

# Effects of Hall Current and Rotation in Modified Couple Stress Generalized Thermoelastic Half Space due to Ramp-Type Heating

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## ABSTRACT

The objective is to study the deformation in a homogeneous isotropic modified couple stress thermoelastic rotating medium in the presence of Hall current and magnetic field due to a ramp-type thermal source. The generalized theories of thermoelasticity developed by Lord Shulman (L-S, 1967) and Green Lindsay (G-L, 1972) are used to investigate the problem. Laplace and Fourier transform technique is applied to obtain the solutions of the governing equations. The displacements, stress components, temperature change and mass concentration are obtained in the transformed domain. Numerical inversion technique has been used to obtain the solutions in the physical domain. Effects of Hall current and rotation are shown in a resulting quantities. Some special cases of interest are also deduced.

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**Keywords:** Modified couple stress; Generalized thermoelasticity; Laplace and Fourier transforms; Ramp-Type heating; Hall current and magnetic effect.

## 1 INTRODUCTION

**C**LASSICAL first gradient approaches in continuum mechanics do not address the size dependency that is observed in smaller scales. Consequently, a number of theories that include higher gradients of deformation have been proposed to capture, at least partially, size-effects at the nano-scale. Additionally, consideration of the second gradient of deformation leads naturally to the introduction of the concept of couple-stresses. Thus, in the current form of these theories, the material continuum may respond to body and surface couples, as well as spin inertia for dynamical problems. The existence of couple-stress in materials was originally postulated by [1]. However, [2] were the first to develop a mathematical model to analyze materials with couple stresses. Lacking an internal material length scale parameter, classical elasticity and plasticity cannot be used to interpret the size effect observed in numerous tests at micron and nanometer scales. However, higher-order (non-local) continuum theories contain material length scale parameters and are capable of explaining microstructure related size (and other effects). Couple stress theories represent one class of such higher-order theories. The classical couple stress elasticity theory was proposed by (e.g., [3-6]) contains four material constants two classical and two additional for isotropic elastic materials. The couple stress theory can be viewed as a special format of strain gradient theory which uses rotation as a variable to describe curvature, while the strain gradient theory uses strain as variable to describe curvature. Couple-stress theory is an extended continuum theory that includes the effects of a couple per unit area on

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a material volume, in addition to the classical direct and shear forces per unit area. This immediately admits the possibility of asymmetric stress tensor, since shear stress no longer have to be conjugate in order to ensure rotational equilibrium. The two additional constants are related to the underlying microstructure of the material and are inherently difficult to determine (e.g. [7], [8]). Every physical theory possesses a certain domain of applicability outside which it fails to predict the physical phenomena with reasonable accuracy. Hence, there has been a need to develop higher-order theories involving only one additional material length scale parameter. Recently, [9] developed a modified couple-stress model, in which the couple stress tensor is symmetrical and only one material length parameter is needed to capture the size effect which is caused by micro-structure. [10] studied the Bernoulli- Euler beam model based on a modified couple stress theory. Variational formulation of a modified couple stress theory and its application to a simple shear problem was studied by [11]. [12] investigated the size effect on dynamic stability of functionally graded microbeams based on a modified couple stress theory. [13] presented a modified couple stress model for bending analysis of composite laminated beams with first order shear deformation. [14] studied the geometrically nonlinear micro-plate formulation based on the modified couple stress theory. [15] investigated the bending and vibration of functionally graded microbeams using a new higher order beam theory and the modified couple stress theory. Recently, the size dependent buckling analysis of microbeams based on modified couple stress theory with high order theories and general boundary conditions have been studied by [16]. [17] studied the size-dependent bending analysis of Kirchhoff nano-plates based on a modified couple-stress theory including surface effects. Thermodiffusion in an elastic solid is due to the coupling of the fields of temperature, mass diffusion and strain. Heat and mass exchange with the environment during the process of the thermodiffusion in an elastic solid. The concept of thermodiffusion is used to describe the processes of thermomechanical treatment of metals (carboning, nitriding steel, etc.) and these processes are thermally activated, and their diffusing substances being, e.g. nitrogen, carbon etc. They are accompanied by deformations of the solid. [18-22] developed the theory of thermoelastic with mass diffusion. In this theory, the coupled thermoelastic model is used. This implies infinite speeds of propagation of thermoelastic waves. [23] developed the theory of generalized thermoelastic diffusion that predicts finite speeds of propagation for thermoelastic and diffusive waves. [24] worked on a problem of a thermoelastic half space with a permeating substance in contact with the bounding plane in the context of the theory of generalized thermoelastic diffusion with one relaxation time. Recently, [25] derived the basic equations in generalized thermoelastic diffusion for Green Lindsay (GL-model) theory and discussed the Lamb waves. The foundations of magnetoelasticity were presented by [26] and [27] and developed by [28]. An increasing attention is being de-voted to the interaction between magnetic field and strain field in a thermoelastic solid due to its many applications in the fields of geophysics, plasma physics and related topics. In all papers quoted above it was assumed that the interactions between the two fields take place by means of the Lorentz forces appearing in the equations of motion and by means of a term entering Ohm's law and describing the electric field produced by the velocity of a material particle, moving in a magnetic field. When the magnetic field is very strong, the conductivity will be a tensor and the effect of Hall current cannot be neglected. The conductivity normal to the magnetic field is reduced due to the free spiraling of electrons and ions about the magnetic lines of force before suffering collisions and a current is induced in a direction normal to both the electric and magnetic fields. This phenomenon is called the Hall effect. In all of the above investigations, the effects of Hall current have not been considered. Effects of Hall current and rotation on magneto-micropolar generalized thermoelasticity due to ramp-type heating was studied by [29]. [30] also investigated the effect of Hall current on generalized magneto-thermoelasticity micropolar solid subjected to ramp-type heating.

The objective of this paper is to consider two dimensional modified couple stress generalized thermoelastic with mass diffusion in the presence of a uniform strong magnetic field acts in  $x_2$  direction taken into consideration the effects of Hall current and rotation. This new model is applied to generalization, Lord – Shulman theory and Green – Lindsay theory solved by using Laplace and Fourier transform technique. The ramp-type heating application is employed to our problem to get the solution in the complete form. The normal stress, tangential stress, couple stress, temperature change and mass concentration are computed and presented graphically for different values of distance. Some particular cases are also derived from the present investigation.

## 2 BASIC EQUATIONS

Following ([9], [25], [29]) the constitutive relations and the equations of motion in a modified couple-stress generalized thermoelastic elastic with mass diffusion in the absence of body forces, body couples, heat and mass diffusion sources are

(i) Constitutive relations

$$t_{ij} = \lambda e_{kk} \delta_{ij} + 2\mu e_{ij} - \frac{1}{2} e_{kij} m_{lk,l} - \beta_1 \left(1 + \tau_1 \frac{\partial}{\partial t}\right) T \delta_{ij} - \beta_2 \left(1 + \tau^1 \frac{\partial}{\partial t}\right) C \delta_{ij}, \quad (1)$$

$$m_{ij} = 2\alpha \chi_{ij}, \quad (2)$$

$$\chi_{ij} = \frac{1}{2} (\omega_{ij} + \omega_{ji}), \quad (3)$$

$$\omega_i = \frac{1}{2} e_{ipq} u_{q,p}. \quad (4)$$

(ii) Equations of motion in the rotation frame of reference are

$$K \Delta T - \rho c_e \left( \frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2} \right) T - a T_0 \left( \frac{\partial}{\partial t} + \gamma \frac{\partial^2}{\partial t^2} \right) C = T_0 \beta_1 \left( \frac{\partial}{\partial t} + \tau_0 \eta_0 \frac{\partial^2}{\partial t^2} \right) (\nabla \cdot \mathbf{u}), \quad (5)$$

(iii) Equation of heat conduction

$$\left( \lambda + \mu + \frac{\alpha}{4} \Delta \right) \nabla (\nabla \cdot \mathbf{u}) + \left( \mu - \frac{\alpha}{4} \Delta \right) \nabla^2 \mathbf{u} - \beta_1 \left(1 + \tau_1 \frac{\partial}{\partial t}\right) \nabla T - \beta_1 \left(1 + \tau^1 \frac{\partial}{\partial t}\right) \nabla C + F = \rho (\ddot{\mathbf{u}} + \Omega \times (\Omega \times \mathbf{u}) + 2(\Omega \times \dot{\mathbf{u}})), \quad (6)$$

(iv) Equation of mass diffusion

$$D \beta_2 \Delta (\nabla \cdot \mathbf{u}) + Da \left(1 + \tau_1 \frac{\partial}{\partial t}\right) \Delta T + \left( \frac{\partial}{\partial t} + \tau^0 \eta_0 \frac{\partial^2}{\partial t^2} \right) C - Db \Delta \left(1 + \tau^1 \frac{\partial}{\partial t}\right) C = 0, \quad (7)$$

$\mathbf{u} = (u_1, u_2, u_3)$  is the components of displacement vector, where  $t_{ij}$  are the components of stress tensor,  $\lambda$  and  $\mu$  are material constants,  $\delta_{ij}$  is Kronecker's delta,  $e_{ij}$  are the components of strain tensor,  $e_{ijk}$  is alternate tensor,  $m_{ij}$  are the components of couple-stress,  $\beta_1 = (3\lambda + 2\mu)\alpha_t$ ,  $\beta_2 = (3\lambda + 2\mu)\alpha_c$ , Here  $\alpha_t, \alpha_c$  are the coefficients of linear thermal expansion and diffusion expansion respectively,  $T$  is the temperature change,  $C$  is the mass concentration,  $\Omega$  is the rotation,  $\alpha$  is the couple stress parameter,  $\chi_{ij}$  is symmetric curvature,  $\omega_i$  is the rotational vector,  $b$  is the coefficient describing the measure of mass diffusion effects,  $a$  is the coefficient describing the measure of thermoelastic diffusion.  $\mathbf{u} = (u_1, u_2, u_3)$  is the components of displacement vector,  $\rho$  is the density,  $\Delta$  is the Laplacian operator,  $\nabla$  is del operator.  $K$  is the coefficient of the thermal conductivity,  $c_e$  is the specific heat at constant strain,  $T_0$  is the reference temperature assumed to be such that  $T/T_0 \ll 1$ .  $D$  is the thermoelastic diffusion constant, Here  $\tau^0, \tau^1$  are the diffusion relaxation times with  $\tau^1 \geq \tau^0 \geq 0$  and  $\tau_0, \tau_1$  are thermal relaxation times with  $\tau_1 \geq \tau_0 \geq 0$ . Here  $\tau_1 = \tau^1 = 0$ ,  $\eta_0 = 1, \gamma = \tau_0$ , for Lord-Shulman (L-S) model and  $\eta_0 = 0, \gamma = \tau^0$ , for Green Lindsay (G-L) model.

Following Zakaria [29], the generalized Ohm's law including Hall current:

$$\mathbf{J} = \frac{\sigma_0}{1+m^2} \left[ \mathbf{E} + \mu_0 (\dot{\mathbf{u}} \times \mathbf{H}) - \frac{\mu_0}{en_e} (\mathbf{J} \times \mathbf{H}) \right], \quad (8)$$

and  $F = \mu_0 (J \times H)$  is the Lorentz force.

where  $J$  is the current density vector,  $\mu_0$  is the magnetic permeability,  $H$  is the total magnetic field vector,  $E$  is the intensity vector of the magnetic field.

### 3 FORMULATION AND SOLUTION OF THE PROBLEM

A homogeneous isotropic, modified couple stress generalized thermoelastic elastic body with mass diffusion occupying the region of a half space  $x_3 \geq 0$  is taken. We consider a rectangular Cartesian coordinate system  $(x_1, x_2, x_3)$  having origin on the surface  $x_3 = 0$ . We consider a plane deformation problem with all the field quantities depending only on  $(x_1, x_3, t)$ . The half surface is subjected to ramp-type heating on the bounding plane  $x_3 = 0$  along with traction-free and iso-concentrated boundary.

For two dimensional problem, we take

$$u_i = (u_1(x_1, x_3, t), 0, u_3(x_1, x_3, t)), T(x_1, x_3, t), C(x_1, x_3, t), \quad (9)$$

Let us assume that the magnetic field  $H$  and the angular velocity  $\Omega$  acts in the direction of  $x_2$  axis as:

$$H = (0, H_0, 0), \quad (10)$$

$$\Omega = (0, \Omega, 0), \quad (11)$$

We also assume that  $E = 0$  and the generalized Ohm's law  $J_2 = 0$  everywhere in the medium. With these considerations, the current density components  $J_1$  and  $J_3$  are given by

$$J_1 = \frac{\sigma_0 B_0}{1+m^2} \left( m \frac{\partial u_1}{\partial t} - \frac{\partial u_3}{\partial t} \right) \quad (12)$$

$$J_3 = \frac{\sigma_0 B_0}{1+m^2} \left( \frac{\partial u_1}{\partial t} + m \frac{\partial u_3}{\partial t} \right) \quad (13)$$

We define the dimensionless quantities:

$$\begin{aligned} x_i' &= \frac{\omega^*}{c_1} x_i, u_i' = \frac{\omega^*}{c_1} u_i, t' = \omega^* t, t_{ij}' = \frac{t_{ij}}{\beta_1 T_0}, m_{ij}' = \frac{m_{ij}}{c_1 \beta_1 T_0}, \gamma' = \omega^* \gamma, \tau_1' = \omega^* \tau_1, \tau_0' = \omega^* \tau_0, \tau^{0'} = \omega^* \tau^0, \\ \tau^{1'} &= \omega^* \tau^1, T' = \frac{\beta_1 T}{\rho c_1^2}, C' = \frac{\beta_2 C}{\rho c_1^2}, \Omega' = \frac{\Omega}{\omega^*}, M = \frac{\sigma_0 B_0^2}{\rho \omega^*}, c_1'^2 = \frac{\lambda + 2\mu}{\rho}, \omega^{*2} = \frac{\lambda}{(\mu t^2 + \rho \alpha)}. \end{aligned} \quad (14)$$

where  $\omega^*$  and  $c_1$  are characteristic frequency and longitudinal wave velocity in the media and  $M$  is the Hartmann number or magnetic parameter respectively.

Upon introducing (14) in Eqs. (5)-(7) with aid of (9)-(13), after suppressing the primes, we obtain

$$\begin{aligned} \frac{\partial^2 u_1}{\partial t} &= \frac{\lambda + \mu}{\rho c_1^2} \left( \frac{\partial e}{\partial x_1} \right) + \frac{\mu}{\rho c_1^2} \nabla^2 u_1 + \frac{\alpha \omega^{*2}}{4 \rho c_1^4} \left( \frac{\partial e}{\partial x_1} - \nabla^2 u_1 \right) - \left( 1 + \tau_1 \frac{\partial}{\partial t} \right) \frac{\partial T}{\partial x_1} - \left( 1 + \tau^1 \frac{\partial}{\partial t} \right) \frac{\partial C}{\partial x_1} \\ &- \frac{M}{(1+m^2)} \left( \frac{\partial u_1}{\partial t} + m \frac{\partial u_3}{\partial t} \right) + \Omega^2 u_1 - 2\Omega \frac{\partial u_3}{\partial t}, \end{aligned} \tag{15}$$

$$\begin{aligned} \frac{\partial^2 u_3}{\partial t} &= \frac{\lambda + \mu}{\rho c_1^2} \left( \frac{\partial e}{\partial x_3} \right) + \frac{\mu}{\rho c_1^2} \nabla^2 u_3 + \frac{\alpha \omega^{*2}}{4 \rho c_1^4} \left( \frac{\partial e}{\partial x_3} - \nabla^2 u_3 \right) - \left( 1 + \tau_1 \frac{\partial}{\partial t} \right) \frac{\partial T}{\partial x_3} - \left( 1 + \tau^1 \frac{\partial}{\partial t} \right) \frac{\partial C}{\partial x_3} \\ &+ \frac{M}{(1+m^2)} \left( m \frac{\partial u_1}{\partial t} - \frac{\partial u_3}{\partial t} \right) + \Omega^2 u_3 + 2\Omega \frac{\partial u_1}{\partial t}, \end{aligned} \tag{16}$$

$$\nabla^2 T - \frac{\rho c_e c_1^2}{K \omega^*} \left( \frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2} \right) T - \frac{a T_0 \beta_1 c_1^2}{\beta_2 K \omega^*} \left( \frac{\partial}{\partial t} + \gamma \frac{\partial^2}{\partial t^2} \right) C = \frac{T_0 \beta_1^2}{\rho K \omega^*} \left( \frac{\partial}{\partial t} + \tau_0 \eta_0 \frac{\partial^2}{\partial t^2} \right) e, \tag{17}$$

$$\nabla^2 e + \frac{a \rho c_1^2}{\beta_1 \beta_2} \left( 1 + \tau_1 \frac{\partial}{\partial t} \right) \nabla^2 T + \frac{\rho c_1^4}{\beta_2^2 D \omega^*} \left( \frac{\partial}{\partial t} + \eta_0 \tau^0 \frac{\partial^2}{\partial t^2} \right) C - \frac{b \rho c_1^2}{\beta_2^2} \nabla^2 \left( 1 + \tau^1 \frac{\partial}{\partial t} \right) C = 0, \tag{18}$$

where

$$\nabla^2 = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_3^2}, e = \frac{\partial u_1}{\partial x_1} + \frac{\partial u_3}{\partial x_3}.$$

The displacement components  $u_1(x_1, x_3, t)$  and  $u_3(x_1, x_3, t)$  are related to the scalar potentials  $\phi(x_1, x_3, t)$  and  $\psi(x_1, x_3, t)$  in dimensionless form as:

$$u_1 = \frac{\partial \phi}{\partial x_1} - \frac{\partial \psi}{\partial x_3}, u_3 = \frac{\partial \phi}{\partial x_3} + \frac{\partial \psi}{\partial x_1}. \tag{19}$$

with the aid of (19), Eqs. (15)-(18) yield

$$\left[ \nabla^2 - \frac{M}{1+m^2} \left( \frac{\partial}{\partial t} \right) - \frac{\partial^2}{\partial t^2} + \Omega^2 \right] \phi - \left[ \frac{M}{1+m^2} + 2\Omega \right] \frac{\partial \psi}{\partial t} - \tau_t T - \tau_t^1 C = 0, \tag{20}$$

$$\left[ \frac{M}{1+m^2} + 2\Omega \right] \frac{\partial \phi}{\partial t} + \left[ a_1 \nabla^2 - a_2 \nabla^4 - \frac{M}{1+m^2} \left( \frac{\partial}{\partial t} \right) - \frac{\partial^2}{\partial t^2} + \Omega^2 \right] \psi = 0, \tag{21}$$

$$\left( -a_5 \nabla^2 \tau_{n_0}^0 \right) \phi + \left( \nabla^2 - a_3 \tau_t^0 \right) T - a_4 \tau_t^0 C = 0, \tag{22}$$

$$\nabla^4 \phi + a_6 \tau_t \nabla^2 T + \left( a_7 \tau_t^{10} - a_8 \tau_t^1 \nabla^2 \right) C = 0, \tag{23}$$

where

$$a_1 = \frac{\mu}{\rho c_1^2}, a_2 = \frac{\alpha \omega^{*2}}{4\rho c_1^4}, a_3 = \frac{\rho c_e c_1^2}{K \omega^*}, a_4 = \frac{a T_0 \beta_1 c_1^2}{\beta_2 K \omega^*}, a_5 = \frac{T_0 \beta_1^2}{\rho K \omega^*}, a_6 = \frac{a \rho c_1^2}{\beta_1 \beta_2}, a_7 = \frac{\rho c_1^4}{\beta_2^2 D \omega^*}, a_8 = \frac{b \rho c_1^2}{\beta_2^2},$$

$$\tau_t = \left(1 + \tau_1 \frac{\partial}{\partial t}\right), \tau_t^1 = \left(1 + \tau^1 \frac{\partial}{\partial t}\right), \tau_t^0 = \left(\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2}\right), \tau_{\eta_0}^0 = \left(\frac{\partial}{\partial t} + \eta_0 \tau_0 \frac{\partial^2}{\partial t^2}\right), \tau_\gamma^0 = \left(\frac{\partial}{\partial t} + \gamma \frac{\partial^2}{\partial t^2}\right),$$

$$\tau_t^{10} = \left(\frac{\partial}{\partial t} + \eta_0 \tau_0 \frac{\partial^2}{\partial t^2}\right), e = \nabla^2 \phi, \frac{\partial u_1}{\partial x_3} - \frac{\partial u_3}{\partial x_1} = \nabla^2 \psi.$$

We define Laplace and Fourier transform as:

$$\bar{f}(x_1, x_3, s) = \int_0^\infty f(x_1, x_3, t) e^{-st} dt$$

$$\hat{f}(\xi, x_3, s) = \int_{-\infty}^\infty \bar{f}(x_1, x_3, s) e^{i\xi x_1} dx_1 \quad (24)$$

Applying the Laplace and Fourier transform defined by (24) on (20)-(23), after simplification, we obtain

$$\{AD^{10} + BD^8 + CD^6 + ED^4 + FD^2 + G\} \{\hat{\phi}, \hat{\psi}, \hat{T}, \hat{C}\} = 0 \quad (25)$$

where

$$A = a_1 (\delta_{12} - \delta_3),$$

$$B = -a_1 \delta_{12} (\xi^2 + \delta_1) - \left( (\delta_{12} - \delta_3) (2a_9 \xi^2 + 1) + a_1 (\delta_{11} + \delta_{12} (2\xi^2 + a_9) - \delta_2 \delta_{10} - 2\xi^2 \delta_3) \right),$$

$$C = (\xi^2 + \delta_1) (\delta_{12} (2a_9 \xi^2 + 1) + a_1 (\delta_{11} + \delta_{12} (2\xi^2 + a_9) - \delta_2 \delta_{10})) + (2a_9 \xi^2 + 1) (\delta_{11} - \delta_2 \delta_{10} + \delta_{12} (2\xi^2 + a_9))$$

$$+ a_1 (\xi^2 (\delta_{11} + \delta_9 \delta_{12} + \xi^2 \delta_{12}) + \delta_9 \delta_{11}) + \delta_{12} (\delta_5 + \xi^2 + \delta_6 \xi^4) + a_1 (\delta_8 \delta_{11} - \delta_2 \delta_{10} \xi^2)$$

$$+ (\delta_8 \delta_{12} - \delta_{10}) ((2a_9 \xi^2 + 1) + 2a_1 \xi^2) + \delta_3 ((\delta_2 \delta_8 - \xi^2 - \delta_9) (2a_9 \xi^2 + 1) - (\delta_5 + \xi^2 + \delta_6 \xi^4) - a_1 \xi^4)$$

$$- 2\xi^2 \delta_3 ((2a_9 \xi^2 + 1) - a_1 (\delta_2 \delta_8 + \xi^2 + \delta_9)),$$

$$E = -(\xi^2 + \delta_1) \left\{ \begin{aligned} & \left( \delta_{11} + \delta_{12} (2\xi^2 + a_9) \right) (2a_9 \xi^2 + 1) + \delta_{12} (\delta_5 + \xi^2 + \delta_6 \xi^4) + a_1 (\xi^2 (\delta_{11} - \delta_2 \delta_{10} + \delta_{12} (\xi^2 + \delta_9)) + \delta_9 \delta_{11}) \\ & - \delta_2 \delta_{10} \end{aligned} \right\}$$

$$+ (2a_9 \xi^2 + 1) (\delta_{12} \xi^4 + (\delta_2 (1 - \delta_{10}) + \delta_9 \delta_{12}) \xi^2 + \delta_9 \delta_{11}) - (\delta_5 + \xi^2 + \delta_6 \xi^4) (\delta_{11} + \delta_{12} (2\xi^2 + a_9) - \delta_2 \delta_{10})$$

$$+ \delta_4 \delta_7 \delta_{12} + (2a_9 \xi^2 + 1) (2\xi^2 (\delta_{10} - \delta_8 \delta_{12}) - \delta_8 \delta_{11}) + (\delta_5 + \xi^2 + \delta_6 \xi^4) ((\delta_{10} - \delta_8 \delta_{12}) - (\delta_2 \delta_8 - \xi^2 - \delta_9) + 2\xi^2)$$

$$+ a_1 \xi^2 (\xi^2 (\delta_{10} - \delta_8 \delta_{12}) - \delta_8 \delta_{11}) + \xi^4 ((2a_9 \xi^2 + 1) - a_1 (\delta_2 \delta_8 + \xi^2 + \delta_9)) - 2\xi^2 (2a_9 \xi^2 + 1) (\delta_2 \delta_8 - \xi^2 - \delta_9),$$

$$F = -(\xi^2 + \delta_1) \left\{ \begin{aligned} & (-\delta_{12} \xi^4 + (\delta_2 \delta_{10} - \delta_9 \delta_{12} - \delta_{11}) \xi^2 - \delta_9 \delta_{11}) (2a_9 \xi^2 + 1) - (\delta_5 + \xi^2 + \delta_6 \xi^4) (\delta_{11} + \delta_{12} (2\xi^2 + a_9) - \delta_2 \delta_{10}) \\ & + (\delta_5 + \xi^2 + \delta_6 \xi^4) (\delta_{12} \xi^4 + (\delta_{11} + \delta_9 \delta_{12} - \delta_2 \delta_{10}) \xi^2 + \delta_9 \delta_{11}) - \delta_4 \delta_7 (\delta_{11} - \delta_2 \delta_{10} + \delta_{12} (\xi^2 + \delta_9)) \\ & + (2a_9 \xi^2 + 1) (\delta_8 \delta_{11} \xi^2 + (\delta_8 \delta_{12} - \delta_{10}) \xi^4) + (\delta_5 + \xi^2 + \delta_6 \xi^4) (\delta_8 \delta_{11} + 2\xi^2 (\delta_8 \delta_{12} - \delta_{10})) \\ & + \xi^4 ((\delta_2 \delta_8 - \xi^2 - \delta_9) (2a_9 \xi^2 + 1) - (\delta_5 + \xi^2 + \delta_6 \xi^4)) + 2\xi^2 (\delta_2 \delta_8 - \xi^2 - \delta_9) (\delta_5 + \xi^2 + \delta_6 \xi^4), \end{aligned} \right\}$$

$$G = (\delta_{12}\xi^4 + (\delta_{11} + \delta_9\delta_{12} - \delta_2\delta_{10})\xi^2 + \delta_9\delta_{11})\left(\delta_4\delta_7 - (\xi^2 + \delta_1)(\delta_5 + \xi^2 + \delta_6\xi^4)\right) + (\delta_5 + \xi^2 + \delta_6\xi^4)\left(-\delta_8\delta_{11}\xi^2 + \xi^4\left((\delta_{10} - \delta_8\delta_{12}) - (\delta_2\delta_8 - \xi^2 - \delta_9)\right)\right),$$

and

$$a_9 = \frac{a_2}{a_1}, \delta_1 = s^2 + \left(\frac{M}{1+m^2}\right)s - \Omega^2, \delta_2 = (1 + \tau_1s), \delta_3 = (1 + \tau^1s), \delta_4 = s\left(\frac{M}{1+m^2} + 2\Omega\right),$$

$$\delta_5 = \frac{1}{a_1}\left(s^2 + \left(\frac{M}{1+m^2}\right)s - \Omega^2\right), \delta_6 = \frac{1}{a_9}, \delta_7 = -\frac{s}{a_1}\left(\frac{M}{1+m^2} + 2\Omega\right), \delta_8 = a_5(s + \tau_0\eta_0s^2),$$

$$\delta_9 = a_3(s + \tau_0s^2), \delta_{10} = a_4(s + \gamma s^2), \delta_{11} = a_7(s + \tau^0\eta_0s^2), \delta_{12} = a_8\delta_3.$$

The solution of the Eq. (25) satisfying the radiation conditions that  $\hat{\phi}$ ,  $\hat{\psi}$ ,  $\hat{T}$  and  $\hat{C}$  tend to zero as  $x_3$  tends to infinity can be written as:

$$(\hat{\phi}, \hat{\psi}, \hat{T}, \hat{C})(x_3, \xi, s) = \sum_{i=1}^5 (1, R_i, S_i, P_i) A_i e^{-m_i x_3}, \tag{26}$$

where

$$R_i = \sum_{i=1}^5 \frac{\delta_7}{\left[ (2a_9\xi^2 + 1)m_i^2 - a_1m_i^4 - (\delta_5 + \delta_6\xi^4 + \xi^2) \right]},$$

$$S_i = \sum_{i=1}^5 \frac{\left[ \delta_8(m_i^2 - \xi^2)(\delta_{11} - \delta_{12}(m_i^2 - \xi^2)^2) + \delta_{10}(m_i^2 - \xi^2)^2 \right]}{\left[ (m_i^2 - \xi^2 - \delta_9)(\delta_{11} - \delta_{12}(m_i^2 - \xi^2)) - \delta_2\delta_{10}(m_i^2 - \xi^2) \right]},$$

$$P_i = \sum_{i=1}^5 \frac{\left[ -(m_i^2 - \xi^2)^2 (\delta_2\delta_8 + (m_i^2 - \xi^2 - \delta_9)) \right]}{\left[ (m_i^2 - \xi^2 - \delta_9)(\delta_{11} - \delta_{12}(m_i^2 - \xi^2)) - \delta_2\delta_{10}(m_i^2 - \xi^2) \right]}, \quad i = 1, 2, 3, 4, 5.$$

#### 4 BOUNDARY CONDITIONS

Mechanical boundary conditions, we consider the traction free plane boundary  $x_3 = 0$ , so

$$t_{33} = t_{31} = m_{32} = 0. \tag{27}$$

Mass concentration boundary condition, we consider the boundary plane  $x_3 = 0$  is iso-concentrated surface, so

$$C = 0. \tag{28}$$

Thermal boundary condition: we suppose that the boundary plane  $x_3 = 0$  is subjected to ramp-type heating, which depends on the coordinate  $x_1$  and the time  $t$  of the form

$$T(x_1, t) = G(t)\delta(x), \quad (29)$$

where

$$G(t) = \begin{cases} 0 & t \leq 0 \\ T_1 \frac{t}{t_0} & 0 < t \leq t_0 \\ T_1 & t > t_0 \end{cases} \quad (30)$$

where  $T_1$  is constant and  $t_0$  is the ramp-type parameter.

Applying Laplace and Fourier transforms defined by (24) on (29) and with the aid of (30), we obtain

$$\hat{T}(\xi, s) = T_1 \frac{(1 - e^{-st_0})}{t_0 s^2} = F_1(\xi, s). \quad (31)$$

where

$$t_{33} = \frac{\lambda}{\beta_1 T_0} \left[ \left( \frac{\partial u_1}{\partial x_1} \right) + \left( 1 + \frac{2\mu}{\lambda} \right) \left( \frac{\partial u_3}{\partial x_3} \right) - \frac{\rho c_1^2}{\lambda} \left\{ \left( 1 + \tau_1 \frac{\partial}{\partial t} \right) T + \left( 1 + \tau^1 \frac{\partial}{\partial t} \right) C \right\} \right], \quad (32)$$

$$t_{31} = \frac{\mu}{\beta_1 T_0} \left( \frac{\partial u_1}{\partial x_3} + \frac{\partial u_3}{\partial x_1} \right) - \frac{\alpha \omega^{*2}}{4c_1^2 \beta_1 T_0} \left[ \left( \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_3^2} \right) \left( \frac{\partial u_1}{\partial x_3} - \frac{\partial u_3}{\partial x_1} \right) \right], \quad (33)$$

$$m_{32} = \frac{\alpha \omega^{*2}}{2c_1^2 \beta_1 T_0} \left( \frac{\partial^2 u_1}{\partial x_3^2} - \frac{\partial^2 u_3}{\partial x_1 \partial x_3} \right). \quad (34)$$

Making use of (26) in the boundary conditions (27)-(29) and with the aid of (19), (24) and (31)-(34), we obtain the expressions for components of displacement, stresses, temperature change and mass concentration as:

$$\hat{u}_1 = \frac{F_1(\xi, s)}{\Delta} \left[ K_1 \Delta_1 e^{-m_1 x_3} - K_2 \Delta_2 e^{-m_2 x_3} + K_3 \Delta_3 e^{-m_3 x_3} - K_4 \Delta_4 e^{-m_4 x_3} + K_5 \Delta_5 e^{-m_5 x_3} \right], \quad (35)$$

$$\hat{u}_3 = \frac{-F_1(\xi, s)}{\Delta} \left[ T_1 \Delta_1 e^{-m_1 x_3} - T_2 \Delta_2 e^{-m_2 x_3} + T_3 \Delta_3 e^{-m_3 x_3} - T_4 \Delta_4 e^{-m_4 x_3} + T_5 \Delta_5 e^{-m_5 x_3} \right], \quad (36)$$

$$\hat{t}_{33} = b_1 \frac{F_1(\xi, s)}{\Delta} \left[ L_1 \Delta_1 e^{-m_1 x_3} - L_2 \Delta_2 e^{-m_2 x_3} + L_3 \Delta_3 e^{-m_3 x_3} - L_4 \Delta_4 e^{-m_4 x_3} + L_5 \Delta_5 e^{-m_5 x_3} \right], \quad (37)$$

$$\hat{t}_{31} = \frac{F_1(\xi, s)}{\Delta} \left[ M_1 \Delta_1 e^{-m_1 x_3} - M_2 \Delta_2 e^{-m_2 x_3} + M_3 \Delta_3 e^{-m_3 x_3} - M_4 \Delta_4 e^{-m_4 x_3} + M_5 \Delta_5 e^{-m_5 x_3} \right], \quad (38)$$

$$\hat{m}_{32} = b_2 \frac{F_1(\xi, s)}{\Delta} \left[ N_1 \Delta_1 e^{-m_1 x_3} - N_2 \Delta_2 e^{-m_2 x_3} + N_3 \Delta_3 e^{-m_3 x_3} - N_4 \Delta_4 e^{-m_4 x_3} + N_5 \Delta_5 e^{-m_5 x_3} \right], \quad (39)$$

$$\hat{T} = \frac{F_1(\xi, s)}{\Delta} \left[ S_1 \Delta_1 e^{-m_1 x_3} - S_2 \Delta_2 e^{-m_2 x_3} + S_3 \Delta_3 e^{-m_3 x_3} - S_4 \Delta_4 e^{-m_4 x_3} + S_5 \Delta_5 e^{-m_5 x_3} \right], \quad (40)$$



$$\hat{C} = \frac{F_1(\xi, s)}{\Delta} \left[ P_1 \Delta_1 e^{-m_1 x_3} - P_2 \Delta_2 e^{-m_2 x_3} + P_3 \Delta_3 e^{-m_3 x_3} - P_4 \Delta_4 e^{-m_4 x_3} + P_5 \Delta_5 e^{-m_5 x_3} \right], \tag{41}$$

where

$$\begin{aligned} \Delta = & g_1(L_1 h_2 - L_2 h_1 + L_3 h_3) + g_2(L_1 h_4 - L_2 h_5 + L_4 h_3) + g_3(L_1 h_6 - L_2 h_{12} + L_5 h_3) + g_4(L_1 h_8 + L_3 h_9 - L_4 h_{10}) \\ & + g_5(L_1 h_{11} + L_3 h_{12} + L_5 h_{10}) + g_6(L_1 h_{14} - L_5 h_9 - L_4 h_{12}) + g_7(L_2 h_{14} - L_4 h_6 - L_5 h_4) + g_8(L_2 h_8 + L_3 h_4 + L_4 h_2) \\ & + g_9(L_2 h_{11} + L_3 h_6 - L_5 h_2) + g_{10}(L_3 h_{14} + L_4 h_{11} + L_5 h_8), \end{aligned}$$

$\Delta_i$  ( $i = 1, \dots, 5$ ) are obtain by replacing 1<sup>st</sup>, 2<sup>nd</sup>, 3<sup>rd</sup>, 4<sup>th</sup> and 5<sup>th</sup> column by  $[0, 0, 0, F_1(\xi, s), 0]^T$  in  $\Delta_i$ .  
and

$$\begin{aligned} K_i = & (-i\xi + m_i R_i), T_i = (m_i + i\xi R_i), L_i = \left( (m_i^2 - \xi^2) + \frac{2\mu}{\lambda} m_i (m_i + i\xi R_i) - \frac{\rho c_1^2}{\lambda} (\delta_2 S_i + \delta_3 P_i) \right), \\ M_i = & V_1 \left( 2i\xi m_i - (m_i^2 - \xi^2) R_i \right) - V_2 \left( (m_i^2 - \xi^2) \left( (m_i i\xi - R_i m_i^2) + i\xi (m_i + i\xi R_i) \right) \right), \\ N_i = & m_i (m_i^2 - \xi^2) R_i, b_1 = \frac{\lambda}{\beta_1 T_0}, V_1 = \frac{\mu}{\beta_1 T_0}, V_2 = \frac{\alpha \omega^{*2}}{4c_1^2 \beta_1 T_0}, g_1 = (S_4 P_5 - P_4 S_5), g_2 = (S_3 P_5 - P_3 S_5), \\ g_3 = & (S_3 P_4 - P_3 S_4), g_4 = (S_3 P_4 - P_3 S_4), g_5 = (S_2 P_4 - P_2 S_4), g_6 = (S_2 P_3 - P_2 S_3), g_7 = (S_3 P_1 - P_3 S_1), \\ g_8 = & (S_5 P_1 - P_5 S_1), g_9 = (S_4 P_1 - P_4 S_1), g_{10} = (S_1 P_2 - P_1 S_2), h_1 = (M_1 N_3 - M_3 N_1), h_2 = (M_2 N_3 - M_3 N_2), \\ h_3 = & (M_1 N_2 - M_2 N_1), h_4 = (M_4 N_2 - M_2 N_4), h_5 = (M_4 N_1 - M_1 N_4), h_6 = (M_2 N_5 - M_5 N_2), \\ h_7 = & (M_1 N_5 - M_5 N_1), h_8 = (M_3 N_4 - M_4 N_3), h_9 = (M_4 N_1 - M_1 N_4), h_{10} = (M_3 N_1 - M_1 N_3), \\ h_{11} = & (M_5 N_3 - M_3 N_5), h_{12} = (M_4 N_5 - M_5 N_4), \end{aligned} \tag{41}$$

### 5 PARTICULAR CASES

If  $m = 0$ , in Eqs. (35)-(41), we obtain the components of displacement and stresses in a modified couple stress thermoelastic with mass diffusion with rotating medium without Hall current effect.

If the effect of rotation is absent ( $\Omega = 0$ ), in Eqs. (35)-(41), we obtain the components of displacement and stresses in a modified couple stress thermoelastic with mass diffusion with the following changed values of  $\delta_1, \delta_4, \delta_5$  and  $\delta_7$  as:

$$\delta_1 = \left( \frac{M}{1+m^2} s + s^2 \right), \delta_4 = s \left( \frac{Mm}{(1+m^2)} \right), \delta_5 = -\frac{1}{a_1} \left( \frac{M}{1+m^2} s + s^2 \right), \delta_7 = -\frac{s}{a_1} \left( \frac{Mm}{(1+m^2)} \right).$$

If  $\tau_1 = \tau^1 = 0, \eta_0 = 1, \gamma = \tau_0$  in Eqs. (35)-(41), we obtain the corresponding results for modified couple stress thermoelastic with mass diffusion under the influence of Hall current and rotation for Lord Shulman (L-S) model.

If  $\eta_0 = 0, \gamma = \tau^0$  in Eqs. (35)-(41), we obtain the corresponding results for modified couple stress thermoelastic with mass diffusion under the influence of Hall current and rotation for Green Lindsay (G-L) model.

## 6 INVERSION OF THE TRANSFORMATION

To obtain the solution of the problem in physical domain, we must invert the transforms in (35)-(41). Here the displacement components, normal and tangential stresses, couple stress, temperature change and mass concentration are functions of  $x_3$ , the parameters of Laplace and Fourier transforms  $s$  and  $\xi$  respectively and hence are of the form  $f(\xi, x_3, s)$ . To obtain the function  $f(x_1, x_3, t)$  in the physical domain, we first invert the Fourier transform using

$$\bar{f}(x_1, x_3, s) = \int_{-\infty}^{\infty} e^{-i\xi x_1} \bar{f}(\xi, x_3, s) d\xi = \int_{-\infty}^{\infty} (\cos(\xi x) f_e - i \sin(\xi x) f_0) d\xi \quad (42)$$

where  $f_e$  and  $f_0$  are respectively the odd and even points of  $\hat{f}(x, \xi, s)$ . Thus the expression (42) gives the Laplace transform  $\bar{f}(x_1, x_3, s)$  of the function  $f(x_1, x_3, t)$ . Following [31], the Laplace transform function  $\bar{f}(x_1, x_3, s)$  can be inverted to  $f(x_1, x_3, t)$ .

The last step is to calculate the integral in Eq. (42). The method for evaluating this integral is described by [32]. It involves the use of Romberg's integration with adaptive step size. This also uses the results from successive refinements of the extended trapezoidal rule followed by extrapolation of the results to the limit when the step size tends to zero.

## 7 NUMERICAL RESULTS AND DISCUSSION

For numerical computations, following [24], we take the copper material (thermoelastic diffusion solid) as:

$$\begin{aligned} \lambda &= 7.76 \times 10^{10} \text{ Kg m}^{-1} \text{ s}^{-2}, \mu = 3.86 \times 10^{10} \text{ Kg m}^{-1} \text{ s}^{-2}, T_0 = 0.293 \times 10^3 \text{ K}, c_e = 0.3831 \times 10^3 \text{ JK g}^{-1} \text{ K}^{-1}, \\ \alpha_t &= 1.78 \times 10^{-5} \text{ K}^{-1}, \alpha_c = 1.98 \times 10^{-4} \text{ m}^3 \text{ Kg}^{-1}, a = 1.02 \times 10^4 \text{ m}^2 \text{ s}^{-2} \text{ K}^{-1}, b = 9 \times 10^5 \text{ Kg}^{-1} \text{ m}^5 \text{ s}^{-2}, \\ D &= 0.85 \times 10^{-8} \text{ Kg s m}^{-3}, \rho = 8.954 \times 10^3 \text{ Kg m}^{-3}, K = 0.386 \times 10^3 \text{ W m}^{-1} \text{ K}^{-1}, \alpha = .05 \text{ Kg m s}^{-2}, \\ t &= 0.5 \text{ s}, \tau_0 = .01 \text{ s}, t_0 = 0.02 \text{ s}, \tau_1 = 0.07 \text{ s}, \tau^1 = 0.08 \text{ s}. \end{aligned}$$

The Hall current parameters are given by [29]

$$\sigma_0 = 9.36 \times 10^5 \text{ Col}^2 \text{ sec} / \text{Kg m}^3, H_0 = 10^5 \text{ Col} / \text{msec}, B_0 = 0.5 \text{ Kg Col}^{-1} \text{ sec}^{-1}$$

The software Matlab 7.10.4 has been used to determine the normal stress, tangential stress, couple stress, temperature change and mass concentration for different values of Hall current parameters and rotation for both L-S and G-L theories are computed numerically and shown graphically in Figs. 1-10 respectively.

In Figs. 1-5, solid line (—), solid line with centre symbol (—\*—) and solid line with centre symbol (—○—) corresponds to L-S theory for  $m = 0, 0.25, 0.75$  and keeping  $\Omega = 0.5$ , respectively. Similarly, small dash line (----), small dash line with centre symbol (----\*----) and small dash line with centre symbol (----○----) corresponds to G-L theory for  $m = 0, 0.25, 0.75$  and keeping  $\Omega = 0.5$  respectively.

From Figs. 6-10, solid line (—), solid line with centre symbol (—\*—) and solid line with centre symbol (—○—) corresponds to L-S theory for  $\Omega = 0, 0.25, 0.75$  and keeping  $m = 0.5$  respectively. Similarly, small dash line (----), small dash line with centre symbol (----\*----) and small dash line with centre symbol (----○----) corresponds to G-L theory for  $\Omega = 0, 0.25, 0.75$  and keeping  $m = 0.5$  respectively.

7.1 Effect of Hall parameter

Fig. 1 shows the variations of normal stress  $t_{33}$  with distance  $x$  for both L-S and G-L theories for different values of Hall parameter. The values of  $t_{33}$  increase monotonically in the range  $0 \leq x \leq 1.3$  and then decrease as  $x$  increase further for both the theories of thermoelasticity.

Fig. 2 represents the variations of tangential stress with different values of Hall parameter  $m = 0, 0.25, 0.75$ . It is noticed that the values of  $t_{31}$  first oscillate in the range  $0 \leq x < 1.1$ , increase rapidly in the range  $1.1 \leq x < 1.3$ , decrease in the range  $1.3 \leq x \leq 1.8$  and then oscillates for the remaining values of  $x$ . It is evident that the values of tangential stress for  $m = 0$  is higher than in comparison to  $m = 0.25, 0.75$  for both the theories.

Fig. 3 depicts that the variations of couple stress  $m_{32}$  with distance  $x$  for Hall parameter  $m = 0, 0.25, 0.75$ . Its values initially oscillate in the range  $0 \leq x \leq 1.0$ , increase sharply in the range  $1.0 \leq x < 1.3$ , decrease rapidly in the range  $1.3 \leq x \leq 1.8$  and then increase further for remaining values of  $x$ . The values of couple stress component remain oscillatory for all values of  $x$  for both L-S and G-L theories.

Fig. 4 shows that the variations of temperature change  $T$  with distance  $x$  for both L-S and G-L theories. Its values initially oscillates in the range  $0 \leq x < 0.9$ , increase monotonically in the range  $0.9 \leq x \leq 1.3$  and then decrease sharply for the remaining values of  $x$ . The values of temperature change with Hall parameter ( $m = 0$ ) for L-S theory is higher than that of G-L theory in the whole range, whereas the values of temperature change with hall parameter ( $m = 0.25, 0.75$ ) for L-S theory is greater than that of G-L theory in the range  $0 \leq x \leq 0.8$  and less in the remaining values of  $x$ .

Fig. 5 represents the variations of massconcentration  $C$  with different values of Hall parameter  $m = 0, 0.25, 0.75$ . The values of  $C$  oscillate in the whole range for both L-S and G-L theories.

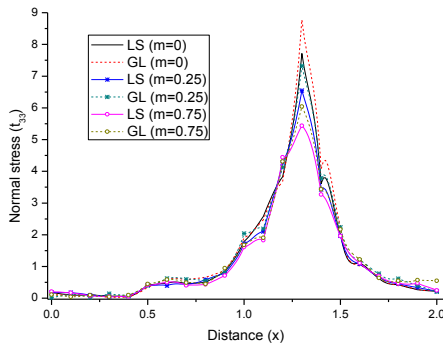


Fig.1 Variation of normal stress with Hall parameter  $m$ .

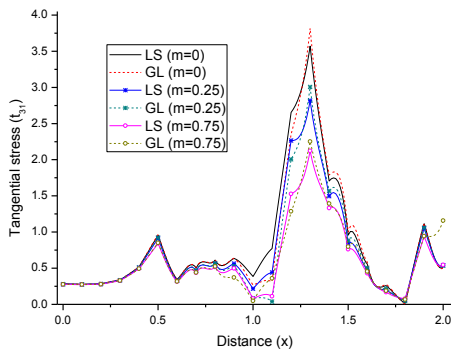
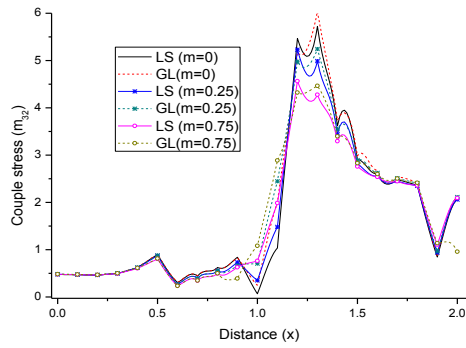
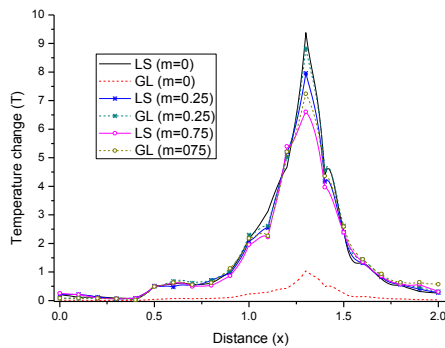


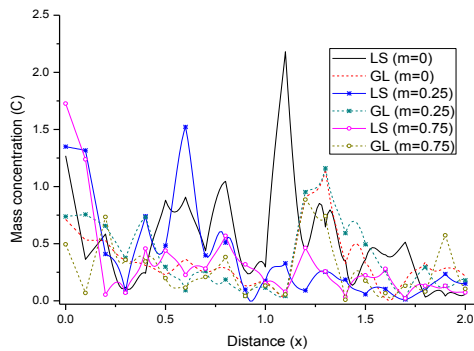
Fig.2 Variation of tangential stress with Hall parameter  $m$ .



**Fig.3**  
Variation of couple stress with Hall parameter  $m$ .



**Fig.4**  
Variation of temperature change with Hall parameter  $m$ .



**Fig.5**  
Variation of mass concentration with Hall parameter  $m$ .

## 7.2 Effect of rotation

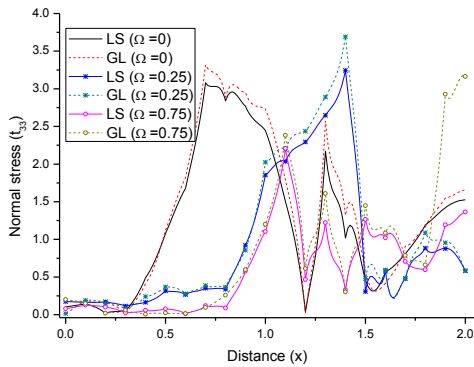
Fig. 6 depicts the variations of normal stress  $t_{33}$  with distance  $x$  for rotation ( $\Omega = 0, 0.25, 0.75$ ). The values of  $t_{33}$  for rotation ( $\Omega = 0, 0.75$ ) increase and decrease alternately with distance  $x$  for both L-S and G-L theories, whereas the value of  $t_{33}$  for rotation ( $\Omega = 0.25$ ) for L-S theory is more in the range  $0 \leq x < 0.5$  and less in the range  $0.5 \leq x \leq 2$  in comparison to G-L theory.

Fig. 7 shows that the variations of tangential stress  $t_{31}$  with distance  $x$  for rotation ( $\Omega = 0, 0.25, 0.75$ ). The values of  $t_{31}$  for  $\Omega = 0$  for G-L theory is higher in comparison to L-S theory for  $\Omega = 0$ , whereas for  $\Omega = 0.25, 0.75$ , its values increase and decrease alternately as the values of  $x$  increase further for both L-S and G-L theories.

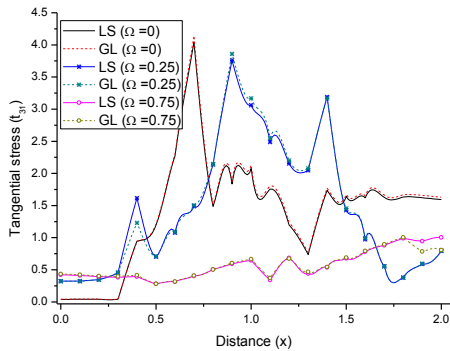
Fig. 8 represents the variations of couple stress  $m_{32}$  with different values of rotation  $\Omega = 0, 0.25, 0.75$ . For  $\Omega = 0$ , the values of couple stress initially oscillate in the range  $0 \leq x < 1$  and then increase for remaining values of  $x$  and for  $\Omega = 0.25$ , its values increase monotonically in the range  $0 \leq x < 1$ , decrease in the range  $0 \leq x < 1.4$  and then oscillate as  $x$  increase further, whereas for  $\Omega = 0.75$ , the values of couple stress firstly decrease in the range  $0 \leq x < 0.7$ , increase in the range  $0.7 \leq x < 1.2$  and then oscillate for remaining values of  $x$ . The values of couple stress for G-L theories are greater than in comparison to L-S theories for  $\Omega = 0, 0.25$  and  $0.75$ .

Fig. 9 shows that the variations of temperature change  $T$  with distance  $x$  for both L-S and G-L theories. The values of  $T$  for rotation  $\Omega = 0$ , its values initially increase in the range  $0 \leq x < 0.8$ , decrease in the range  $0.8 \leq x < 1.2$ , oscillate in the range  $1.2 \leq x < 1.5$  and then increase as  $x$  increase further for both the theories. For  $\Omega = 0.25$ , its values initially oscillate in the range  $0 \leq x < 0.9$ , increase in the range  $0.9 \leq x < 1.5$  and oscillates in the range  $1.5 \leq x \leq 2.0$ , whereas the values of  $T$  for  $\Omega = 0.75$ , oscillates in the range  $0 \leq x < 0.8$ , increase in the range  $0.8 \leq x < 1.2$  and then again oscillatory behaviour is noticed for remaining values of  $x$  for both L-S and G-L theories.

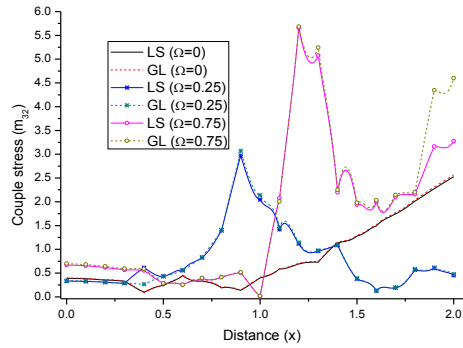
Fig. 10 depicts the variations of mass concentration  $C$  with distance  $x$  for both L-S and G-L theories for different values of rotation. The behavior of variations of mass concentration for L-S ( $\Omega = 0$ ) is more in the range  $0 \leq x < 0.5$ , less in the range  $0.5 \leq x \leq 2.0$  in comparison for G-L ( $\Omega = 0$ ), whereas for LS ( $\Omega = 0.25$ ), its values more in the range  $0 \leq x < 0.2$  and increase and decrease alternately with remaining values of  $x$  for G-L ( $\Omega = 0.25$ ). The values of mass concentration for ( $\Omega = 0.75$ ), increase and decrease alternately in the range  $0 \leq x < 1.0$ , and for remaining values of  $x$ , its values for L-S theory is more in comparison to G-L theory.



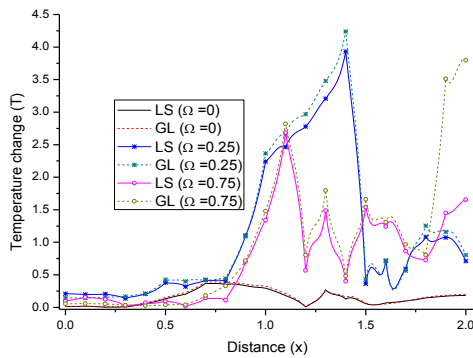
**Fig.6**  
Variation of normal stress with rotation  $\Omega$ .



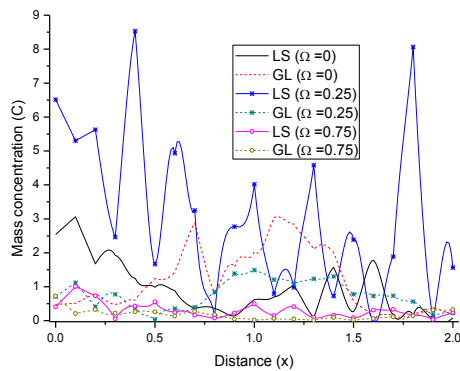
**Fig.7**  
Variation of tangential stress with rotation  $\Omega$ .



**Fig.8**  
Variation of couple stress with rotation  $\Omega$ .



**Fig.9**  
Variation of temperature change with rotation  $\Omega$ .



**Fig.10**  
Variation of mass concentration with rotation  $\Omega$ .

## 8 CONCLUSIONS

Analysis of stresses, temperature change and mass concentration due to ramp-type heating is a significant problem of continuum mechanics. The result obtained from above study are summarized as.

The resulting quantities depicted graphically are observed to be very sensitive towards the Hall and rotation parameters. Figures show that the Hall and rotation parameters have oscillatory effects on the numerical values of the physical quantities obtained after computational process. It is also observed that the physical quantities are also effected by the different non-classical theories of thermoelasticity. It is observed that the values of stress components  $t_{33}, t_{31}$  and  $m_{32}$  for G-L theory are more in comparison to L-S theory due to the effect of Hall parameter and rotation. It is also observed that initially the values of temperature change for L-S theory is more in

comparison to G-L theory as the Hall parameter increases and reverse behavior is noticed due to the effect of rotation. Appreciable effect of Hall parameter and rotation is observed on the mass concentration. The results obtained in the study should be beneficial for people working on modified couple stress thermoelastic solid with mass diffusion. By introducing the Hall parameter and rotation to the assumed model present a more realistic mode for future study.

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