)\ddot{v}_{0t}+\left(\frac{11}{210}\rho_c c^2\right)\ddot{w}_{b,y}-\left(\frac{13}{420}\rho_c c^2\right)\ddot{w}_{ty}-\left(\frac{3\rho_c c^2}{140G_{cy}}\right)\ddot{\tau}_{yz}$$
$$-\left(\frac{13}{70}\rho_c c d_b\right)\ddot{\varphi}_{yb}+\left(\frac{9}{140}\rho_c c d_t\right)\ddot{\varphi}_{yt} \qquad (15d)$$

$$M_{xx,x}^{(t)}+M_{xy,y}^{(t)}-Q_{xz}^{(t)}+\tau_{xz}\frac{d_t}{2}=\left(-\frac{9}{280}c d_t d_b\rho_c\right)\ddot{\varphi}_{xb}+\left(\frac{13}{140}\rho_c d_t^2 c+I_{2t}\right)\ddot{\varphi}_{xt}+\left(\frac{13}{840}\rho_c c^2 d_t\right)\ddot{w}_{b,x}$$
$$-\left(\frac{11}{420}\rho_c c^2 d_t\right)\ddot{w}_{\mathrm{t,x}}+\left(\frac{3\rho_c c^2 d_t}{280G_x}\right)\ddot{\tau}_{xz}+\left(\frac{9}{140}c\rho_c d_t\right)\ddot{u}_{0b}+\left(\frac{13}{70}c\rho_c d_t\right)\ddot{u}_{0t} \qquad (15e)$$

$$M_{xx,x}^{(b)}+M_{xy,y}^{(b)}-Q_{xz}^{(b)}+\tau_{xz}\frac{d_b}{2}=\left(-\frac{9}{280}c d_t d_b\rho_c\right)\ddot{\varphi}_{xt}+\left(\frac{13}{140}\rho_c d_b^2 c+I_{2b}\right)\ddot{\varphi}_{xt}+\left(\frac{13}{840}\rho_c c^2 d_b\right)\ddot{w}_{t,x}$$
$$-\left(\frac{11}{420}\rho_c c^2 d_b\right)\ddot{w}_{b,x}+\left(\frac{3\rho_c c^2 d_b}{280G_x}\right)\ddot{\tau}_{xz}-\left(\frac{9}{140}c\rho_c d_b\right)\ddot{u}_{0t}-\left(\frac{13}{70}c\rho_c d_b\right)\ddot{u}_{0b} \qquad (15f)$$

$$M_{yy,y}^{(t)}+M_{xy,x}^{(t)}-Q_{yz}^{(t)}+\tau_{yz}\frac{d_t}{2}=\left(-\frac{9}{280}c d_t d_b\rho_c\right)\ddot{\varphi}_{yb}+\left(\frac{13}{140}\rho_c d_t^2 c+I_{2t}\right)\ddot{\varphi}_{yt}+\left(\frac{13}{840}\rho_c c^2 d_t\right)\ddot{w}_{b,y}$$
$$-\left(\frac{11}{420}\rho_c c^2 d_t\right)\ddot{w}_{t,y}+\left(\frac{3\rho_c c^2 d_t}{280G_y}\right)\ddot{\tau}_{yz}+\left(\frac{9}{140}c\rho_c d_t\right)\ddot{v}_{0b}+\left(\frac{13}{70}c\rho_c d_t\right)\ddot{v}_{0t} \qquad (15g)$$

$$M_{yy,y}^{(b)}+M_{xy,x}^{(b)}-Q_{yz}^{(b)}+\tau_{yz}\frac{d_b}{2}=\left(-\frac{9}{280}c d_t d_b\rho_c\right)\ddot{\varphi}_{yt}+\left(\frac{13}{140}\rho_c d_b^2 c+I_{2b}\right)\ddot{\varphi}_{yb}+\left(\frac{13}{840}\rho_c c^2 d_b\right)\ddot{w}_{t,y}$$
$$-\left(\frac{11}{420}\rho_{\mathrm{c}} c^2 d_b\right)\ddot{w}_{b,y}+\left(\frac{3\rho_{\mathrm{c}} c^2 d_b}{280G_y}\right)\ddot{\tau}_{\mathrm{yz}}-\left(\frac{9}{140}c\rho_{\mathrm{c}} d_{\mathrm{b}}\right)\ddot{v}_{0t}-\left(\frac{13}{70}c\rho_c d_b\right)\ddot{v}_{0b} \qquad (15h)$$

$$\bar{N}_{xx}^{t}w_{t,xx}+\bar{N}_{yy}^{t}w_{t,yy}+2\bar{N}_{xy}^{t}w_{t,xy}+Q_{xz,x}^{(t)}+Q_{yz,y}^{(t)}-\frac{E_c}{c}\left(w_t-w_b\right)+\frac{\mathrm{c}}{2}\left(\tau_{xz,x}+\tau_{yz,y}\right)=-\frac{1}{105}\rho_{\mathrm{c}}\mathrm{c}^3\left(\ddot{w}_{\mathrm{t,xx}}+\ddot{w}_{\mathrm{t,yy}}\right)$$
$$+\frac{1}{140}\rho_c c^3\left(\ddot{w}_{b,xx}+\ddot{w}_{b,yy}\right)+\frac{1}{420}\rho_c c^3\left(\frac{\ddot{\tau}_{xz,x}}{G_x}+\frac{\ddot{\tau}_{yz,y}}{G_y}\right)+\frac{11}{210}\rho_c c^2\left(\ddot{u}_{0t,x}+\ddot{v}_{0t,y}\right)+\frac{13}{420}\rho_c c^2\left(\ddot{u}_{0b,x}+\ddot{v}_{0b,y}\right)$$
$$+\frac{11}{420}\rho_c c^2 d_t\left(\ddot{\varphi}_{xt,x}+\ddot{\varphi}_{yt,y}\right)-\frac{13}{840}\rho_c c^2 d_b\left(\ddot{\varphi}_{xb,x}+\ddot{\varphi}_{yb,y}\right)+I_{0t}\ddot{w}_t \qquad (15i)$$

$$\bar{N}^b_{xx} w_{b,xx} + \bar{N}^b_{yy} w_{b,yy} + 2\bar{N}^b_{xy} w_{b,xy} + Q^{(b)}_{xz,x} + Q^{(b)}_{yz,y} + \frac{E_c}{c}\left(w_t - w_b\right) + \frac{c}{2}\left(\tau_{xz,x} + \tau_{yz,y}\right) = -\frac{1}{105}\rho_c c^3\left(\ddot{w}_{b,xx} + \ddot{w}_{b,yy}\right)$$
$$+\frac{1}{140}\rho_c c^3\left(\ddot{w}_{t,xx} + \ddot{w}_{t,yy}\right) + \frac{1}{420}\rho_c c^3\left(\frac{\ddot{\tau}_{xz,x}}{G_x} + \frac{\ddot{\tau}_{yz,y}}{G_y}\right) - \frac{11}{210}\rho_c c^2\left(\ddot{u}_{0b,x} + \ddot{v}_{0b,y}\right) - \frac{13}{420}\rho_c c^2\left(\ddot{u}_{0t,x} + \ddot{v}_{0t,y}\right) \tag{15j}$$
$$+\frac{11}{420}\rho_c c^2 d_b\left(\ddot{\varphi}_{xb,x} + \ddot{\varphi}_{yb,y}\right) - \frac{13}{840}\rho_c c^2 d_t\left(\ddot{\varphi}_{xt,x} + \ddot{\varphi}_{yt,y}\right) + I_{0b}\ddot{w}_b$$

$$-\frac{c^3}{12E_c}\left(\tau_{xz,xx} + \tau_{yz,yx}\right) + \frac{c}{G_{cx}}\tau_{xz} - \frac{c}{2}\left(w_{t,x} + w_{b,x}\right) + u_{0t} - u_{0b} + \frac{d_t}{2}\varphi_{xt} + \frac{d_b}{2}\varphi_{xb}$$
$$= \ddot{w}_{t,x}\left(-\frac{\rho_c c^3}{420\ G_{cx}} + \frac{\rho_c c^3}{120\ E_c} + \frac{\rho_c c^2 d_t}{20\ E_c}\right) + \ddot{w}_{b,x}\left(-\frac{\rho_c c^3}{420\ G_{cx}} + \frac{\rho_c c^3}{120\ E_c} + \frac{\rho_c c^2 d_b}{20\ E_c}\right) + \ddot{\tau}_{xz}\left(\frac{\rho_c c^3}{210\ G^2_{cx}} - \frac{\rho_c c^3}{10\ E_c G_{cx}}\right) \tag{15k}$$
$$+\ddot{u}_{0t}\left(\frac{3\rho_c c^2}{140 G_{cx}} - \frac{\rho_c c^2}{10 E_c}\right) + \ddot{u}_{0b}\left(-\frac{3\rho_c c^2}{140 G_{cx}} + \frac{\rho_c c^2}{10 E_c}\right) + \ddot{\varphi}_{xt}\left(\frac{3}{280}\frac{\rho_c c^2 d_t}{G_{cx}}\right) + \ddot{\varphi}_{xb}\left(\frac{3}{280}\frac{\rho_c c^2 d_b}{G_{cx}}\right)$$

$$-\frac{c^3}{12E_c}\left(\tau_{xz,xy} + \tau_{yz,yy}\right) + \frac{c}{G_{cy}}\tau_{yz} - \frac{c}{2}\left(w_{t,y} + w_{b,y}\right) + v_{0t} - v_{0b} + \frac{d_t}{2}\varphi_{yt} + \frac{d_b}{2}\varphi_{yb} =$$
$$\ddot{w}_{t,y}\left(-\frac{\rho_c c^3}{420\ G_{cy}} + \frac{\rho_c c^3}{120\ E_c} + \frac{\rho_c c^2 d_t}{20\ E_c}\right) + \ddot{w}_{b,y}\left(-\frac{\rho_c c^3}{420\ G_{cy}} + \frac{\rho_c c^3}{120\ E_c} + \frac{\rho_c c^2 d_b}{20\ E_c}\right) + \ddot{\tau}_{yz}\left(\frac{\rho_c c^3}{210\ G^2_{cy}} - \frac{\rho_c c^3}{10\ E_c G_{cy}}\right) + \tag{15l}$$
$$\ddot{v}_{0t}\left(\frac{3\rho_c c^2}{140 G_{cy}} - \frac{\rho_c c^2}{10 E_c}\right) + \ddot{v}_{0b}\left(-\frac{3\rho_c c^2}{140 G_{cy}} + \frac{\rho_c c^2}{10 E_c}\right) + \ddot{\varphi}_{yt}\left(\frac{3}{280}\frac{\rho_c c^2 d_t}{G_{cy}}\right) + \ddot{\varphi}_{yb}\left(\frac{3}{280}\frac{\rho_c c^2 d_b}{G_{cy}}\right)$$

The structural relationship between the resultants of stress in multi-layered composite layers with respect to the displacement is as follows:

$$\begin{Bmatrix} N^{(i)}_{xx} \\ N^{(i)}_{yy} \\ N^{(i)}_{xy} \end{Bmatrix} = \begin{bmatrix} A^{(i)}_{11} & A^{(i)}_{12} & A^{(i)}_{16} \\ A^{(i)}_{21} & A^{(i)}_{22} & A^{(i)}_{26} \\ A^{(i)}_{16} & A^{(i)}_{26} & A^{(i)}_{66} \end{bmatrix} \begin{Bmatrix} u_{0i,x} \\ v_{0i,y} \\ u_{0i,y} + v_{0i,x} \end{Bmatrix} + \begin{bmatrix} B^{(i)}_{11} & B^{(i)}_{12} & B^{(i)}_{16} \\ B^{(i)}_{21} & B^{(i)}_{22} & B^{(i)}_{26} \\ B^{(i)}_{16} & B^{(i)}_{26} & B^{(i)}_{66} \end{bmatrix} \begin{Bmatrix} \varphi_{xi,x} \\ \varphi_{yi,y} \\ \varphi_{xi,y} + \varphi_{yi,x} \end{Bmatrix}$$

$$\begin{Bmatrix} M^{(i)}_{xx} \\ M^{(i)}_{yy} \\ M^{(i)}_{xy} \end{Bmatrix} = \begin{bmatrix} B^{(i)}_{11} & B^{(i)}_{12} & B^{(i)}_{16} \\ B^{(i)}_{21} & B^{(i)}_{22} & B^{(i)}_{26} \\ B^{(i)}_{16} & B^{(i)}_{26} & B^{(i)}_{66} \end{bmatrix} \begin{Bmatrix} u_{0i,x} \\ v_{0i,y} \\ u_{0i,y} + v_{0i,x} \end{Bmatrix} + \begin{bmatrix} D^{(i)}_{11} & D^{(i)}_{12} & D^{(i)}_{16} \\ D^{(i)}_{21} & D^{(i)}_{22} & D^{(i)}_{26} \\ D^{(i)}_{16} & D^{(i)}_{26} & D^{(i)}_{66} \end{bmatrix} \begin{Bmatrix} \varphi_{xi,x} \\ \varphi_{yi,y} \\ \varphi_{xi,y} + \varphi_{yi,x} \end{Bmatrix} \tag{16}$$

$$\begin{Bmatrix} Q^{(i)}_{yz} \\ Q^{(i)}_{xz} \end{Bmatrix} = k \begin{bmatrix} A^{(i)}_{44} & A^{(i)}_{45} \\ A^{(i)}_{45} & A^{(i)}_{55} \end{bmatrix} \begin{Bmatrix} w_{i,y} + \varphi_{yi} \\ w_{i,x} + \varphi_{xi} \end{Bmatrix}$$

where A_{ij}, B_{ij} and D_{ij} are respectively tensile stiffness, coupling stiffness and bending stiffness matrices with 3*3 dimensions whose elements are obtained using following relations:

$$A_{\mathrm{ij}} = \sum_{\mathrm{k=1}}^{\mathrm{N}}\left(\bar{Q}_{\mathrm{ij}}\right)_{\mathrm{k}}\left(z_{\mathrm{k}} - z_{\mathrm{k-1}}\right) \qquad B_{\mathrm{ij}} = \frac{1}{2}\sum_{\mathrm{k=1}}^{\mathrm{N}}\left(\bar{Q}_{\mathrm{ij}}\right)_{\mathrm{k}}\left({z^2}_{\mathrm{k}} - {z^2}_{\mathrm{k-1}}\right) \qquad D_{\mathrm{ij}} = \frac{1}{3}\sum_{\mathrm{k=1}}^{\mathrm{N}}\left(\bar{Q}_{\mathrm{ij}}\right)_{\mathrm{k}}\left({z^3}_{\mathrm{k}} - {z^3}_{\mathrm{k-1}}\right) \tag{17}$$

3 FREE VIBRATION AND BUCKLING RESPONSE OF THE SANDWICH SHEET WITH SIMPLE SUPPORT

Boundary conditions of the problem at each of the four edges of the sandwich panel are assumed to be simple support. Following relations, called Navier response, satisfied the boundary conditions. Constant coefficients in these series are obtained from the equilibrium equations.

$$\begin{Bmatrix} u_{oj}(x,y,t) \\ v_{oj}(x,y,t) \\ w_{oj}(x,y,t) \\ \varphi_{xj}(x,y,t) \\ \varphi_{yj}(x,y,t) \\ \tau_{yz}(x,y,t) \end{Bmatrix} = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \begin{Bmatrix} u_{ojmn}.\cos(\alpha_m x).\sin(\beta_n y) \\ v_{ojmn}.\sin(\alpha_m x).\cos(\beta_n y) \\ w_{ojmn}.\sin(\alpha_m x).\sin(\beta_n y) \\ X_{jmn}.\cos(\alpha_m x).\sin(\beta_n y) \\ Y_{jmn}.\sin(\alpha_m x).\cos(\beta_n y) \\ T_{cxmn}.\cos(\alpha_m x).\sin(\beta_n y) \\ T_{cymn}.\sin(\alpha_m x).\cos(\beta_n y) \end{Bmatrix} \tag{18}$$

According to the symmetry of the problem with respect to the middle plane, interactions of membrane forces, bending curvatures, and bending forces with middle surface strain become zero, and in this condition we have:

$$A_{16}^j = A_{26}^j = B_{16}^j = B_{26}^j = D_{16}^j = D_{26}^j = 0 \qquad (j = t, b) \tag{19}$$

Substituting relation (18) and relation (15), different x and y factors are eliminated from the equations and the differential equation system governing the sandwich sheet's vibration are converted into a algebraic coordinate system with following general form:

$$\left([k]_{12*12} - \omega^2 [M]_{12*12}\right)\{\Delta\}_{12*12} = 0 \tag{20}$$

where k,M and Δ are respectively stiffness matrix, mass and matrix of the unknowns and Δ is written as follows:

$$\{\Delta\}_{12*12} = \{u_{ot}, v_{ot}, u_{ob}, v_{ob}, w_{ot}, w_{ob}, \varphi_{xt}, \varphi_{xb}, \varphi_{yt}, \varphi_{yb}, T_{cx}, T_{cy}\} \tag{21}$$

To analyze the buckling, we can write:

$$N_{xx} = -N_o, N_{yy} = -KN_o \qquad N_{xy} = q_t = q_b = 0 \tag{22}$$

In this article, each layer bears some part of the load depending on its intra-axial stiffness. Therefore, each of the top and bottom layers reaches their critical buckling loads at the same time. The contribution of each of the top and bottom layers of the applied axial load can be obtained as [9]:

$$\begin{aligned} \bar{N}_{xx}^j &= \frac{\alpha_j}{\alpha_t + \alpha_b} N_{xx} \qquad & \bar{N}_{yy}^j &= \frac{\beta_j}{\beta_t + \beta_b} N_{yy} \\ \alpha_j &= \frac{A_{22j}}{A_{11j}A_{22j} - A_{12j}^2} \qquad & \beta_j &= \frac{A_{11j}}{A_{11j}A_{22j} - A_{12j}^2} \end{aligned} \tag{23}$$

In this condition, the algebraic coordinates system would take the following form:

$$\left([k]_{12*12}\right)\{\Delta\}_{12*12} = 0 \tag{24}$$

Matrix coefficients include constant numbers and loading parameters of the buckling which can be written as a multiplier of $\bar{N}_{xx}$. Dimensionless parameter of the buckling load which is widely used in many references to compare the results of buckling analysis in sandwich sheets, is defined as follows:

$$N_{cr} = \frac{\bar{N}_{xx} b^2}{E_2 H^3} \tag{25}$$

3.1 Results of sandwich panel's free vibration

In this section, results of free vibration of a 5-layered sandwich panel (0.90/core/0.90) are compared with that reported by the references and presented in Table 1-3.

Mechanical properties related to the sheet's layers are [2]:

$$E_1 = 131Gpa, E_2 = E_3 = 10.34Gpa \qquad G_{12} = G_{13} = 6.985Gpa, G_{23} = 6.205Gpa$$
$$\nu_{12} = \nu_{13} = 0.22, \nu_{23} = 0.49 \qquad \rho_t = \rho_b = 1627(kg/m^3)$$

And the core:

$$E_c = 689\times10^{-3}\,Gpa \qquad \nu_c \approx 0 G_c = 3.45\times10^{-3}\,Gpa, \rho_c = 97(kg/m^3)$$

In Table 1., results of the present method are compared with the exact solution [18], finite element analysis [19], and equivalent single-layer model [2] for different a/H. Theoretical results in the present study are in good agreement with the finite element method [19]. In this method, 8-needed elements with non-linear acceleration along the *x* and *y* axes and constant acceleration in *z* direction were assumed which caused 1.01% error for $a/H = 4$. Also, the maximum difference of the present method was related to the exact solution method [18] and equivalent single layer theory [2] with errors respectively equal to 6.85% and 59.32%.

In Table 2., dimensionless frequency is compared with different references for different *a/b* ratio. The greatest difference in the results of the present theory in Table 2. was respectively, with finite element method (2.52%), exact solution method (6.89) and equivalent single layer theory (64.85%).

As shown in Tables 1 and 2., this theory has acceptable accuracy, also on the contrary, to reference [5], in order to overcome the problem of inconsistency between acceleration and velocity fields, equation of motions (13) were used to obtain the acceleration field. The main benefit of this theory is its simplicity and limited number of equations with respect to the second method of Frostig's higher order theory [5]. To extract the dynamic equations for the core, 3-D elasticity theory was utilized.

Table 1
Comparison of the first non-dimensional fundamental frequencies of (0/90/core/0/90) sandwich plate with different a/H.

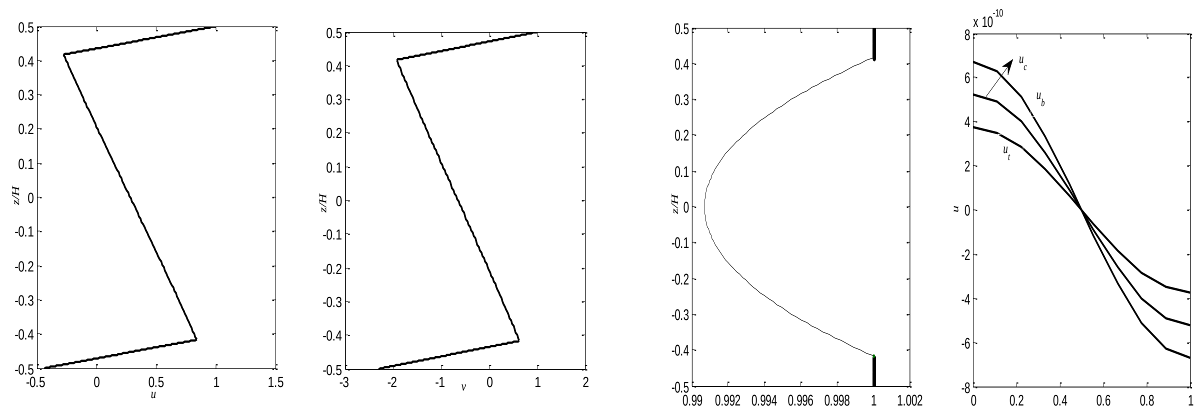
a/H	Present theory	[18]	[19]	[2]
2	0.7379	0.7141	0.7368	1.1734
4	0.9804	0.9363	0.9904	1.0913
10	1.9736	1.8480	1.9712	4.8519
20	3.6870	3.4791	3.6836	8.5838
30	5.3084	5.0371	5.3034	11.0788
40	6.7792	6.4634	6.777	12.6555
50	8.0703	7.7355	8.0698	13.6557
60	9.1993	8.8492	9.1929	14.3133
70	10.1561	9.8118	10.153	14.7583
80	11.0010	10.6368	10.9672	15.0702
90	11.6574	11.3408	11.6552	15.2946
100	12.2361	11.9400	12.2358	15.4647

Table 2
Comparison of the first non-dimensional fundamental frequencies of (0/90/core/0/90) sandwich plate with different a/b .

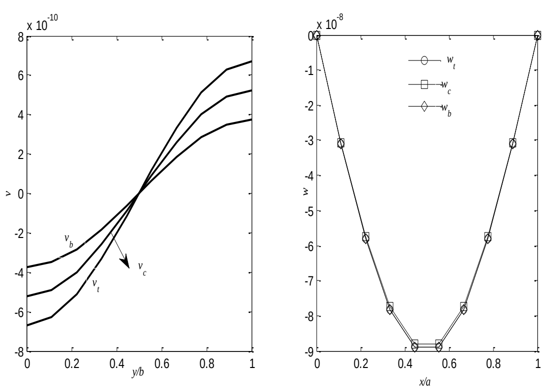
a/b	Present theory	[18]	[19]	[2]
1	1.9736	1.8464	1.9712	4.8519
1.5	1.1658	1.0900	1.1644	2.8130
2	0.8601	0.8048	0.8584	2.4469
2.5	0.7073	0.6627	0.7045	1.5660
3	0.6189	0.5804	0.6145	1.2976
5	0.4794	0.4494	0.4676	0.8102

In Figs. 2 and 3, two first modes ($m = n = 1$) are plotted for transverse, longitudinal and vertical displacements of the square sandwich panel (0.90/core/0.90). In the first mode (Fig. 2), the motions are in-phase with each other. The entire sandwich panel has one vibration. The sheets move up and down and the top and bottom layers move vertically in the same direction. However, in the second mode (Fig. 3), the motions are out-of-phase with each other and the vertical displacement is opposite to each other. Plane and transverse displacement of the sandwich panel are plotted in Figs. 2 and 3 (first row) with respect to the dimensionless thickness (*z/H*).

In-plane changes *u* and *v*, and vertical displacement *w* at points ($x = 0$ and $y = a/2$), ($x = a/2, y = 0$) and ($x = a/2, y = b/2$) are also studied. In the first mode, it is observed that displacements *u* and *v* have linear distribution, while w has non-linear distribution; on the contrary, in the second mode, *u* and *v* have non-linear distribution, while *w* has linear distribution.

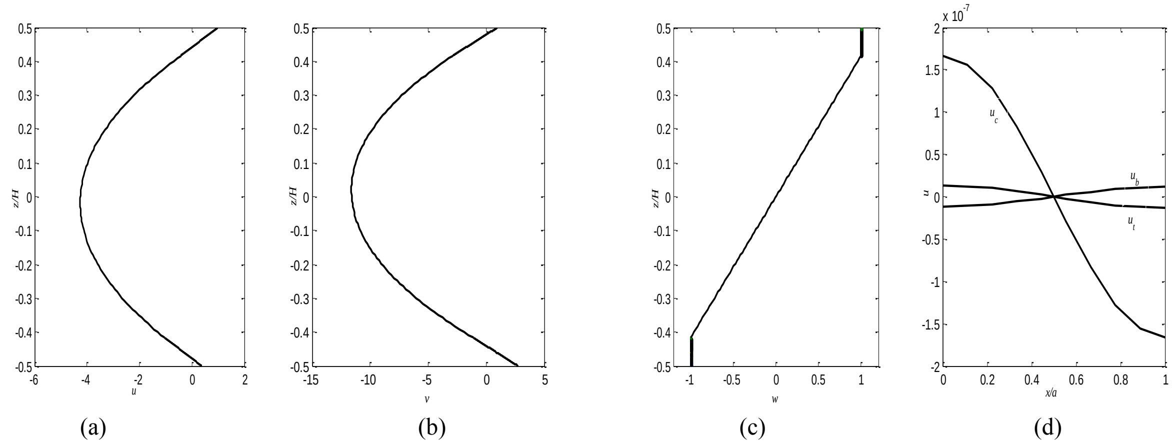


(a) (b) (c) (d)



(e) (f)

Fig.2
Through-the-thickness variations of the normalized displacements (first row) and normalized mode shapes (second row) corresponding to the Eigen-frequency $\Omega_1 = 2.1063$ of wave number ($m = n = 1$) for the (0/90/core/0/90) sandwich plate with $a/H = 10, c/d_t = 10$:
(a) *U*(*x*=0, *y*=b/2); (b) *v*(*x*=*a*/2, *y*=0); (c) *w*(*x*=*a*/2, *y*=*b*/2);
(d) *U*(*y*=*b*/2, $z_t = 0, z_c = c/2, z_b = 0$);
(e) *v*(*x*=*a*/2, $z_t = 0, z_c = c/2, z_b = 0$);
(f) *w*(*x*=a/2, $z_t = 0, z_c = c/2, z_b = 0$).



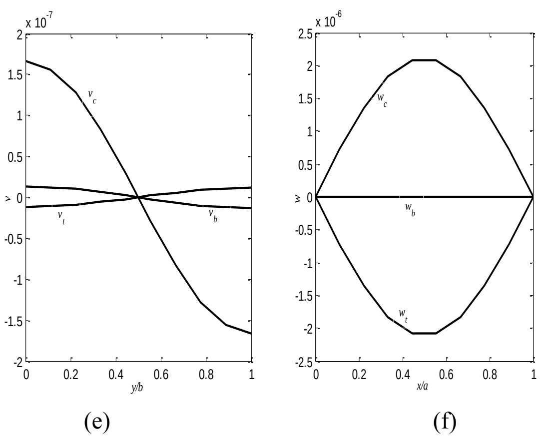


Fig.3
Through-the-thickness variations of the normalized displacements (first row) and normalized mode shapes (second row) corresponding to the Eigen-frequency $\Omega_2 = 13.8556$ of wave number ($m = n = 1$) for the (0/90/core/0/90) sandwich plate with $a/H = 10, c/d_t = 10$:
(a) *U*(*x*=0, *y*=b/2); (b) *v*(*x*=*a*/2, *y*=0); (c) *w*(*x*=*a*/2, *y*=*b*/2);
(d) *U*(*y*=*b*/2, $z_t = 0, z_c = c/2, z_b = 0$);
(e) *v*(*x*=*a*/2, $z_t = 0, z_c = c/2, z_b = 0$);
(f) *w*(*x*=a/2, $z_t = 0, z_c = c/2, z_b = 0$).

3.2 Results of sandwich panel buckling

Results of uniaxial buckling in the square 5-layered sandwich sheet (0.90/core/0.90) are compared with other references, as presented in Table 3. (material of the sandwich sheet is the same as previous section).

The results of the present work have good agreement with the results of layerwise method in reference [11] where the equations were extracted based on non-linear higher order theory. Furthermore, the results of the method used by Shari'at [16] had a slight difference with that of the present method. In this method, Shari'at [16] considered the non-linear strain-displacement relationship which satisfied the continuity of all the normal transverse components.

In addition, results obtained by the method used in reference [20], which is based on the linear theory, were not very similar to that of the present work because the stresses and normal transverse strains were ignored when extracting the equations.

In Table 4., buckling load of the 5-layered sandwich sheet (0.90/core/0.90) are obtained with different thicknesses (*a/H*=20,10,20/3,5), different length to width ratio (*a/b*=1,2,5,10) and different load ratios (*k*).

As presented in this table, for a sheet with fixed geometry, increased *k* decreases the buckling load, because the increase in the compressive load in other in-plane direction contributed to earlier buckling of the sheet and therefore decreased the buckling load. Investigating the results showed that at a constant *a/b* ratio, increase in the thickness of the sheet decreased the dimensionless buckling load.

3.3 Investigation of vibration with the axial load

In order to study the effect of external uniaxial force on vibration behavior, the 5-layered square sandwich sheet (0.90/core/0.90) was assumed to have material properties similar to the first example.

Dimensionless frequency response with respect to the dimensionless load is plotted in Figs. 4(a) and (b). According to the figures, increasing the load decreases the system frequency to the extent that the frequency would tend to zero at the critical load.

Table 3
Comparison dimensionless biaxial overall buckling load for rectangular sandwich plate with different a/H .

a/H	Present theory	[16]	[11]	[20]
2	0.1089	0.0105	0.0109	0.01173
4	0.0189	0.0181	0.0190	0.02083
10	0.07488	0.0708	0.0749	0.08313
20	0.2659	0.2513	0.2659	0.2952
30	0.5576	0.5269	0.5576	0.6190
40	0.9182	0.8707	0.9181	1.0128
50	1.3151	1.2535	1.3150	1.4380
60	1.7210	1.6492	1.7209	1.8643
70	2.1157	2.0299	2.1156	2.2870
80	2.4863	2.3923	2.4862	2.6739
90	2.8261	2.7261	2.8260	3.0257
100	3.1324	3.0282	3.1324	3.3408

Table 4
Dimensionless biaxial overall buckling load for rectangular sandwich plate [0/90/Core/0/90].

a/b	a/H	$k=0$	$k=0.1$	$k=0.2$	$k=0.5$	$k=1$	$k=2$	$k=5$	$k=10$
1	20/1	0.2750	0.2500	0.2292	0.1834	0.1375	0.0917	0.0458	0.0250
	20/2	0.0816	0.0742	0.0680	0.0544	0.0408	0.0272	0.0136	0.0084
	20/3	0.0448	0.0405	0.00371	0.0297	0.0223	0.0148	0.0074	0.0040
	20/4	0.0314	0.0286	0.0262	0.0210	0.0157	0.0105	0.0052	0.0029
2	20/1	0.7332	0.5237	0.4073	0.2444	0.1466	0.0815	0.0349	0.0179
	20/2	0.2627	0.1876	0.1459	0.0876	0.0525	0.0292	0.0125	0.0064
	20/3	0.1741	0.1244	0.0967	0.0580	0.0348	0.0193	0.0083	0.0042
	20/4	0.1428	0.1020	0.0793	0.0476	0.0286	0.0159	0.0068	0.0035
5	20/1	6.4631	1.8466	1.0772	0.04784	0.2486	0.1267	0.0513	0.0257
	20/2	4.0308	1.1517	0.6718	0.2986	0.1550	0.0790	0.0320	0.0161
	20/3	3.5397	1.0113	0.5900	0.2622	0.1361	0.0694	0.0281	0.0141
	20/4	3.3223	0.9492	0.5537	0.2461	0.1278	0.0651	0.0264	0.0132
10	20/1	10.6362	5.6942	2.9827	1.2282	0.6202	0.3116	0.1250	0.0626
	20/2	8.6859	4.6987	2.4612	1.0134	0.5117	0.2571	0.1032	0.0516
	20/3	7.232	4.2930	2.2487	0.9259	0.4676	0.2349	0.0943	0.0472
	20/4	5.4557	3.9505	2.0693	0.8521	0.4303	0.2162	0.0867	0.0434

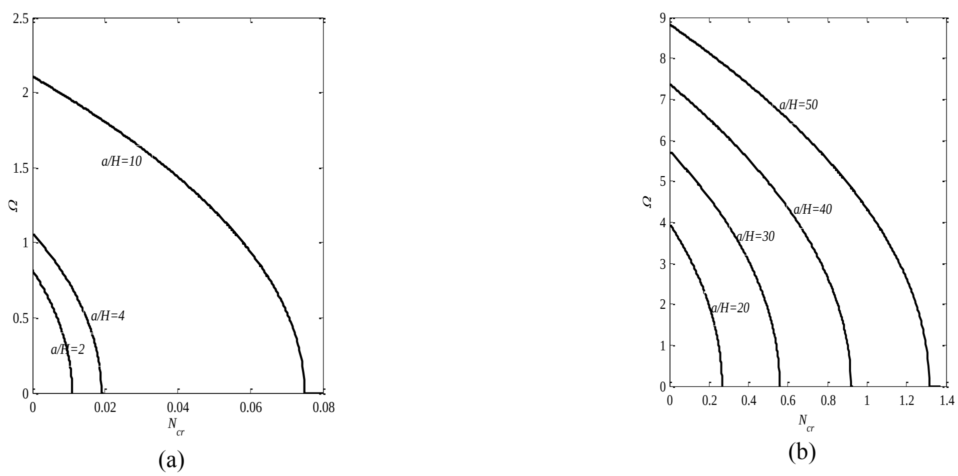


Fig.4
Effect of increasing compressive loading on the fundamental frequency a) thick sandwich plate b) thin sandwich plate.

4 CONCLUSIONS

In this article, equations of motion were formulated based on shear stresses in the core. First order shear stress theory was applied to the layers. In this theory, problem of inconsistencies between acceleration and velocity fields in the Frostig's first theory was simply resolved. The main benefit of this theory is its simplicity and limited number of equations with respect to the second method of Frostig's higher order theory. To extract the dynamic equations for the core, 3-D elasticity theory was utilized. To derive the governing dynamic equation of the entire system, Hamilton's principle was used. In free vibration analysis, the panel was assumed to be under the effect of initial plane compressive pre-loads. Results showed that by approaching the plane pre-loads to the values of critical buckling load, the natural frequency of the panel tends to zero. Results obtained from the present theory are in good agreement with the results reported by the recent articles. Speed, simplicity and high accuracy of the analytical solution proposed in this article are among its advantages in comparison to other methods. In this study, free vibration and buckling behaviors of a sandwich panel with a flexible core and simple supports for all the boundaries were studied using the improved theory with the following results:

1. Increasing the ratio a/b increases the length and reduces the width and the sheet would approach to a beam. On the contrary to the beams, in this case, all four edges of the sheet have simple support boundary condition and therefore vertical and shear deformations of the sheet decrease and stiffness and couplings between the membrane-bending and shear effects increase. As a result, natural frequency increases and the dimensionless frequency decreases.
2. Increase in ratio q/H, i.e. making the sheet thinner, increase the sheet dimensionless frequency.
3. At a constant d_t/H, increase in the sheet thickness decreases the dimensionless buckling load.
4. Increase in the ratio a/b increases the dimensionless buckling load.
5. Natural frequency decreases by increasing the axial load and reaches zero at critical buckling load.
6. In this research, for the first time, in addition to resolving the inconsistency in the mathematical logic of the Frostig's first theory, theory of the layers and the results were also improved.

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