

# Generation of Love Wave in a Media with Temperature Dependent Properties Over a Heterogeneous Substratum

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Received 28 May 2019; accepted 2 July 2019

## ABSTRACT

The present paper deals with the generation of Love waves in a layer of finite thickness over an initially stressed heterogeneous semi-infinite media. The rigidity and density of the layer are functions of temperature, i.e. they are temperature dependent. The lower substratum is an initially stressed medium and its rigidity and density vary linearly with the depth. The frequency relation of Love waves has been acquired in compact form. Numerical calculations are accomplished and a number of graphs for non-dimensional phase velocity versus non-dimensional wave number are plotted to display the influence of intrinsic parameters like initial stress and inhomogeneity factors on the generation of Love waves. It is initiated that the non-dimensional phase velocity of Love wave decreases with increase in the non-dimensional wave number and is strongly influenced by the initial stress of the substratum and the inhomogeneity factors of the layer and the substratum. This study may provide effective information in the field of industrial engineering, civil engineering as well as geophysics and seismology.

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**Keywords:** Love waves, Heterogeneity, Temperature-dependent properties, Initial-stress.

## 1 INTRODUCTION

LOVE waves generation within several special physical circumstances which are probable to situate in the interior of the earth is discussed in this article. The appearance of initial stresses in solid substances can have a significant influence on their following repercussion to applied loads that is vary divers from the analogous consequence in the inexistence of initial stresses. The stresses, which subsist in an elastic body even though external forces are not present, are designate as initial stresses and the body is mentioned as initially stressed. Rock mechanics, mechanics of materials and structural elements, mechanics of composites, seismology, geophysics and similar fields are fundamental scientific areas in which it is necessary to study the effect of initial stresses or strains as applied to elastic waves. The fact that the earth is in a state of high initial stress was first predicted by Love [21]. Due to atmospheric pressure, gravity variation, creep, difference in temperature, large initial stresses may exist inside the earth. The high stress developed below the

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earth's surface due to gravity has a strong influence on the generation of elastic waves propagate due to earthquake, explorations and impacts. Thus, it is necessary to study the properties of wave propagation in the presence of initial stress. Biot [5] showed that the elastic wave propagation in the presence of initial stress was different from the stress free case and could not be described with the classical linear theory of elasticity and stress-dependent elastic coefficients. Biot [6] described the theory of incremental deformation in his celebrated book 'Mechanics of Incremental Deformation' and later many researchers applied this theory to study the propagation of surface waves in pre-stressed elastic bodies. Sun and Luo [30] studied propagation of waves in thermal dependent graded materials. Guz [13] discussed three-dimensional linearized theory of elastic waves propagating in initially stressed solids. Du et al. [11] investigated love wave propagation in layered magneto-electro-elastic structures with initial stress. Ahmed and Abo-Dahab [2] studied propagation of Love waves in an orthotropic granular layer under initial stress overlying a semi-infinite granular medium. Abd-Alla et al. [1] discussed the propagation of shear waves in an inhomogeneous anisotropic incompressible and initially stressed medium under the influence of gravity field. Chattaraj et al. [7] studied propagation of a Love waves in an initially stressed fluid-saturated anisotropic porous layer with an irregular boundary sandwiched between two isotropic half-spaces. Kakar et al. [15] investigated propagation of Love waves in crustal layer having temperature dependent inhomogeneity. A number of authors have investigated these models with different approaches, including Alam and Kundu [3], Gupta et al. [12], Kundu et al. [17, 18], Sethi et al. [29], and Sarkar and Lahiri [27]. Pal et al. [25] studied propagation of Love and SH-waves in initially stressed and inhomogeneous medium of different geometry.

The study of the behavior of stresses in elastic materials with temperature-dependent properties is of importance for many engineering applications. Last few decades, maximum of the superintendences in the field of solid mechanics and wave propagation were done without contemplating the reliance of material properties on temperature, which limit the pertinency of the solutions acquired to particular ranges of temperature. At rampant temperature the material properties like the Poisson's ratio, the coefficient of thermal expansion, the modulus of elasticity and the thermal conductivity are no longer constants. Many results of theoretical and experimental studies have revealed that the earth is considerably more complicated than the models presented earlier. In current years, due to the development in several fields in science and technology, the prerequisite of taking into consideration the real behavior of the material characteristics became necessary. Das et al. [9] investigate the interaction of a homogeneous and isotropic perfect conducting half space with rotation, in the context of Lord-Shulman theory. Tomar and Khurana [32] also studied the the elastic field of thermo-chiral medium by extending the governing equations and constitutive relations of hemitropic micropolar material to include temperature field.

The theory of thermoelasticity of materials with temperature-dependent properties is enormously suitable for modeling of the interaction between mechanical and thermal fields. Matysiak et al. [23] investigated SH-waves propagation in an isotropic layer having shear modulus and mass density linearly dependent on temperature. Prasad et al. [26] discussed the influence of inhomogeneity due to temperature on the generation of shear waves in an anisotropic media. Nowinski [24] was one of the first researchers who notably demonstrate the theoretical basis for the exploration of elastic bodies with temperature-dependent modulus. After that many theoretical analyses of wave propagation on elastic solid with temperature-dependent properties has been subjected by many researchers. Hata [14] pointed out thermoelastic problem for a Griffith crack in a plate with temperature-dependent properties. The influence of high temperature and pressure on P and S waves velocities in the crustal and mantle rocks was calibrated by Kern [16]. Matysiak [22] elaborated the effect of material inhomogeneity caused by exponential temperature dependent properties of the elastic bodies on the propagation of wave fronts. Aouadi and El-Karamany [4] delineated plane waves in generalized thermo viscoelastic material with relaxation time and temperature-dependent properties.

In the existing literature, the effect of temperature on the mechanical properties of materials have been proposed under several novelty distributions by many researchers (Czaplewski et al. [8]; Deliktas and Teymur [10], Lee and Saravanos [19]; Tao [31]; Tillmann et al. [33]; Xu et al. [35]). A well amount of experimental results concerning reliance of the mechanical possessions of solids on temperature can be initiated by Schreiber et al. [28]. The problems of repercussion to the temperature disparities by wave velocities in granulate are outlined by Lokajíček et al. [20]. The aforementioned investigations are impel by numerous geotechnical problems during choosing of suitable underground sites for burying nuclear waste. Keeping the above facts in mind an effort is made to explore the generation of Love waves in a layer over an initially stressed inhomogeneous substratum. The material properties of the layer are assumed to be temperature dependent whereas the rigidity and density of the lower substratum vary linearly with the depth. The frequency relation of Love waves has been acquired in compact form. Numerical calculations are accomplished and a number of graphs are plotted to display the influence of inhomogeneity factors and initial stress on the phase velocity of Love waves.

## 2 FORMULATION AND SOLUTION OF THE PROBLEM

In this paper, we consider a temperature dependent layer of finite thickness  $d$  is lying over an initially stressed inhomogeneous substratum. Consider the rectangular coordinate system  $(x, y, z)$  in such a way that  $x$  and  $y$ -axes are taken on the horizontal plane and the  $z$ -axis is pointing vertically downward. Geometry of the problem is displayed in Fig. 1. Origin  $O$  lies at the common interface of the layer and the substratum. Let  $\mu_1$  and  $\rho_1$  be the rigidity and density of the lower substratum, which vary with the depth as follow:

$$\mu_1(z) = \mu_{10}(1+lz), \quad \rho_1(z) = \rho_{10}(1+mz) \quad (1)$$

where  $l$  and  $m$  are constants having dimension that is inverse of length.

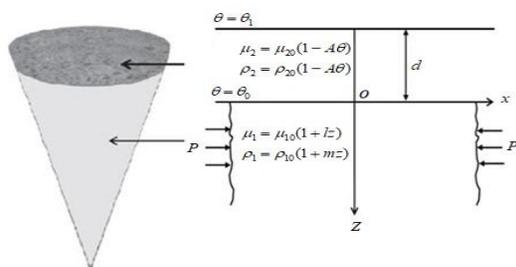
Let  $\mu_2$  and  $\rho_2$  be the variable rigidity and density of the layer. Let  $\theta$  designates as the temperature. The upper free surface and lower boundary plane of the layer are bestowed at constant temperature  $\theta_1$  and  $\theta_0$ . The distribution of temperature in the layer of small thickness is taken into account

$$\theta(z) = \theta_0 + \frac{\theta_0 - \theta_1}{d}z, \quad z \in [0, -d] \quad (2)$$

The variance of the rigidity  $\mu_2$  and the density  $\rho_2$  on the temperature is considered by

$$\mu_2(z) = \mu_{20}(1 - A\theta), \quad \rho_2(z) = \rho_{20}(1 - A\theta) \quad (3)$$

where  $A$  is constant.



**Fig.1**  
Geometry of the problem.

Let  $(u_1, v_1, w_1)$  and  $(u_2, v_2, w_2)$  are the displacement vectors in the substratum and in the layer respectively. In two-dimension  $x - z$  plane and for Love waves, we have

$$u_j = w_j = 0, \quad v_j \equiv v_j(x, z, t) \quad (j = 1, 2) \quad (4)$$

### 2.1 Governing equation and solution for the semi-infinite media

Let  $P$  be the compressive initial stress acting along  $x$ - axis in the semi-infinite media. The equation of motion for the inhomogeneous semi-infinite media under the compressive initial stress  $P$  in the absence of body forces is given by Biot [6]

$$\frac{\partial \sigma_{yx}}{\partial x} + \frac{\partial \sigma_{yz}}{\partial z} - \frac{P}{2} \left( \frac{\partial^2 v_1}{\partial x^2} \right) = \rho_1 \frac{\partial^2 v_1}{\partial t^2} \quad (5)$$

Using the stress-strain relation

$$\sigma_{yx} = 2\mu_1 e_{yx} = \mu_1 \left( \frac{\partial v_1}{\partial x} + \frac{\partial u_1}{\partial y} \right), \sigma_{yz} = 2\mu_1 e_{yz} = \mu_1 \left( \frac{\partial v_1}{\partial z} + \frac{\partial w_1}{\partial y} \right)$$

and Eqs. (1) and (4), the equation of motion (5) takes the form

$$\left[ 1 - \frac{P}{2\mu_{10}(1+lz)} \right] \frac{\partial^2 v_1}{\partial x^2} + \frac{\partial^2 v_1}{\partial z^2} + \frac{l}{1+lz} \frac{\partial v_1}{\partial z} = \frac{\rho_{10}(1+mz)}{\mu_{10}(1+lz)} \frac{\partial^2 v_1}{\partial t^2} \quad (6)$$

A general trial solution to solve Eq. (6) we use the following substitution

$$v_1 = V(z) \exp\{ik(x-ct)\} \quad (7)$$

where  $k$  is the wave number,  $V$  is the unknown displacement amplitude,  $i = \sqrt{-1}$ , and  $c$  is the phase velocity. Using Eq. (7) into Eq. (6), we get

$$\frac{d^2 V}{dz^2} + \frac{l}{1+lz} \frac{dV}{dz} + \left[ \frac{\rho_{10}(1+mz)}{\mu_{10}(1+lz)} c^2 - \left\{ 1 - \frac{P}{2\mu_{10}(1+lz)} \right\} \right] k^2 V = 0 \quad (8)$$

Introducing  $V = \psi(z)/(1+lz)^{1/2}$  into Eq. (8) to eliminate the term  $dV/dz$ , we have

$$\frac{d^2 \psi(z)}{dz^2} + \left[ \frac{l^2}{4(1+lz)^2} - k^2 \left\{ \left( 1 - \frac{P}{2\mu_{10}(1+lz)} \right) - \frac{c^2(1+mz)}{c_1^2(1+lz)} \right\} \right] \psi(z) = 0 \quad (9)$$

where  $c_1^2 = \mu_{10}/\rho_{10}$ .

Substituting  $S = \left[ 1 - \frac{P}{2\mu_{10}(1+lz)} - \frac{c^2 m}{c_1^2 l} \right]^{1/2}$ ,  $\eta = \frac{2Sk(1+lz)}{l}$  and  $\omega = kc$  in Eq. (9), we get

$$\frac{d^2 \psi}{d\eta^2} + \left[ \frac{R}{2\eta} + \frac{1}{4\eta^2} - \frac{1}{4} \right] \psi = 0 \quad (10)$$

where  $R = \frac{\omega^2(l-m)}{c_1^2 l^2 k S}$ .

Eq. (10) is Whittaker's equation, solution of which is given by

$$\psi(\eta) = E_1 W_{R/2,0}(\eta) + E_2 W_{-R/2,0}(\eta) \quad (11)$$

where  $E_1$  and  $E_2$  are arbitrary constants and  $W_{R/2,0}, W_{-R/2,0}$  are the Whittaker's functions.

Keeping in the mind the solution of Eq. (11) satisfies the boundary condition  $V(z) \rightarrow 0$  as  $z \rightarrow \infty$  i.e.,  $\psi(\eta) \rightarrow 0$  as  $\eta \rightarrow \infty$ , the appropriate solution becomes  $\psi(\eta) = E_1 W_{R/2,0}(\eta)$ .

Hence, the displacement component in the lower substratum in view of the above relations is given by

$$v_1(x, z, t) = V(z) \exp\{ik(x-ct)\} = \frac{E_1 W_{R/2,0}(\eta)}{(1+lz)^{1/2}} \exp\{ik(x-ct)\}$$

Expanding the Whittaker's function (Whittaker and Watson [34]) up to linear term, we get

$$v_1(x, z, t) = E_1 \exp\left\{\frac{-Sk(1+lz)}{l}\right\} \left(\frac{2Sk}{l}\right)^{1/2} \left[1 + (1-R)\frac{Sk(1+lz)}{l}\right] \exp\{ik(x-ct)\} \quad (12)$$

## 2.2 Governing equation and solution for the layer

The equation of motion for the layer governing Love waves in the absence of any body force is given by

$$\frac{\partial}{\partial x} \left( \mu_2 \frac{\partial v_2}{\partial x} \right) + \frac{\partial}{\partial z} \left( \mu_2 \frac{\partial v_2}{\partial z} \right) = \rho_2 \frac{\partial^2 v_2}{\partial t^2} \quad (13)$$

Substituting Eqs. (2) and (3) into Eq. (13) we get

$$\frac{\partial^2 v_2}{\partial x^2} + \frac{\partial^2 v_2}{\partial z^2} + \frac{\beta}{\alpha + \beta z} \frac{\partial v_2}{\partial z} = \frac{1}{c_2^2} \frac{\partial^2 v_2}{\partial t^2} \quad (14)$$

where

$$\alpha = 1 - A\theta_0, \beta = A \left( \frac{\theta_1 - \theta_0}{d} \right), c_2^2 = \frac{\mu_{20}}{\rho_{20}} \quad (15)$$

Substituting  $\alpha + \beta z = \xi$  Eq. (14) transform to

$$\frac{1}{\beta^2} \frac{\partial^2 v_2}{\partial x^2} + \frac{\partial^2 v_2}{\partial \xi^2} + \frac{1}{\xi} \frac{\partial v_2}{\partial \xi} = \frac{1}{c_2^2 \beta^2} \frac{\partial^2 v_2}{\partial t^2} \quad (16)$$

For the surface wave changing harmonically, the solution for the displacements of Eq. (16) can be assumed as:

$$v_2(x, \xi, t) = U(\xi) \exp\{ik(x-ct)\} \quad (17)$$

Applying Eq. (17) into Eq. (16), takes the form

$$\frac{d^2 U}{d\xi^2} + \frac{1}{\xi} \frac{dU}{d\xi} - \frac{k^2}{\beta^2} \left(1 - \frac{c^2}{c_2^2}\right) U = 0 \quad (18)$$

The general solution of Eq. (18) may be given as:

$$U = B_1 J_0 \left( \frac{ik\xi}{\beta} \sqrt{1 - \frac{c^2}{c_2^2}} \right) + B_2 Y_0 \left( \frac{-ik\xi}{\beta} \sqrt{1 - \frac{c^2}{c_2^2}} \right)$$

where  $B_1$  and  $B_2$  are constants and  $J_0$  and  $Y_0$  are Bessel functions of first and second kind respectively of order zero. Putting the value of  $U$  in Eq. (17), we have

$$v_2(x, \xi, t) = \left[ B_1 J_0 \left( \frac{ik\xi}{\beta} \sqrt{1 - \frac{c^2}{c_2^2}} \right) + B_2 Y_0 \left( \frac{-ik\xi}{\beta} \sqrt{1 - \frac{c^2}{c_2^2}} \right) \right] \exp\{ik(x-ct)\}$$

Finally the displacement component in the layer can be found as:

$$v_2(x, z, t) = \left[ B_1 J_0 \left( \frac{ik(\alpha + \beta z)}{\beta} \sqrt{1 - \frac{c^2}{c_2^2}} \right) + B_2 Y_0 \left( \frac{-ik(\alpha + \beta z)}{\beta} \sqrt{1 - \frac{c^2}{c_2^2}} \right) \right] \exp\{ik(x - ct)\} \quad (19)$$

### 3 BOUNDARY CONDITIONS AND FREQUENCY EQUATION

The boundary conditions to be requiring in the present problem are as follows:

- i. The upper surface of the layer is stress free, i.e.,  $\mu_2 \frac{\partial v_2}{\partial z} = 0$  at  $z = -d$
- ii. Displacements are continuous at the interface, i.e.,  $v_1 = v_2$  at  $z = 0$
- iii. Stresses are continuous at the interface, i.e.,  $\mu_1 \frac{\partial v_1}{\partial z} = \mu_2 \frac{\partial v_2}{\partial z}$  at  $z = 0$

Using Eqs. (12) and (19) in the above boundary conditions, the subsequent three equations are given as:

$$B_1 J_1(ikd(\gamma - 1)\Upsilon) - B_2 Y_1(-ikd(\gamma - 1)\Upsilon) = 0 \quad (20)$$

$$B_1 J_0(ikd\gamma\Upsilon) + B_2 Y_0(-ikd\gamma\Upsilon) - E_1 \left( 2S_0 \frac{k}{l} \right)^{1/2} \left[ 1 + (1 - R_0) S_0 \frac{k}{l} \right] \exp(-S_0 \frac{k}{l}) = 0 \quad (21)$$

$$B_1 i\mu_{20} (1 - A\theta_0) \Upsilon J_1(ikd\gamma\Upsilon) - B_2 i\mu_{20} (1 - A\theta_0) \Upsilon Y_1(-ikd\gamma\Upsilon) + E_1 \mu_{40} \left( 2 \frac{k}{l} \right)^{1/2} \exp(-S_0 \frac{k}{l}) \Delta = 0 \quad (22)$$

where

$$\Delta = \left( \frac{1}{2} \zeta S_0^{-1} + S_0 \right) \left[ \left( \frac{k}{l} R_0 S_0 - 1 \right) S_0^{1/2} - \frac{k}{l} S_0^{3/2} \right] + (1 - R_0) S_0^{3/2} + \frac{3}{4} \zeta S_0^{-1/2} + \frac{1}{4} \zeta \left( \frac{l}{k} - R_0 S_0 \right) S_0^{-3/2}$$

$$\zeta = \frac{P}{2\mu_{10}}, \quad \gamma = \frac{\alpha}{\beta d}, \quad \Upsilon = \sqrt{1 - \frac{c^2}{c_2^2}}, \quad R_0 = \frac{\omega^2 (l - m)}{c_1^2 l^2 k S_0} \quad \text{and} \quad S_0 = \left[ 1 - \frac{P}{2\mu_{10}} - \frac{c^2 m}{c_1^2 l} \right]^{1/2}$$

Eliminating  $B_1$ ,  $B_2$  and  $E_1$  from Eqs. (20)–(22), the frequency relation of Love waves as follows:

$$\frac{J_1(ikd(\gamma - 1)\Upsilon) Y_0(-ikd\gamma\Upsilon) + Y_1(-ikd(\gamma - 1)\Upsilon) J_0(ikd\gamma\Upsilon)}{J_1(ikd(\gamma - 1)\Upsilon) Y_1(-ikd\gamma\Upsilon) - Y_1(-ikd(\gamma - 1)\Upsilon) J_1(ikd\gamma\Upsilon)} = \frac{i\mu_{20} (1 - A\theta_0) S_0^{1/2}}{\mu_{40} \Delta} \left[ 1 + (1 - R_0) S_0 \frac{k}{l} \right] \Upsilon$$

### 4 NUMERICAL COMPUTATION AND DISCUSSION

For the numerical calculation and graphical illustration the effect of inhomogeneity and initial stress of the substratum and inhomogeneity of the layer, for the propagation of a Love-type wave, non-dimensional phase velocity  $c/c_2$  has been plotted against non-dimensional wave number  $kd$  for different values of inhomogeneity parameters and initial stress. In Figs. 2(a)–5(a) compressive initial stress has been taken whereas in Figs. 2(b)–5(b) tensile initial stress has been taken. The value of initial stress parameter  $\zeta (= P/2\mu_{10})$  in Figs. 2(a)–4(a) is taken as  $\zeta = 0.3$ , whereas in Figs.

2(b)–4(b) it is taken as  $\zeta = -0.3$ . Furthermore, following values of affecting parameters are taken unless otherwise stated:

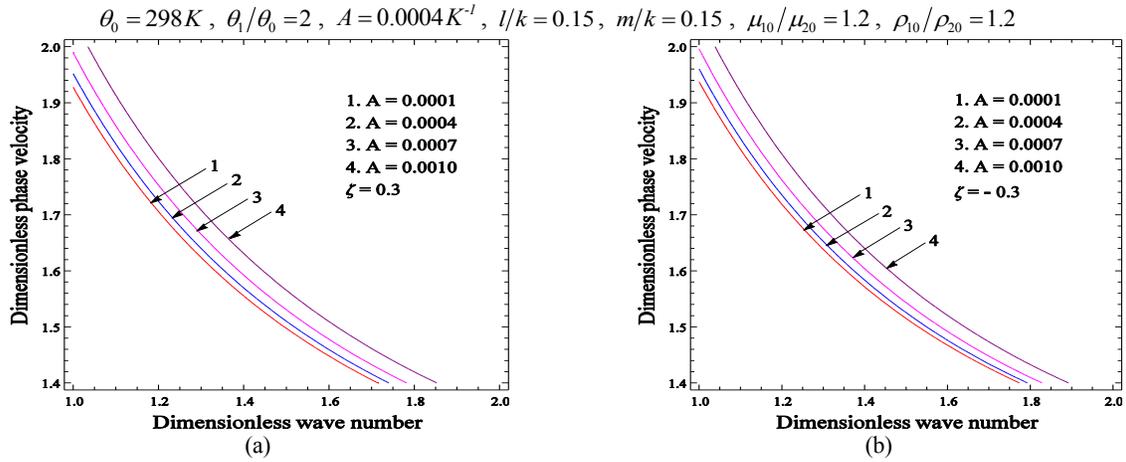


Fig.2

Dimensionless phase velocity  $c/c_2$  versus dimensionless wave number  $kd$  demonstrating the effect of inhomogeneity parameter  $A$  of the layer.

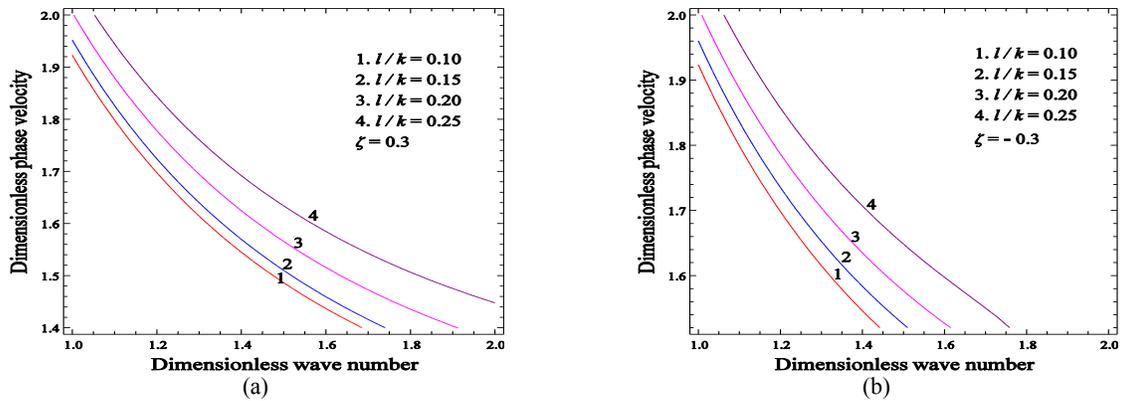


Fig.3

Dimensionless phase velocity  $c/c_2$  versus dimensionless wave number  $kd$  demonstrating the effect of inhomogeneity associated with the rigidity of the lower substratum.

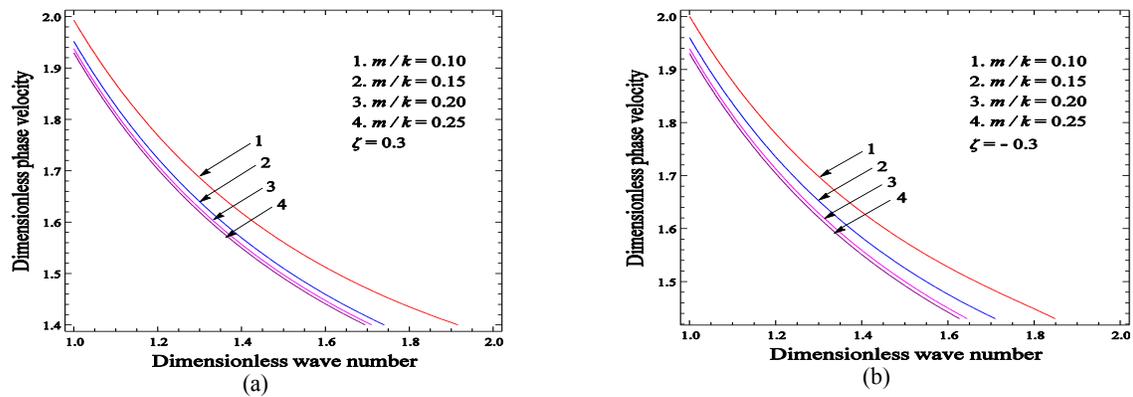


Fig.4

Dimensionless phase velocity  $c/c_2$  versus dimensionless wave number  $kd$  demonstrating the effect of inhomogeneity associated with the density of the lower substratum.

The variations of  $c/c_2$ , the non-dimensional phase velocity with non-dimensional wave number  $kd$  for different values of initial stress and inhomogeneity parameters are shown through Figs. 2-5. All the Figures reflect the dependency that as the non-dimensional wave number increases, the non-dimensional phase velocity of Love wave decreases.

Fig. 2 shows the seesaw of  $c/c_2$  with  $kd$  for distinct values of the inhomogeneity parameter  $A$ . The curves labeled as 1, 2, 3 and 4 corresponds to  $A = 0.0001 K^{-1}$ ,  $0.0004 K^{-1}$ ,  $0.0007 K^{-1}$  and  $0.0010 K^{-1}$  respectively. It is rendered from Fig. 2 that the value of dimensionless phase velocity increases with increase in the value of the parameter  $A$ .

Fig. 3 demonstrates the effect of non-dimensional inhomogeneity parameter  $l/k$  associated with the rigidity of the lower substratum. It is observed from Fig. 3 that the phase velocity increases with increase in the value of the rigidity inhomogeneity parameter of the lower substratum.

Fig. 4 depicts the influence of  $m/k$ , the density inhomogeneity parameter of the lower substratum on the non-dimensional phase velocity. From Fig. 4 we see that the phase velocity decreases with increase in the value of the density inhomogeneity parameter of the lower substratum.

The effect of initial stress parameter  $\zeta (= P/2\mu_0)$  on non-dimensional phase velocity is shown in Fig. 5. From Fig. 5(a) it is observed that as compressive initial stress parameter increases, the phase velocity of Love wave decreases. From Fig. 5(b) we see that as tensile initial stress parameter increases, the phase velocity of Love wave increases.

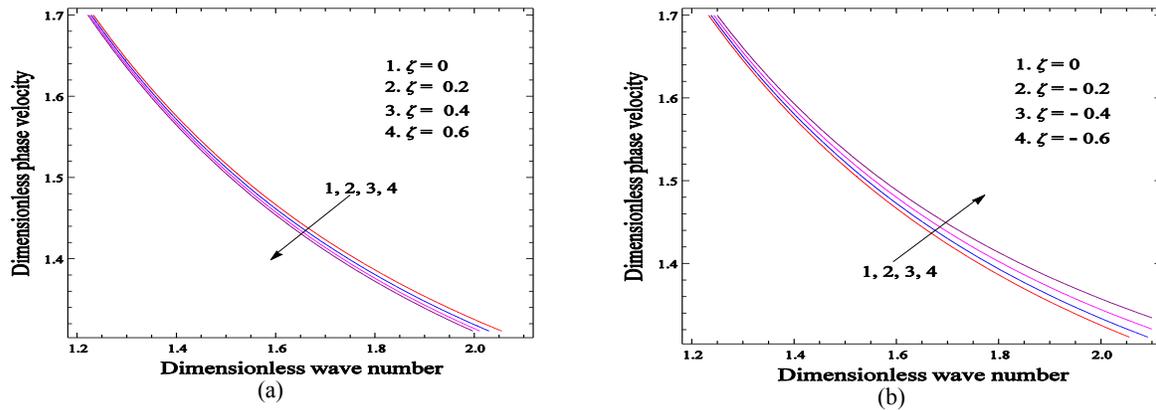


Fig.5

Dimensionless phase velocity  $c/c_2$  versus dimensionless wave number  $kd$  demonstrating the effect of initial-stress of the lower substratum.

## 5 CONCLUSIONS

In the present paper, using an analytical approach, an attempt is made to investigate the propagation of Love waves in an inhomogeneous layer of finite thickness over an initially stressed inhomogeneous substratum. The heterogeneity in the layer is caused due to dependence of material properties on the temperature, and the rigidity and the density of the initially stressed substratum vary linearly with depth. Solutions for displacement in the layer and the substratum are derived and the frequency equation of Love waves has been obtained in closed form. Phase velocity has been demarcated numerically and the effect of inhomogeneity and the initial stress is observed. From the numerical calculations we conclude that

- Phase velocity of Love wave increases with increase in the inhomogeneity parameter of the layer.
- Phase velocity of Love wave increases with increase in the magnitude of the inhomogeneity parameter associated with the rigidity of the substratum, whereas phase velocity decreases with increase in the inhomogeneity parameter associated with the density of the substratum.
- Phase velocity of Love wave decreases with increase in the magnitude of the compressive initial stress and increases with increase in the magnitude of the tensile initial stress of the substratum.
- Also, the results of this analysis show that there is a strong correlation between the velocity dispersion and wave number of Love wave and all the figures show that phase velocity decreases with the increase of non-

dimensional wave numbers, which is the well-known nature of seismic waves, i.e., as depth increases, the velocity of surface waves decreases.

Mathematical modelling and theoretical investigations of elasto-dynamic problems have eminent importance in almost all branches of the natural sciences and modern engineering. The results obtained in the present article may provide workable information for experimental researchers working in the field of geophysics and earthquake engineering and seismologist working in the field of mining tremors and drilling into the crust of the earth.

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