

# Fracture Analysis of Externally Semi-Elliptical Crack in a Spherical Pressure Vessel with Hoop-Wrapped Composite

H. Eskandari<sup>\*</sup>, Gh. Rashed, F. Mirzade

*Abadan Institute of Technology, Petroleum University of Technology, Abadan, Iran*

Received 4 February 2018; accepted 26 March 2018

## ABSTRACT

In this paper the effect of composite hoop-wrapped on stress intensity factor for semi-elliptical external crack which located in spherical pressure vessel, were investigated through the Finite Element Analysis. In order to find the effect of some parameters such as composite thickness and width, internal pressure and crack geometry, comparisons between different cases were done and discussed in detail. The result show that repairing crack with composite hoop-wrapped, can significantly reduce the stress intensity factor along the crack front. © 2018 IAU, Arak Branch. All rights reserved.

**Keywords:** Stress intensity factor; Semi-elliptical crack; Spherical pressure vessel; Composite layer.

## 1 INTRODUCTION

THE term pressure vessel referred to those reservoirs or containers, which are subjected to internal or external pressures. These pressure vessels are used in variety applications in oil and gas industry to store fluids under pressure, chemical industry, thermal and nuclear power plant, in space and ocean depth, and in water, steam, gas and air supply system in industries. Spherical pressure vessels though less common than cylindrical ones, but this type of vessel is preferred for storage of high pressure fluid. Theoretically, a spherical pressure vessel has approximately twice the strength of a cylindrical pressure vessel with the same wall thickness[1] and is the ideal shape to hold internal pressure[2]. Due to presence of corrosion agent, residual tensile stress, and cyclic pressure loading condition these pressure vessels are susceptible to cracking. Also application of pressure and thermal shock can cause a crack in this structure. Another common reason for initiating crack is Stress Corrosion Cracking (SCC) which is the growth of crack formation in a corrosive environment. As a result, unexpected sudden failure of normally ductile metals subjected to a tensile stress, may be occurred. To avoid the catastrophic explosion it should pay more attention to static fracture endurance. This is possible with the calculation of the Stress Intensity Factor (SIF) along the crack front. In the past many researchers have worked on SIF evaluation for different cases. There are many studies which carried out in considering semi-elliptical crack on cylindrical pressure vessels [3-6]. In 1989 the stress intensity factors for complete internal and external crack in spherical shells are determined by Chao et al.[7]. The 3D solution for a single internal semi-elliptical crack in spherical pressure vessel done by Hakimi et al.[8]. In the study which carried out in 2010 Perl et al [9] determined the 3D stress intensity factors for arrays of inner radial lunular or crescentic cracks in a typical spherical pressure vessel via finite element method employing singular elements along the crack front. In 2012, three-dimensional stress intensity factors for ring cracks and arrays of coplanar cracks emanating from the inner surface of a spherical pressure vessel studied by Perl et al. [10]. Recent research indicates

<sup>\*</sup>Corresponding author. Tel.: +98 9173430348.  
E-mail address: [eskandari@put.ac.ir](mailto:eskandari@put.ac.ir) (H.Eskandari).

that the use of composite patch is an efficient way for repairing cracks in the fractured structures by increasing the fatigue life of the damaged component. The repair of cracks using composite patch material has proven significant in decreasing the stress intensity at the tip of cracks consequently the service life of the cracked structural components is extended [11-13]. Some work has been done on fracture properties of steel-lined hoop wrapped cylinders. The effect of composite wrapping on the fracture behavior of steel-lined hoop wrapped cylinder with internal axial crack was investigated by Sue [14]. Shahani and kheirkhah [15] evaluated SIF for internal semi-elliptical circumferential crack in steel-lined hoop wrapped cylinder. In 2013 the stress intensity factor of semi-elliptical surface crack in a cylinder with hoop wrapped composite layer were investigated by Chen [16] .

The aim of this work is to investigate the effect of composite layer on SIF for semi-elliptical external crack which located in spherical pressure vessel. Also the effect of crack geometry, thickness and width of composite layer, fiber orientation and composite material on the values of the SIF's along the crack front are discussed in detail.

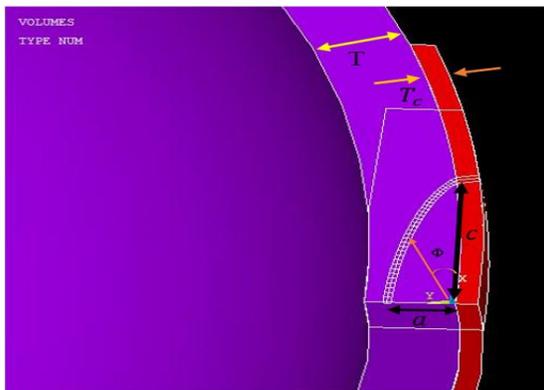
## 2 GEOMETRY AND PROPERTIES

A typical spherical pressure vessel with an internal diameter  $R_i$  and external diameter  $R_o$  with  $R_o / R_i = 1.1$  is considered. This vessel is subjected to the internal pressure  $P_i$  and contains an external semi-elliptical crack. The parameters 'a' and 'c' are respectively illustrated the depth and half length of the crack. A hoop wrapped composite which is wound only around the crack location is shown in Fig.1 .The composite width is depicted by  $W$  and the thickness of composite and metallic linear are shown by  $T_c$  and  $T$  respectively.

The metallic linear material is Cr-Mo steel with Young modulus and Poisson's ratio equal to 200 GPa and 0.28 respectively. Different composite materials can be used to cover the steel liner around the crack. The mechanical properties of different composite materials are listed in Table1.[17]

**Table 1**  
Mechanical properties of composite materials[17].

Material	$E_1$ (GPa)	$E_2$ (GPa)	$\nu_{12}$	$\nu_{23}$	$G_{12}$ (GPa)
Graphite-Epoxy	172.4	10.34	0.3	0.18	4.82
Carbon-Epoxy	132	10.3	0.25	0.25	12
Kevlar-Epoxy	76	5.5	0.34	0.34	2.2
Glass-Epoxy	45	12	0.25	0.25	4.4

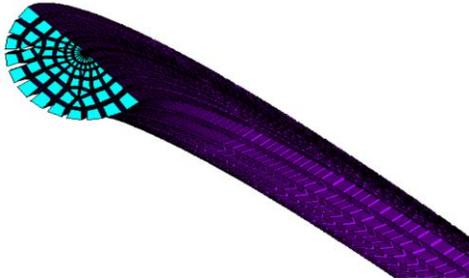


**Fig.1**  
Configuration of semi-elliptical external crack in the spherical pressure vessel.

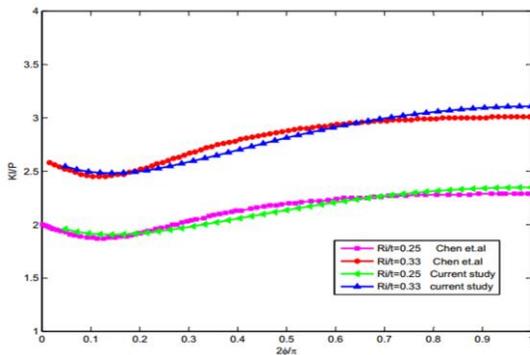
## 3 FINITE ELEMENT MODELING

The finite element model of the steel linear is extracted by employing 20 node-solid186 elements and to discretize the composite hoop wrapped, layered solid 186 elements in the ANSYS 16.0 [18] standard code were used. The square root singularity of stresses and strains is modelled by shifting the mid-point nodes to the quarter-point locations in the region around the crack front. Such elements are called singular elements. (See Fig.2) Due to the

symmetry, only a quarter of the cylinder was analyzed by the finite element method. Around the crack front the local mesh is refined enough to achieve the stress and strain distribution accurately. The whole finite element model has nearly 35,000 elements. To validate the FE-model, a hoop wrapped cylinder with axial semi-elliptical inner crack is employed in order to compare the results with those reported in Ref.[16]. Here E-glass fiber/epoxy composite is chosen as the hoop wrapped layered material. Stress intensity factors are normalized by internal pressure. As can be seen from Fig.3 the results are in a good agreement with those of Ref.[16].



**Fig.2**  
Singular elements around the crack tip.



**Fig.3**  
Verification of the normalized stress intensity factors along the crack front.

#### 4 RESULTS AND DISCUSSION

The non-dimensionalised stress intensity factors are plotted for different crack geometries, composite materials, thickness and width of composite layer and fiber orientation to determine the influence of the each parameters. The stress intensity factor are normalized with respected to  $k_0$  given by:

$$K_0 = \sigma_{\theta\theta} \sqrt{\pi a / Q} \tag{1}$$

Here  $\sigma_{\theta\theta}$ , is the average meridional/hoop stress through the spherical pressure vessel thickness, given by:

$$\sigma_{\theta\theta} = \frac{PR_i^2}{R_o^2 - R_i^2} \tag{2}$$

and  $Q$  refers to the shape factor for an elliptical crack [19]and approximated by:

$$Q = 1 + 1.464(a/c)^{1.65}, \quad a/c < 1 \tag{3}$$

$$Q = 1 + 1.464(c/a)^{1.65}, \quad a/c > 1 \tag{4}$$

Distribution of  $K_I / K_0$  as a function of  $2\Phi / \pi$  are presented for various cases.

4.1 Effect of crack geometry

The distribution of the normalized stress intensity factors  $K_I / K_o$  for various aspect ratio  $a/c = 0.33, 0.66, 1$  and relative crack depth  $a/t = 0.5$  is depicted in Fig.4. The composite material is Graphite-Epoxy with its mechanical property listed in Table1. The ratio of composite thickness to metal thickness  $T_c / T$  and composite width to half-length of crack  $w/c$  are 0.25 and 2, respectively. As seen, for a constant relative crack depth  $a/t$ , the stress intensity factor increased by increasing the aspect ratio  $a/c$  and the composite layer influenced the values of the SIFs. As shown in Fig.4 for slender cracks ( $a/c < 1$ ), the maximum stress intensity factor occurs in the deepest point but for penny shaped cracks ( $a/c = 1$ ) the location of the maximum stress intensity factor shifted from the deepest point to corner points. It means that for  $a/c = 1$  the crack will propagate from corners sooner than other points but by employing composite layer maximum stress intensity factor occurs in deepest point.

Variation of the SIF at the deepest point is plotted with respect to aspect ratio (see Fig.5). As can be observed the rate of reduction is decreased with increasing the aspect ratio. In other words, composite layer can affect the SIF in slender crack (smaller  $a/c$ ) more than others.

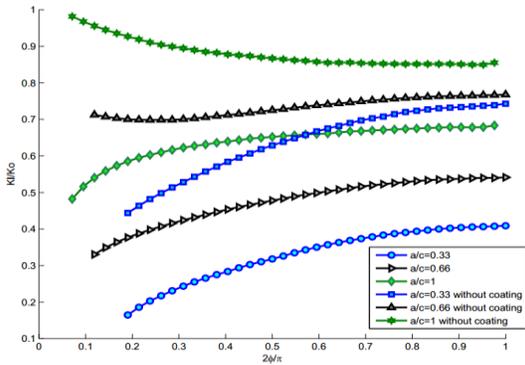


Fig.4 Variation of normalized SIF along the crack front for  $a/t = 0.5$  and different aspect ratio ( $a/c$ ).

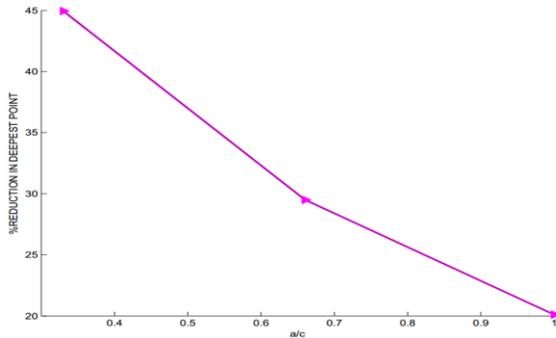
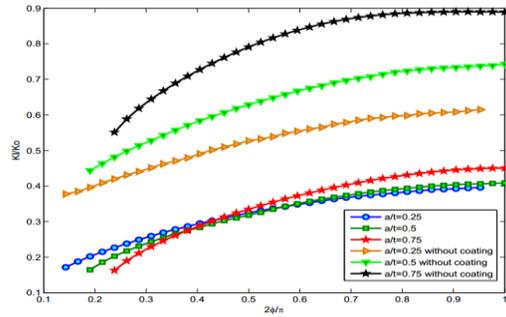
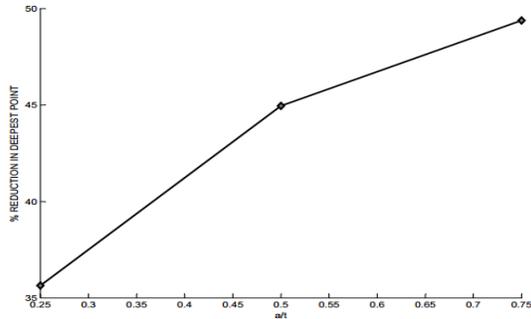


Fig.5 Reduction of SIF (in percent) at the deepest point with respect to aspect ratio ( $a/t = 0.5$ ).

Fig.6 Shows the distribution of the normalized stress intensity factors  $K_I / K_o$  of a semi-elliptical crack in metallic part of the spherical vessel. The aspect ratio  $a/c$  is 0.33. The stress intensity factors for crack depth to wall thickness  $a/t = 0.25, 0.5, 0.75$  are calculated. For constant values of the aspect ratio, the stress intensity factor increased with increasing the relative crack depth. In other words, cracks with larger  $a/t$  are more dangerous than others. The maximum stress intensity factor occurs in the deepest point of the crack front. It means that the deepest point will propagate sooner than corner. Fig.7 the reduction percent of the SIF at the deepest point of the crack front plotted with respect to the relative crack depth  $a/t$ . It can be observed that by increasing the relative crack depth,  $a/t$  the SIF in the deepest point of the crack front increased. In other words, composite layer causes more reduction in SIFs for larger  $a/t$ .



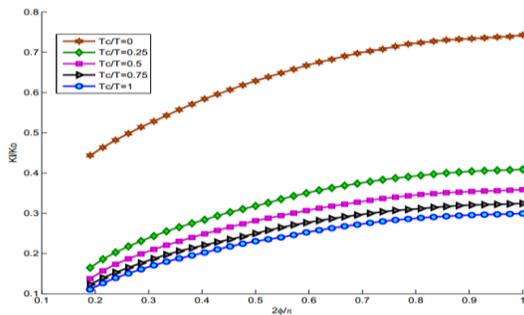
**Fig.6**  
Variation of normalized SIF along the crack front for  $a/c = 0.33$  and different relative crack depth ( $a/t$ ).



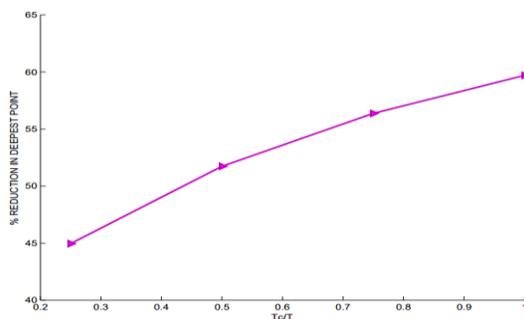
**Fig.7**  
Reduction of SIF (in percent) at the deepest point with respect to the relative crack depth ( $a/c = 0.33$ ).

4.2 Effect of composite thickness

In order to determine the effect of composite thickness, different values of  $T_c/T$  from 0 to 1, are considered. As shown in Fig. 8 by increasing the ratio of composite thickness to metal thickness  $T_c/T$  stress intensity factor along the crack front decreased, also it can be observed that max reduction in SIF occurs at deepest point. Fig. 9 illustrated variation of % reduction of SIF at the deepest point with respect to  $T_c/T$ . It can be seen that there is gradual decreasing along the line slope. In other words for larger  $T_c/T$  rate of reduction of SIF decreased.



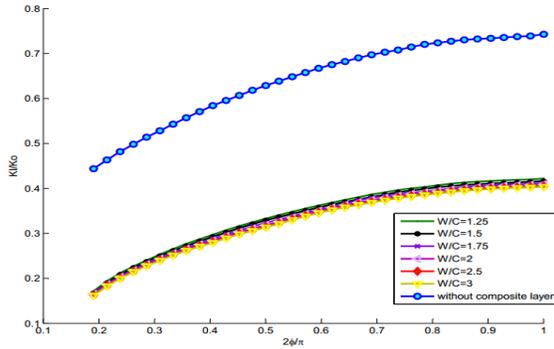
**Fig.8**  
Variation of normalized SIF along the crack front for different composite thickness.



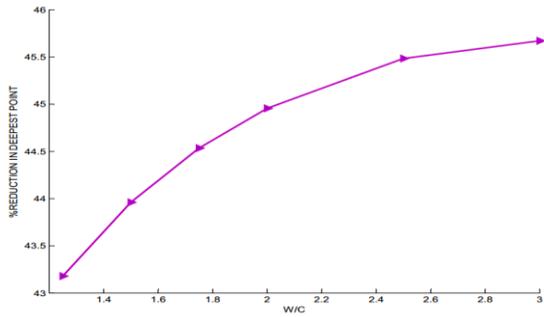
**Fig.9**  
Reduction of SIF (in percent) at the deepest point with respect to  $T_c/T$ .

### 4.3 Effect of composite width

Different values of  $w/c$  from 1.25 to 3 are considered to find the effect of composite width. From Fig.10 it can be seen that increasing the ratio of composite width to half-length of crack  $w/c$  leads to decrease the stress intensity factor along the crack front. The maximum reduction of SIF as a function of  $w/c$  is depicted in Fig.11. The rate of reduction is decreased by increasing the ratio of  $w/c$ . Here a gradual decreasing can be seen through the line slope. It can be say that for larger  $w/c$  reduction percent is decreased, so increasing of composite width would not be affordable.



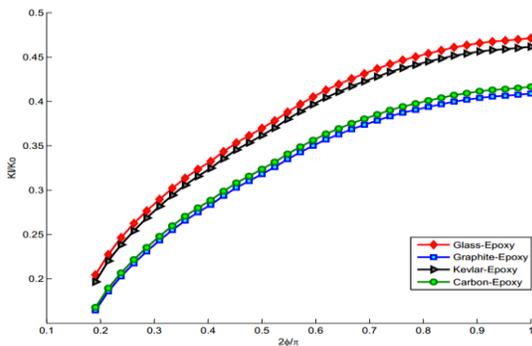
**Fig.10** Variation of normalized SIF along the crack front for different composite width.



**Fig.11** Reduction of SIF (in percent) at the deepest point with respect to  $w/c$ .

### 4.4 Effect of composite material

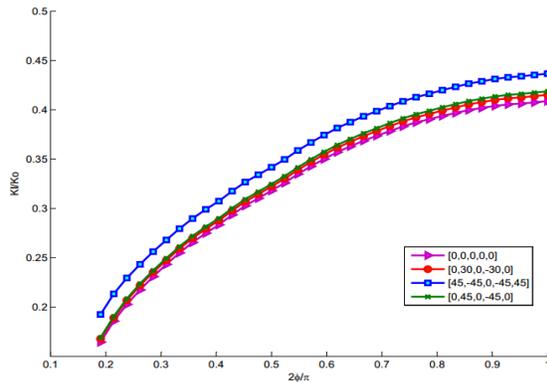
As can be seen in Fig. 12 the Graphite-Epoxy could have more reduction in stress intensity factor compared with four selected composite material. So, it can be concluded that larger the  $E_1$ , higher the strength of composite material against the fracture endurance.



**Fig.12** Variation of normalized SIF along the crack front for different composite material.

#### 4.5 Effect of fiber orientation

In order to study the effect of fiber orientation different layer stacking are considered. As can be seen from the Fig. 13 fiber orientation has no considerable effect on the stress intensity factor through the crack front. The best layer stacking is occurred for  $[0,0,0,0,0]$  which fibers are normal to the crack front. Here the stress intensity factors along the crack front experience the lowest value.



**Fig.13**

Variation of normalized stress intensity factor along the crack front for different fiber orientation.

## 5 CONCLUSIONS

In the present paper the three-dimensional finite element calculations are performed to investigate the effect of hoop-wrapped composite on SIFs along the crack front. The effects of the crack geometry, composite thickness and width, the composite property and fiber orientation on the stress intensity factor along the crack front were determined and the following results were obtained:

- Composite hoop-wrapped, can significantly reduce the stress intensity factor along the crack front. Higher the composite thickness, higher the reduction in SIF.
- By increasing the relative crack depth,  $a/t$  the SIF reduction in the deepest point of the crack front is increased.
- There is a decrease in reduction of SIF at the deepest point by increasing the aspect ratio. It means that the composite layer is more effective for slender crack.
- For aspect ratio  $a/c < 1$ , the maximum stress intensity factor occurs at the deepest point. It means that deepest point is more dangerous than corners.
- Graphite epoxy Composite layer with higher Young modulus has maximum reduction in values of the SIF through the crack front.

## REFERENCES

- [1] Hearn E.J., 1997, *Mechanics of Materials 2*, The mechanics of elastic and plastic deformation of solids and structural materials: Butterworth-Heinemann.
- [2] Nilsen K., 2011, *Development of Low Pressure Filter Testing Vessel and Analysis of Electrospun Nanofiber Membranes for Water Treatment*, Wichita State University.
- [3] Shahani A., Nabavi S., 2006, Closed form stress intensity factors for a semi-elliptical crack in a thick-walled cylinder under thermal stress, *International Journal of Fatigue* **28**(8): 926-933.
- [4] Aydin L., Artem H.S.A, 2008, Axisymmetric crack problem of thick-walled cylinder with loadings on crack surfaces, *Engineering Fracture Mechanics* **75**(6): 1294-1309.
- [5] Miura N., 2008, Comparison of stress intensity factor solutions for cylinders with axial and circumferential cracks, *Nuclear Engineering and Design* **238**(2): 423-434.
- [6] Shahani A., Habibi S., 2007, Stress intensity factors in a hollow cylinder containing a circumferential semi-elliptical crack subjected to combined loading, *International Journal of Fatigue* **29**(1): 128-140.
- [7] Chao Y. J., Chen H., 1989, Stress intensity factors for complete internal and external cracks in spherical shells, *International Journal of Pressure Vessels and Piping* **40**(4): 315-326.

- [8] El Hakimi A., Le Grogne P., Hariri S., 2008, Numerical and analytical study of severity of cracks in cylindrical and spherical shells, *Engineering Fracture Mechanics* **75**(5): 1027-1044.
- [9] Perl M., Bernshtein V., 2010, 3-D stress intensity factors for arrays of inner radial lunular or crescentic cracks in a typical spherical pressure vessel, *Engineering Fracture Mechanics* **77**(3): 535-548.
- [10] Perl M., Bernshtein V., 2012, Three-dimensional stress intensity factors for ring cracks and arrays of coplanar cracks emanating from the inner surface of a spherical pressure vessel, *Engineering Fracture Mechanics* **94**: 71-84.
- [11] Baker A., Jones R., 1988, *Bonded Repair of Aircraft Structures*, Martinus Nijhoff, Dordrecht.
- [12] Benyahia F., Albedah A., Bouiadjra B.B., 2014, Stress intensity factor for repaired circumferential cracks in pipe with bonded composite wrap, *Journal of Pressure Vessel Technology* **136**(4): 041201.
- [13] Gu L., Kasavajhala A.R.M., Zhao S., 2011, Finite element analysis of cracks in aging aircraft structures with bonded composite-patch repairs, *Composites Part B: Engineering* **42**(3): 505-510.
- [14] Su B., Bhuyan G., 1998, Effect of composite wrapping on the fracture behavior of the steel-lined hoop-wrapped cylinders, *International Journal of Pressure Vessels and Piping* **75**(13): 931-937.
- [15] Shahani A., Kheirikhah M., 2007, Stress intensity factor calculation of steel-lined hoop-wrapped cylinders with internal semi-elliptical circumferential crack, *Engineering Fracture Mechanics* **74**(13): 2004-2013.
- [16] Chen J., Pan H., 2013, Stress intensity factor of semi-elliptical surface crack in a cylinder with hoop wrapped composite layer, *International Journal of Pressure Vessels and Piping* **110**: 77-81.
- [17] Committee A.I.H., 1990, *Engineered Materials Handbook: Adhesives and Sealants*, CRC.
- [18] 16.0, A., 2016, FE program package, ANSYS Inc.
- [19] Standard, 2007, A. 579-1/ASME FFS-1 Fitness for Service, API.