

# An Exact Solution for Vibration Analysis of Soft Ferromagnetic Rectangular Plates Under the Influence of Magnetic Field with Levy Type Boundary Conditions

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## ABSTRACT

In this paper vibration of ferromagnetic rectangular plates which are subjected to an inclined magnetic field is investigated based on classical plate theory and Maxwell equations. Levy type solution and Finite element method using Comsol software are used to obtain the frequency of the plate subjected to different boundary conditions, good agreements is obtained when computed results are compared with those obtained by Comsol software, the results have shown that the frequency of the plates increases with the magnetic field and the effect of magnetic field is similar to the Winkler's foundation. © 2017 IAU, Arak Branch. All rights reserved.

**Keywords :** Free Vibration; Magnetic field; Ferromagnetic plate; Exact solution; Levy type.

## 1 INTRODUCTION

IN recent years, the increasing use of electromagnetic waves and magnetic fields which are result of electrical activities in a wide range of industries and sciences such as power plants, air industries branch of aerospace, medicine, power transmission lines, medical diagnostic devices, measuring instruments, non-destructive tests etc., have made this research study the importance of the effects of magnetic fields on the sheets which are one of the important components of devices working in presence of magnetic field. Electromagnetic theories such as Maxwell's law and Continuum Mechanics have been used for this type of evaluation. The result has led to derive the suggestion of Magnetic traction and Lorentz force on plates, Moon [1], Eringen [2], Liang et al. [3]. Liang wei et al. [3] omitted have studied Natural frequencies caused by vibrations, suggesting a model for beams in Magnetic fields and examining Cantilever thin beams. Moon and Pao [4] examined the buckling of thin plates under Magnetic fields. Young et al. [5] evaluated the buckling and bending of the thin Magnetoelastic plates and compared the results with the experimental results present. Takahisa et al. [6] omitted studied the application of Lorentz force in evaluating the floating systems in Magnetic fields for non-ferromagnetic thin plates. Hoffmann and Coworkers [7] have found the tensor of Maxwell's tension for Magnetoelastic substance and assessed the classic criteria for failure. Eringen [8] introduced the theory to evaluate the electromagnetic effects of elastic plates, He has attained the strain relations associated with these plates. Liang Wei et al. [11] omitted presented both a mathematical model and frequencies related to a cantilever beam in presence of magnetic field in practice. Chen et al. [13], Ven [15], Wang et al. [17], Goudjo and Maugin [18] omitted have studied free vibration of Non-homogeneous transverse isotropic elastic magneto electric Sheets and also the static & dynamic stability of plates have been studied by other researchers. Golubeva et al. [20] omitted, have studied the thin isotropic conducting ferromagnetic plate with finite electrical

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conductivity in presence of longitudinal magnetic field. In this article free vibration of ferromagnetic plates in constant magnetic fields with boundary conditions of Levy type for rectangular plates were evaluated. Natural frequencies, where extracted by precise calculation of the equations and the results were compared with the results of Finite element Method.

## 2 THEORETICAL FORMULATIONS

### 2.1 Structure

In this article which is based on the classic theory of plates, the governing equation of plate vibration based on Maxwell's equations is as follows [2 , 9]:

$$\nabla \cdot \vec{B} = 0 \tag{1}$$

$$\nabla \times \vec{H} = 0 \tag{2}$$

Also the Lorentz force is omitted explained as:

$$F_i^E = B_{i,j} M_j \tag{3}$$

In which  $B_{i,j}$  is the Intensity of the Magnetic field;  $H$  is the density of the Magnetic field;  $M$  is the magnetization vector. The equation of Plate's motion is extracted by implementing the dynamic equilibrium and Maxwell's relations [11]:

$$\nabla \cdot \sigma + \rho(f + \ddot{u}) + F^E = 0 \tag{4}$$

$${}_E \sigma_{ij} = \lambda \delta_{ij} u_{k,k} + G(u_{i,j} + u_{j,i}) = \lambda \delta_{ij} \epsilon_{kk} + 2G \epsilon_{ij} \tag{5}$$

In which  $\lambda$  and  $G$  are Lamé parameters;  ${}_E \sigma_{ij}$  is elasticity stress tensor; Stresses of the magnetic field regarding the intensity of the magnetic field are equal to  $B$  as follows:

$$\sigma_{xx}^E = -M_x B_x + \frac{1}{\mu_m} (B_x^2 - \frac{1}{2} B^2) + (M_x B_x + M_z B_z) \tag{6}$$

$$\sigma_{yy}^E = \frac{1}{\mu_m} (\frac{1}{2} B^2) + (M_x B_x + M_z B_z) \tag{7}$$

$$\vec{M} = \frac{x_m}{\mu_m} \vec{B} \tag{8}$$

$$\vec{M} = x_m \vec{H} \tag{9}$$

In which  $x_m$  is magnetic susceptibility ;  $\mu_m$  is magnetic permeability and  $\mu_0$  vaccum permeability; Regarding omitted the vector of magnetic field on the plate which creates angle  $\xi$  with as shown in Fig. 1, we will have:

$$\vec{B} = B_0 \cos \xi \hat{i} - B_0 \sin \xi \hat{k} \tag{10}$$

Due to the smaller magnetic strains in comparison with mechanical strains, the magnetic strains can be neglected

Also due to fixed magnetic fields all over the plate:

$$F_i^E = 0 \quad (11)$$

After writing the equilibrium relations of the plate we will have:

$$n_i (t_{ij})_{plate} - n_i (t_{ij})_{air} = 0 \quad (12)$$

So the magnetic tractions of the plate surface are as follows:

$$t_{ij} = n_i \left( \frac{1}{2\mu_0} (2B_i^- B_j^- - B_k^- B_k^- \delta_{ij}) \right) + n_i B_i M_j - n_i \left( \frac{1}{\mu_m} B_i B_j - \frac{1}{2\mu_m} B_k B_k \delta_{ij} \right) + n_i M_k B_k \delta_{ij} \quad (13)$$

In the above equation  $\delta_{ij}$  is kronecherdelta,  $B^-$  stands for the magnitude of the magnetic field out of the plate or air and  $B$  for the magnitude in the plate; with attention to the plate's motion, the amount of magnetic field on the surface of the plate is:

$$B_n^- = B_0 \sin(\phi + \xi) \quad (14)$$

$$\phi = \frac{\partial w}{\partial x} \quad (15)$$

with respect to the difference of magnetic field in the plate and air, by using the below equation the changes in the magnetic field will be obtained :

$$n_i (B_i)_{plate} - n_i (B_i)_{air} = 0 \quad (16)$$

$$B_n^- = B_n \quad (17)$$

$$B_x = B_x^- (1 + x_m) \quad (18)$$

Finally the surface tractions will be declared as follows:

$$t_{nx} = B_n M_x = M_x B_0 \left( \frac{\partial w}{\partial x} \cos \xi + \sin \xi \right) \quad (19)$$

Because this traction is affecting the upper and lower surface of the plate, the couple of the magnetic field is calculated as:

$$m_x = M_x h B_0 \left( \frac{\partial w}{\partial x} \cos \xi + \sin \xi \right) \quad (20)$$

Axial forces and bending moments and shear forces are defined as:

$$N_{ij} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_{ij}^E dz + \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_{ij}^E dz \quad , \quad ij = xx, yy, xy \quad (21)$$

$$M_{ij} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \frac{h}{2} E \sigma_{ij} z dz + \int_{-\frac{h}{2}}^{\frac{h}{2}} \frac{h}{2} \sigma_{ij}^E z dz, \quad ij = xx, yy, xy \tag{22}$$

$$Q_{(x,y)} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \frac{h}{2} \sigma_{(xz,yz)} dz \tag{23}$$

with replacing the upper quantities in the equilibrium equation there will be:

$$\frac{\partial^2 M_{xx}}{\partial x^2} + 2 \frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_{yy}}{\partial y^2} + \frac{\partial m_x}{\partial x} = \rho h \frac{\partial^2 w}{\partial t^2} \tag{24}$$

which finally leads to this equation:

$$D \nabla^4 w - K \frac{\partial^2 w}{\partial x^2} + \rho h \frac{\partial^2 w}{\partial t^2} = 0 \tag{25}$$

In which  $\nabla^4$  is biharmonic operator,  $D$  flexural rigidity of plate,  $\rho$  is density,  $h$  is plate thickness and  $K$  defined as:

$$K = m_x h B_0 \cos \xi \tag{26}$$

Eq. (25) is a partial differential equation of the fourth order degree and linear whose second term shows the effect of the magnetic field, it also shows that the magnetic field has created an effect similar to Winkler foundation as we can see. If the magnetic fields zero, Eq. (25) will change to the vibration equation of an ordinary classic plate, we will solve this equation using the method of separation of variables and the following assumption we will have:

$$w(x, y, t) = W(x, y) T(t) \tag{27}$$

This assumption will lead to two equations:

$$\frac{1}{T(t)} \frac{d^2 T(t)}{dt^2} = -\omega^2 \tag{28}$$

$$\nabla^4 W(x, y) - \frac{\psi_1}{\beta_1^2} \frac{\partial^2 W(x, y)}{\partial x^2} - \frac{\omega^2}{\beta_1^2} W(x, y) = 0 \tag{29}$$

which parameters are defined as:

$$\beta_1^2 := \frac{D}{\rho h} \tag{30}$$

$$\psi_1 := \frac{k}{\rho h} \tag{31}$$

$$\lambda^4 := \frac{\omega^2}{\beta_1^2} = \frac{\rho h \omega^2}{D} \tag{32}$$

$$\psi^2 := \frac{K}{D} \quad (33)$$

$$(\nabla^4 - \lambda^4)W(x, y) - \Psi^2 \frac{\partial^2 W(x, y)}{\partial x^2} = 0 \quad (34)$$

Solving the Eq.(28) will lead to this answer:

$$T(t) = A \cos \alpha t + B \sin \alpha t \quad (35)$$

By using the separation of variables we will have from Eq.(35):

$$W(x, y) = X(x)Y(y) \quad (36)$$

$$X^{(IV)}Y + 2X''Y'' + XY^{(IV)} - \Psi^2 X''Y - \lambda^4 XY = 0 \quad (37)$$

$$Y''(y) = -\beta^2 Y(y) \quad , \quad Y^{(IV)}(y) = -\beta^2 Y''(y) \quad (38)$$

$$X''(x) = -\alpha^2 X(x) \quad , \quad X^{(IV)}(x) = -\alpha^2 X''(x) \quad (39)$$

After replacement:

$$-\alpha^2 X''Y + 2\alpha^2 \beta^2 XY - X \beta^2 Y'' + \Psi^2 \alpha^2 XY - \lambda^4 XY = 0 \quad (40)$$

$$-\beta^2 \frac{Y''}{Y} + 2\alpha^2 \beta^2 + \Psi^2 \alpha^2 - \lambda^4 = \alpha^2 \frac{X''}{X} = \text{Const.} \quad (41)$$

$$X_m(x) = A \sin \alpha_m x \quad m = 1, 2, \dots \quad (42)$$

Finally the applied boundary conditions for simply supported plate lead to:

$$Y(y) = C_1 \sin \delta_1 y + C_2 \cos \delta_1 y + C_3 \sinh \delta_2 y + C_4 \cosh \delta_2 y \quad (43)$$

which

$$\delta_2 = \sqrt{\alpha_m^2 + \sqrt{\lambda^4 - \Psi^2 \alpha_m^2}} \quad (44)$$

$$\delta_1 = \sqrt{-\alpha_m^2 + \sqrt{\lambda^4 - \Psi^2 \alpha_m^2}} \quad (45)$$

By simplifying for natural frequencies and mode shapes, then there will be:

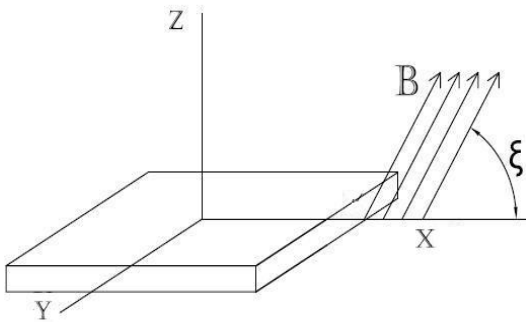
$$\omega_{mn} = \pi^2 \left[ \left( \frac{m^2}{a^2} + \frac{n^2}{b^2} \right)^2 + \frac{m^2}{\pi^2 a^2} \Psi^2 \right]^{\frac{1}{2}} \sqrt{\frac{D}{\rho h}} \quad m, n = 1, 2, \dots \quad (46)$$

$$w_{mn}(x, y, t) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} (A_{mn} \cos \omega_{mn} t + B_{mn} \sin \omega_{mn} t) \quad (47)$$

### 3 RESULTS AND DISCUSSION

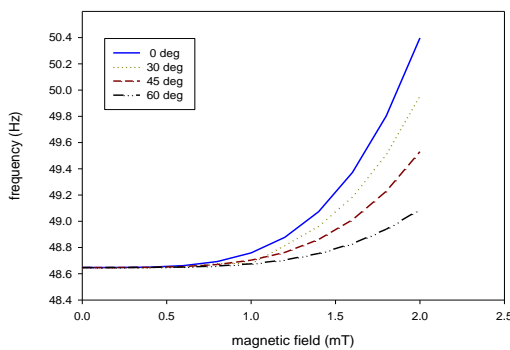
For obtaining the natural frequencies of plates, the ratio of length to width was different and in all the thickness of 1 Cm was fixed . Magnetic field was applied to the plates with different angles of 30-45-60 degree. The silicon steel plate had characteristics of  $\rho = 7800 \frac{kg}{m^3}$ ,  $E = 200Gpa$ ,  $\nu = 0.3$  and  $x = 1000$ ,  $\mu_0 = 4\pi \times 10^{-7} H/m$ . The natural frequencies of the plate were extracted from the Eq.(46).The ratio of frequency changes of rectangular plate with different length to width ratio and Levy type boundary conditions, under magnetic field compared to the situation where the magnetic field is zero, which is shown in Fig. 2 through 8, results have shown that with increasing the radiation angle of magnetic field, the effect of magnetic field on the natural frequency is decreased. Fig. 9 through 14 are the curves showing that first natural frequency varies with the angle of magnetic field. Fig. 15, shows the effect of magnetic field exposure angle on the first natural frequency using finite element method and exact solution. Comparison of two methods with increase of magnetic intensity, shown in Fig. 16, indicates that omitted the results have no significant difference. Fig. 17through 22 illustrates, how with increasing the magnetic intensity, frequency ratio, which defined as natural frequency with magnetic field per natural frequency in the absence of magnetic field, is increased.

According to the results of precise calculation and simulation by using the Finite Element Method by Comsol software, it is observed that there is not a significant error between the two results, the maximum error is 1.98%. The results of both methods reveal that magnetic fields increase the natural frequencies of plates, and there is a rise in natural frequencies as the magnitude of the magnetic field increases. Results show that by changing the angle of the magnetic field, the natural frequencies will also change so that the effect of the Magnetic field on natural frequencies will decrease as the angle is increased, which will reach zero at the angle of 90 degree.



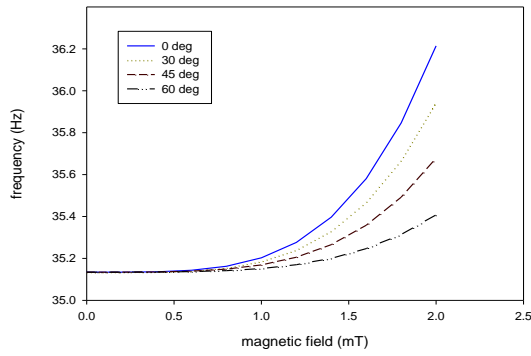
**Fig.1**

The radiation method of the magnetic field to the plate.

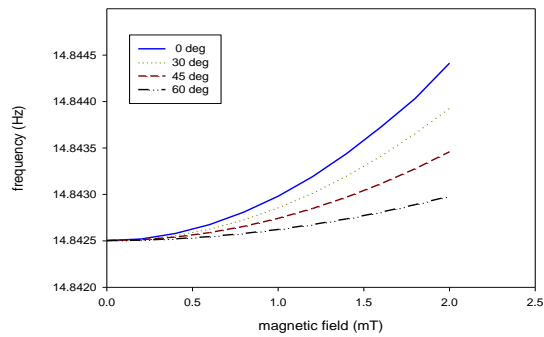


**Fig.2**

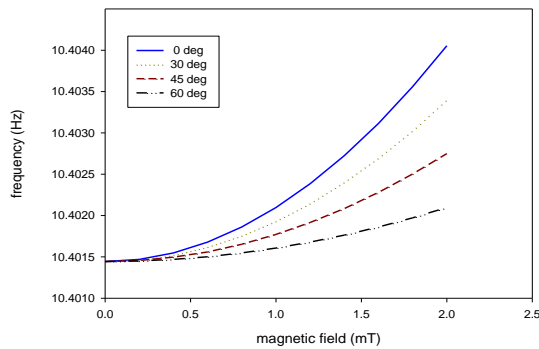
The first natural frequency changes in proportion to the applied magnetic field to the plate with ssss boundary conditions and ratio  $a/b = 1$  at different angles.



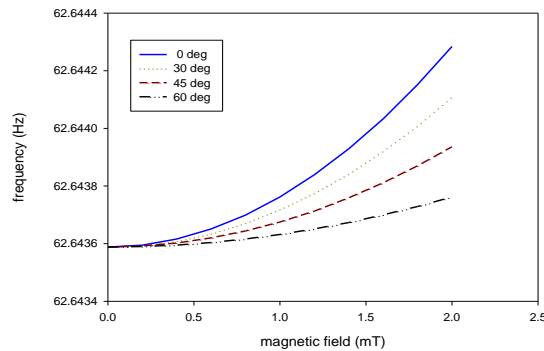
**Fig.3**  
The first natural frequency changes in proportion to the applied magnetic field to the plate with ssss boundary conditions and ratio  $a/b = 1.5$  at different angles.



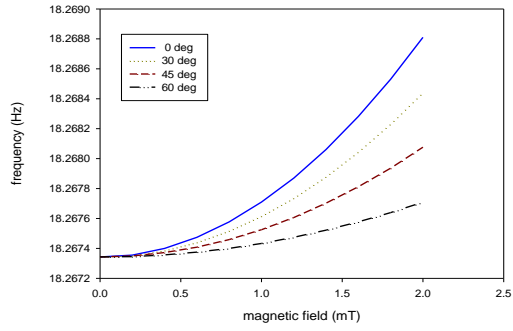
**Fig.4**  
The first natural frequency changes in proportion to the applied magnetic field to the plate with sssf boundary conditions and ratio  $a/b = 1.5$  at different angles.



**Fig.5**  
The first natural frequency changes in proportion to the applied magnetic field to the plate with ssff boundary conditions and ratio  $a/b = 1.5$  at different angles.

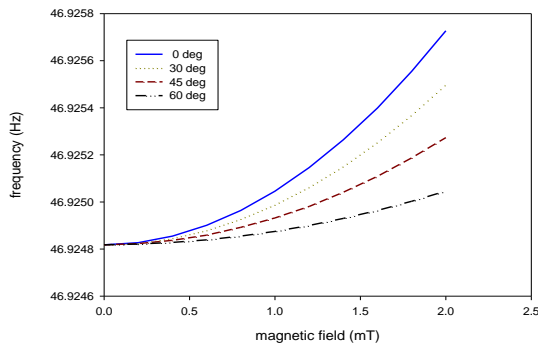


**Fig.6**  
The first natural frequency changes in proportion to the applied magnetic field to the plate with sscc boundary conditions and ratio  $a/b = 1.5$  at different angles.



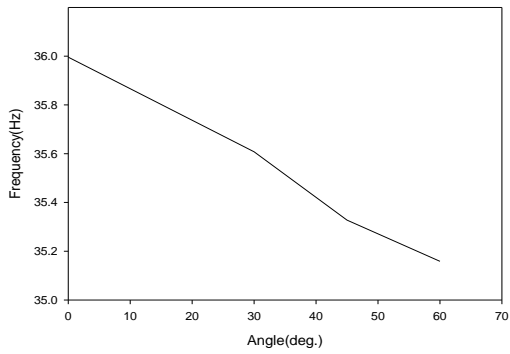
**Fig.7**

The first natural frequency changes in proportion to the applied magnetic field to the plate with sscf boundary conditions and ratio  $a/b = 1.5$  at different angles.



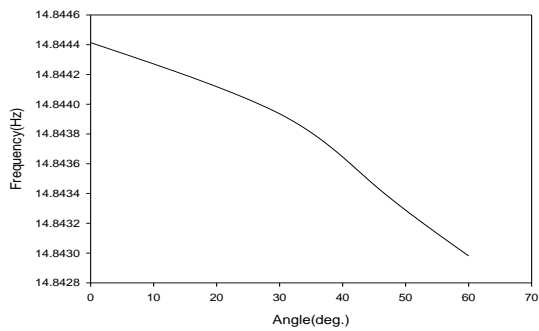
**Fig.8**

The first natural frequency changes in proportion to the applied magnetic field to the plate with sscs boundary conditions and ratio  $a/b = 1.5$  at different angles.



**Fig.9**

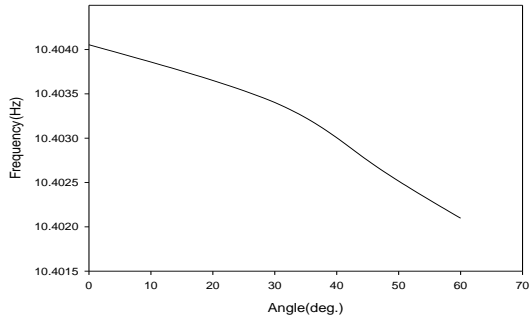
The effect of magnetic field exposure angle on the first natural frequency of the plate with sssf boundary conditions and ratio  $a/b = 1.5$ .



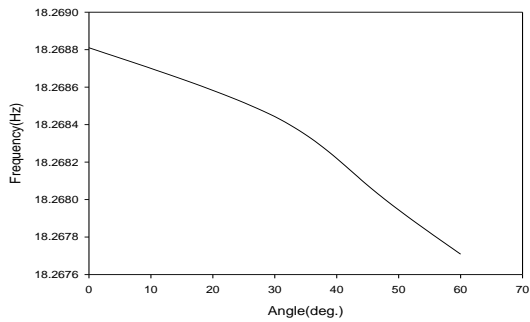
**Fig.10**

The effect of magnetic field exposure angle on the first natural frequency of the plate with sssf boundary conditions and ratio  $a/b = 1.5$ .

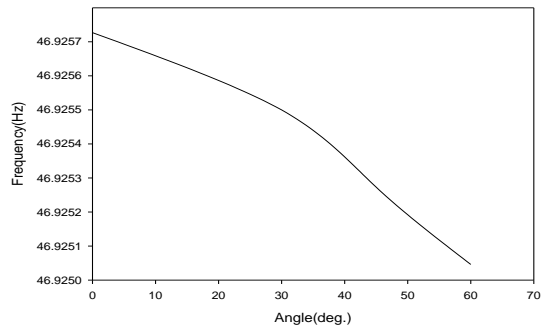




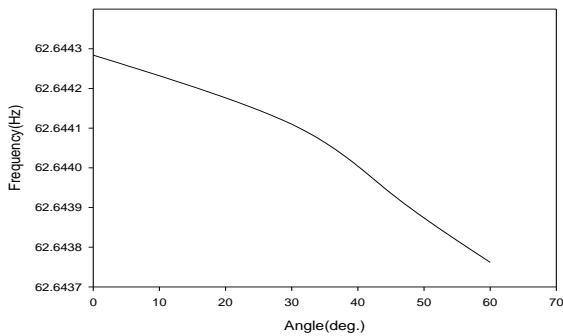
**Fig.11**  
The effect of magnetic field exposure angle on the first natural frequency of the plate with ssfc boundary conditions and ratio  $a/b = 1.5$ .



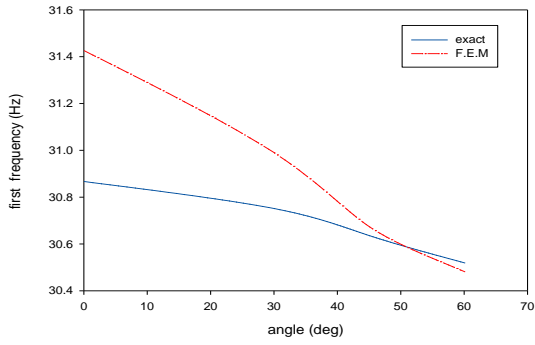
**Fig.12**  
The effect of magnetic field exposure angle on the first natural frequency of the plate with sssc boundary conditions and ratio  $a/b = 1.5$ .



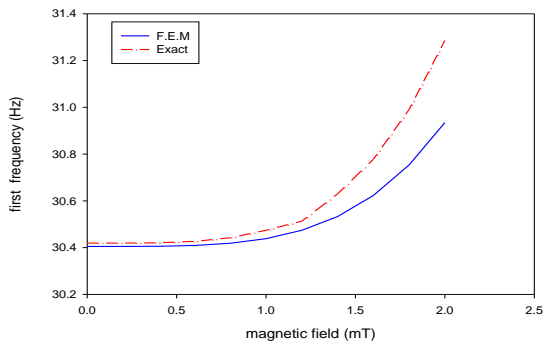
**Fig.13**  
The effect of magnetic field exposure angle on the first natural frequency of the plate with sscs boundary conditions and ratio  $a/b = 1.5$ .



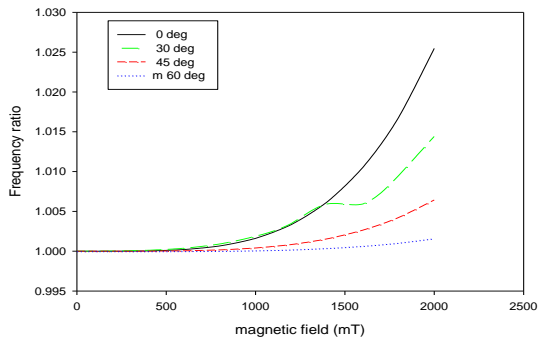
**Fig.14**  
The effect of magnetic field exposure angle on the first natural frequency of the plate with ssc boundary conditions and ratio  $a/b = 1.5$ .



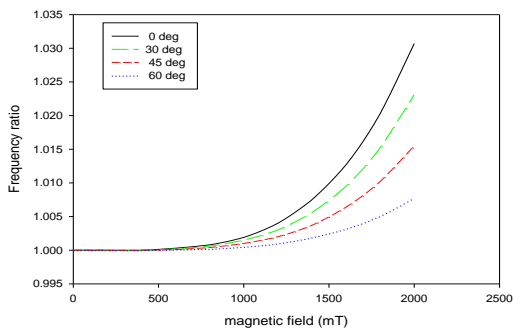
**Fig.15**  
The Effect of exposure angle on the natural frequencies of plate and comparison with finite element method.



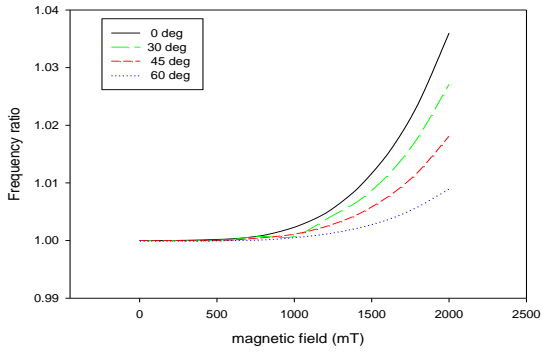
**Fig.16**  
The Effect of magnetic field intensity on the natural frequency of plate and comparison with the finite element method.



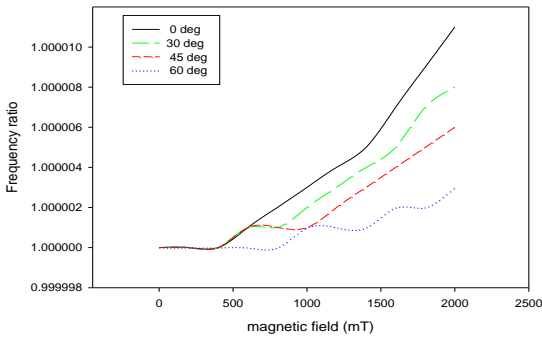
**Fig.17**  
Changes in the frequency ratio of magnetic intensity for different angles relative to the plate with ssss boundary conditions and ratio  $a/b = 1.5$  by finite element method.



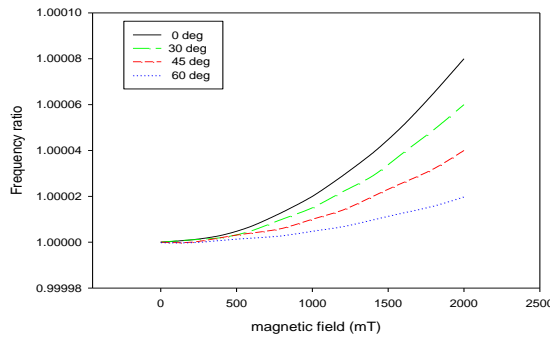
**Fig.18**  
Changes in the frequency ratio of magnetic intensity for different angles relative to the plate with ssss boundary conditions and ratio  $a/b = 1.5$  by exact method.



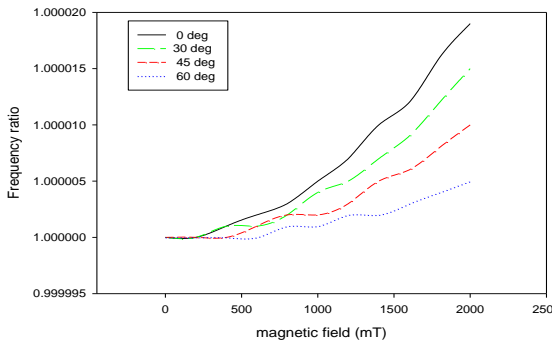
**Fig.19**  
Changes in the frequency ratio of magnetic intensity for different angles relative to the plate with ssss boundary conditions and ratio  $a/b = 1$  by exact method.



**Fig.20**  
Changes in the frequency ratio of magnetic intensity for different angles relative to the plate with sssc boundary conditions and ratio  $a/b = 1.5$  by finite element method.



**Fig.21**  
Changes in the frequency field of magnetic intensity for different angles relative to the plate with ssss boundary conditions and ratio  $a/b = 1.5$  by finite element method.



**Fig.22**  
Changes in the frequency ratio of magnetic intensity for different angles relative to the plate with sssc boundary conditions and ratio  $a/b = 1.5$  by finite element method.

#### 4 CONCLUSIONS

Vibration of rectangular plate subjected to an inclined magnetic field were studied by a closed form solution and finite element method using Comsol software, the results of the two methods are compatible with each other, The

results show that the frequency of the plate increases with increasing applied magnetic field and as the inclined angle increased, the influence of Magnetic field reduces. And the effect of the magnetic field is similar to Winkler elastic foundation on the vibration of the plate.

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