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Free Vibration Analysis of Nonlinear Circular Plates Resting on Winkler and Pasternak Foundations

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ABSTRACT

Dynamic behaviour of nonlinear free vibration of circular plate resting on two-parameter foundation is studied. The governing ordinary differential equation is solved analytically using hybrid Laplace Adomian decomposition method. The analytical solutions obtained are verified with existing results in literature. The analytical solutions are used to determine the influence of elastic foundation, radial and circumferential stress on natural frequency of the plate. In addition, the radial and circumferential stress determined. From the results, it is observed that, increase in elastic foundation parameter increases the natural frequency of the plate. It is recorded that the modal radial and circumferential stress affect the extrema mode of the plate. It is hoped that the present study will contribute to the existing knowledge in the field of vibration analysis of engineering structures.

Keywords: Free vibration; Natural frequency; Fluid; Two-parameters foundations; Laplace Adomian decomposition method.

1 INTRODUCTION

INVESTIGATION into dynamic behaviour of plate resting on elastic foundation is attracting huge attention of the researchers due to its wide application in many areas of engineering. The dynamic behaviour of circular plate on elastic foundation is important to geotechnics and structural engineers so as to guide in the design purposes. On the numerical investigation of plate on elastic foundation, Andrea et al. [1] used finite element method. For vibration analysis of plate coupled with fluid, in 2016, Lamb [2] determined the natural frequency using Rayleigh's method. In another study, Raoa et al. [3] adopted exact method in investigation of circular plate resting on Winkler foundation. Based on the literature review, handling singularity problem and non-trivial solution of circular plate is quiet challenging with semi-analytical method. This is because of the independent variable in the governing equation that always result to infinity in the analysis. Though, numerical method is very effective in handing non-linearity as a result of geometry and singularity issues like this but, the limitation of finding stability and convergence studies which increase the computation time and cost is a huge challenge. Meanwhile, exact method [4]

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also requires good knowledge of mathematics for application, which also suffers the setback of handling nonlinear problem. Therefore, in an attempt to provide approximate solution, Soni et al. [5] analysed nonlinear functional graded plate submerged in fluid using Galerkin method of solution. However, Galerkin method an approximate method of solution is very reliable, simple and easy but having an issue with accuracy and precise result and higher mode natural frequency. It requires extending the assumed polynomial solution to four unknowns, which require in-depth knowledge of mathematics to handle. Kumar and Prashar [6] investigated vibration of circular plate using Rayleigh-Ritz technique. However, Rayleigh-Ritz an approximate method of solution having the limitation of finding deflection function and undetermined coefficients. Furthermore, Homotopy perturbation methods (HPM) [7-9] handles the non-linear problems without any restriction but connected with problem of finding small parameters. Adomian decomposition method (ADM) is proven to be very reliable method of solution for handling non-trivial solutions. It is a closed form solution, with fast convergence and little iterations, its easier to use. The accuracy of the soultion is enhanced with the combination of exact method of solution to handle the linear part of the equation while the rest are handled with ADM.

Previous studies show that, application of Laplace Adomian method to determination of free vibration of nonlinear governing equation of circular plate resting on elastic foundation has not been investigated. Therefore, the present study is on, dynamic investigation of non-linear free vibration of circular plates resting on Winkler and Pasternak foundations using Hybrid Adomian decomposition method. The analytical solutions are used for the parametric study.

2 PROBLEM FORMULATION AND MATHEMATICAL ANALYSIS

Consider a circular plate resting on Winkler and Pasternak foundation in Fig 1. under various boundary conditions simply-supported, free and clamped edge conditions. According to Kirchhoff plate theory the following assumptions are considered in the model of governing equation.

1. Plate thickness is smaller compared to the dimension of the circular plate.
2. Normal stress is assumed negligible in transvers direction of the circular plate.
3. Rotary inertia effect is negligible.
4. Normal to the undeformed middle surface remain straight and normal to the deformed middle surface without length stretching.

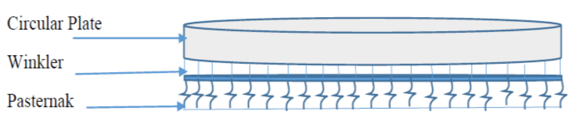


Fig.1
Circular plate resting on Two-Parameter foundation.

The non-dimensionless governing differential equation of the model by [10] is:

$$\frac{d^4f}{dr^4}+\frac{2}{r}\frac{d^3f}{dr^3}-\frac{B}{r^2}\frac{d^2f}{dr^2}+\frac{B}{r^3}\frac{df}{dr}+\frac{A}{r^4}f+k_wf-k_pf^3-g\nabla^2f=\Omega^2f\,, \tag{1}$$

where f is the deflection, r the radius, k_w the linear Winkler model, k_p is the nonlinear Winkler Model, g is the shear of Pasternak foundation and Ω^2 is the natural frequency.

$$A=m^4-4m^2;B=2m^2+1. \tag{2}$$

2.1 The governing equation

Taking care of the singularity issue related to the governing equation. The equation may be transformed into this:

$$g=\frac{1}{r},g^2=\frac{1}{r^2}\Rightarrow\frac{dg}{dr}=-\frac{1}{r^2}, \tag{3}$$

Consequently, we arrived at;

$$\frac{d^4f}{dr^4}+2g\frac{d^3f}{dr^3}-Bg^2\frac{d^2f}{dr^2}+Bg^3\frac{df}{dr}+Ag^4f+k_wf-k_s\frac{d^2f}{dr^2}-k_sg\frac{df}{dr}+k_pf^3=\Omega^2f\,, \tag{4}$$

and,

$$\frac{dg}{dr}+g^2=0, \tag{5}$$

where the initial and boundary conditions are given as:

$$g(r)\big|_{r=0}=g_0, \tag{6}$$

2.2 Boundary conditions

According to classical plate theory, the three boundary conditions considered may be written in dimensionless function $f(r)$ as follows:

Clamped edge

$$f(r)\big|_{r=1}\Rightarrow\left.\frac{df}{dr}\right|_{r=1}=0, \tag{7}$$

Simply Supported

$$f(r)\big|_{r=1}\Rightarrow M_r\big|_{r=1}=-D\left[\frac{d^2f}{dr^2}+v\left(\frac{1}{r}\frac{df}{dr}+\frac{m^2}{r^2}f\right)\right]=0, \tag{8}$$

Free edge Support

$$M_r\big|_{r=1}=-D\left[\frac{d^2f}{dr^2}+v\left(\frac{1}{r}\frac{df}{dr}+\frac{m^2}{r^2}f\right)\right]=0;\quad \mathrm{V}_r\big|_{r=1}=\left[\frac{d^3f}{dr^3}+\frac{1}{r}\frac{d^2f}{dr^2}+\left(\frac{m^2v-2m^2-1}{r^2}\right)\frac{df}{dr}+\left(\frac{3m^2-m^2v}{r^3}\right)f\right]=0 \tag{9}$$

where M_r is radial bending moment and V_r is the radial shear force per unit length. D is the flexural rigidity and v is the Poisson's ratio. The regularity conditions at the centre are given as:

Considering a circular plate without hole or cut hole, the condition at the center of the plate $r=0$ is given as:

Symmetric case

$$\left.\frac{df}{dr}\right|_{r=0}=0,\quad \mathrm{V}_r\big|_{r=0}=\frac{d^3f}{dr^3}=0,\quad for\left(m=0,2,4\cdots\right), \tag{10}$$

Axisymmetric case

$$f(r)\big|_{r=0}=0,\quad M_r\big|_{r=0}=\left.\frac{d^2f}{dr^2}\right|_{r=0}=0\quad for\left(m=1,3,5\cdots\right). \tag{11}$$

3 METHOD OF SOLUTION: LAPLACE TRANSFORM AND ADOMIAN DECOMPOSITION METHOD

3.1 Description of Adomian decomposition method

The Adomian decomposition method (ADM) is developed by George Adomian in 1990s. It is a semi analytical method of solving partial and ordinary nonlinear equation. It uses 'Adomian polynomials' for fast convergence of nonlinear aspect of differential equation. The principle of operation is:

Considering the following equation:

$$Lf + Nf + Rf = g(\text{r}) \tag{12}$$

where L is a linear Operator, N is the nonlinear operator, R is the remaining linear operator and g is the inhomogeneous term. If L is a fourth order operator, it is define by:

$$L = \frac{d^4}{dr^4}, \tag{13}$$

Assuming L is invertible, and then inverse operator L^{-1} is given as;

$$L^{-1}(.) = \int_0^r\int_0^r\int_0^r\int_0^r (.)drdrdrdr, \tag{14}$$

Therefore,

$$L^{-1}Lf = f(r) - f(0) - rf'(0) - \frac{1}{2!}r^2 f''(0) - \frac{1}{3!}r^3 f'''(0), \tag{15}$$

Applying L^{-1} to both sides Eq. (12) gives:

$$f = \Phi_0 - L^{-1}Rf - L^{-1}Nf + L^{-1}g(\text{r}), \tag{16}$$

where

$$\Phi_0 = \begin{cases} f(0), & if \;\; L = \dfrac{d}{dr} \\ f(0) + rf'(0), & if \;\; L = \dfrac{d^2}{dr^2} \\ f(0) + rf'(0) + \dfrac{r^2}{2!}f''(0), & if \;\; L = \dfrac{d^3}{dr^3} \\ f(0) + rf'(0) + \dfrac{r^2}{2!}f''(0) + \dfrac{r^3}{3!}f'''(0), & if \;\; L = \dfrac{d^4}{dr^4} \end{cases} \tag{17}$$

The decomposition principle comprises of decomposing the solution to sum of infinite number of terms defined by the series:

$$f = \sum_{n=0}^{\infty} f_n, \tag{18}$$

The nonlinear term is written as:

$$N(f) = \sum_{n=0}^{\infty} A_n, \tag{19}$$

where A_n 's are the Adomian polynomials.

$$A_n = \frac{1}{n!}\left[\frac{\partial^n}{\partial \xi^n}\left(\left(\sum_{i=0}^{\infty}\xi^i f_i\left(r\right)\right)\right)\right]_{\xi=0} \qquad n = 0,1,2,3\ldots; \tag{20}$$

where ξ is a grouping parameter.

$$\begin{aligned} A_0 &= N(f_0), \\ A_1 &= N(f_0)f_1, \\ A_2 &= N'(f_0)f_2 + \frac{1}{2!}N''(f_0)f_1^{\,2}, \end{aligned} \tag{21}$$

Substituting Eq. (18) and (19) into (16) gives:

$$\sum_{n=0}^{\infty} f_n = \Phi_0 - L^{-1}R\left(\sum_{n=0}^{\infty} f_n\right) - L^{-1}\left(\sum_{n=0}^{\infty} A_n\right) + L^{-1}g(\mathrm{r}), \tag{22}$$

The iterative schemes are:

$$f = \Phi_0 + L^{-1}g(\mathrm{r}), \tag{23}$$

$$f_{n+1} = -L^{-1}Rf_n - L^{-1}A_n, \quad \text{where} \quad n \geq 0, \tag{24}$$

This results to:

$$\begin{aligned} f_0 &= \Phi_0 + L^{-1}g(\mathrm{r}), \\ f_1 &= -L^{-1}Rf_0 - L^{-1}A_0, \quad \text{where} \quad n \geq 0, \\ f_2 &= -L^{-1}Rf_1 - L^{-1}A_1, \end{aligned} \tag{25}$$

3.2 Basic ideal of laplace transform

If f (t) is a function of a variable *t*. $\mathcal{L}\left\{F(t)\right\}$ and is defined by the integral:

$$\mathcal{L}\left\{F(t)\right\} = f(s) = \int_0^{\infty} e^{-st}F(t)dt, \tag{26}$$

Some of the properties used in this study includes:

$$\mathcal{L}\left\{1\right\} = \frac{1}{s}(s > 0), \tag{27}$$

$$\mathcal{L}\left\{t^n\right\} = \frac{n!}{s^{n+1}}(s > 0), \tag{28}$$

$$\mathcal{L}\left\{F^{(n)}(t)\right\} = s^n f(s) - s^{n-1}F(0) - s^{n-2}F'(0) - \cdots F^{(n-1)}(0), \tag{29}$$

where $F^{(n)}(t)$ represents the $n-th$ derivative of $F(t)$ and $\mathcal{L}\{F(t)\}=f(s)$. If Laplace transform of $F(t)$ is $f(s)$, then the inverse Laplace transform of $f(s)$ is expressed by $F(t)=\mathcal{L}^{-1}\{f(s)\}$, where $\mathcal{L}^{-1}$ is called inverse Laplace operator.

The inverse Laplace of Eqs. (27) and (28) are:

$$1=\mathcal{L}^{-1}\left(\frac{1}{s}\right), \tag{30}$$

$$t^{n}=\mathcal{L}^{-1}\left(\frac{n!}{s^{n+1}}\right), \tag{31}$$

3.3 Basic principle of Laplace transform and Adomian decomposition method

LT-ADM is a reliable method of finding the analytical solutions of Linear and nonlinear Ordinary differential equation. Laplace transform is applied on both sides of Eq. (12) to arrive at:

$$\mathcal{L}\{L(f)\}+\mathcal{L}\{[R(f)+N(f)-f(\mathrm{r})]\}=0, \tag{32}$$

Applying the differential property of LT,

$$s^{n}\mathcal{L}\{f\}-s^{n-1}f(0)-s^{n-2}f'(0)-\cdots f^{(n-1)}(0)+\mathcal{L}\{R(f)+N(f)-f(\mathrm{r})\}, \tag{33}$$

Introducing the Laplace boundary condition on the linear part of Eq. (33), then applying inverse Laplace

$$F=\mathcal{L}^{-1}\left\{\left(\frac{1}{s^{n}}\right)\left[s^{n-1}F(0)+s^{n-2}F'(0)+\cdots+F^{(n-1)}(0)\right]+\left(\frac{1}{s^{n}}\right)\mathcal{L}\left[R(f)+N(f)-f(\mathrm{r})\right]\right\} \tag{34}$$

Applying Eq. (18) into (34), we get,

$$\sum_{n=0}^{\infty}f_{n}=\mathcal{L}^{-1}\left\{\left(\frac{1}{s^{n}}\right)\left\{s^{n-1}F(0)+s^{n-2}F'(0)+\cdots+F^{(n-1)}(0)\right\}-\left(\frac{1}{s^{n}}\right)\mathcal{L}\left\{R(f)+\left(\sum_{n=0}^{\infty}A_{n}\right)+f(\mathrm{r})\right\}, \tag{35}$$

$$f_{0}=\mathcal{L}^{-1}\left\{\left(\frac{1}{s^{n}}\right)\left\{s^{n-1}F(0)+s^{n-2}F'(0)+\cdots+F^{(n-1)}(0)\right\}+\left(\frac{1}{s^{n}}\right)\mathcal{L}(f(\mathrm{r}))\right\}, \tag{36}$$

$$f_{i+1}=\mathcal{L}^{-1}\left\{\left(\frac{1}{s^{n}}\right)\mathcal{L}\left\{R(f_{i})+\left(\sum_{n=0}^{\infty}A_{n}\right)_{i}\right\}\right\}, \tag{37}$$

$$f=\lim_{p\to 0}F\Rightarrow F_{0}+F_{1}+F_{2}+\cdots, \tag{38}$$

3.4 Application of LT-ADM to the governing equation

The hybrid method is used in obtaining the analytical solution for the governing equation. As previously mentioned the edge conditions considered are simply supported, free and clamped edge respectively. For brevity sake, only simply supported is reported here while same approach is used in obtaining solutions for clamped and free.

Laplace transformation of coupled Eqs. (4) and (5)

$$\mathcal{L}\left[\bar{F}(r)\right]=\mathcal{L}\left(\frac{d^4f}{dr^4}+2g\frac{d^3f}{dr^3}-Bg^2\frac{d^2f}{dr^2}+Bg^3\frac{df}{dr}+Ag^4f+k_wf-k_s\frac{d^2f}{dr^2}-k_sg\frac{df}{dr}+k_pf^3-\Omega^2f\right)=0, \tag{39}$$

$$\mathcal{L}\left[\bar{g}(r)\right]=\mathcal{L}\left(\frac{dg}{dr}+g^2\right)=0, \tag{40}$$

Laplace the regularity condition at the center of the circular plate Eq. (10) symmetric case.

$$\bar{W}'(0)=\bar{W}'''(0)\Rightarrow 0,\qquad \mathrm{g}(0)=\frac{\mathrm{g}_0}{s}, \tag{41}$$

$$\mathcal{L}\left[\bar{F}(r)\right]=\mathcal{L}\left[\frac{d^4f}{dr^4}\right]+\mathcal{L}\left[2g\frac{d^3f}{dr^3}-Bg^2\frac{d^2f}{dr^2}+Bg^3\frac{df}{dr}+Ag^4f+k_wf-k_s\frac{d^2f}{dr^2}-k_sg\frac{df}{dr}+k_pf^3-\Omega^2f\right]=0, \tag{42}$$

$$\mathcal{L}\left[\bar{g}(r)\right]=\mathcal{L}\left[\frac{dg}{dr}\right]+\mathcal{L}\left[g^2\right]=0, \tag{43}$$

$$\begin{aligned}\mathcal{L}[F(r)]&=s^4\mathcal{L}\left[f_n(r)\right]-f_n'''(0)-sf_n''(0)-s^2f_n'(0)-s^3f_n(0)+\\ &\mathcal{L}\left[2g\frac{d^3f}{dr^3}-Bg^2\frac{d^2f}{dr^2}+Bg^3\frac{df}{dr}+Ag^4f+k_wf-k_s\frac{d^2f}{dr^2}-k_sg\frac{df}{dr}+k_pf^3-\Omega^2f\right]=0,\end{aligned} \tag{44}$$

$$\mathcal{L}[g(r)]=s\mathcal{L}\left[g_n(r)\right]-g_n(0)+\mathcal{L}\left[g^2\right]=0, \tag{45}$$

Substitute the Laplace boundary condition Eq. (41) into Eq. (44) and Eq. (45)

$$\begin{aligned}\mathcal{L}[F(r)]&=s^4\mathcal{L}\left[f_n(r)\right]-sf_n''(0)-s^3f_n(0)+\\ &\mathcal{L}\left[2g\frac{d^3f}{dr^3}-Bg^2\frac{d^2f}{dr^2}+Bg^3\frac{df}{dr}+Ag^4f+k_wf-k_s\frac{d^2f}{dr^2}-k_sg\frac{df}{dr}+k_pf^3-\Omega^2f\right]=0,\end{aligned} \tag{46}$$

$$\mathcal{L}[g(r)]=s\mathcal{L}\left[g_n(r)\right]-\frac{g_0}{s}+\mathcal{L}\left[g^2\right]=0, \tag{47}$$

The unknown are represented as $f(0)\Rightarrow\alpha$ while $f''(0)\Rightarrow\beta$, Eq. (46) becomes:

$$\mathcal{L}[F(r)]=s^4\mathcal{L}\left[f_n(r)\right]-\alpha s^3-s\beta+\mathcal{L}\left[2g\frac{d^3f}{dr^3}-Bg^2\frac{d^2f}{dr^2}+Bg^3\frac{df}{dr}+Ag^4f+k_wf-k_s\frac{d^2f}{dr^2}-k_sg\frac{df}{dr}+k_pf^3-\Omega^2f\right]=0, \tag{48}$$

$$\mathcal{L}[g(r)]=s\mathcal{L}\left[g_n(r)\right]-\frac{g_0}{s}+\mathcal{L}\left[g^2\right]=0, \tag{49}$$

$$\mathcal{L}\left[f_n(r)\right]=\frac{\alpha}{s}+\frac{\beta}{s^3}-\mathcal{L}\frac{1}{s^4}\left[2g\frac{d^3f}{dr^3}-Bg^2\frac{d^2f}{dr^2}+Bg^3\frac{df}{dr}+Ag^4f+k_wf-k_s\frac{d^2f}{dr^2}-k_sg\frac{df}{dr}+k_pf^3-\Omega^2f\right]=0, \tag{50}$$

$$\mathcal{L}\left[g_n(r)\right]=\frac{g_0}{s^2}+\mathcal{L}\frac{1}{s}\left[g^2\right]=0, \tag{51}$$

Applying inverse Laplace on Eqs. (50) and (51), one gets

$$f_n(r)=\alpha+\frac{\beta r^2}{2}-\mathcal{L}^{-1}\left[\frac{1}{s^4}\mathcal{L}\left[2g\frac{d^3f}{dr^3}-Bg^2\frac{d^2f}{dr^2}+Bg^3\frac{df}{dr}+Ag^4f+k_wf-k_s\frac{d^2f}{dr^2}-k_sg\frac{df}{dr}+k_pf^3-\Omega^2f\right]\right]=0, \tag{52}$$

$$g_n(r)=g_0r-\mathcal{L}^{-1}\left[\frac{1}{s}\mathcal{L}\left[g^2\right]\right]=0, \tag{53}$$

Applying ADM on nonlinear term in Eqs. (52) and (53). The Adomian's polynomial, $A's$ are generated using Eq. (20) are;

$$\begin{aligned}
&A_0=2g_0\frac{d^3f_0}{dr^3}-\mathrm{B}g_0^2\frac{d^2f_0}{dr^2}+\mathrm{B}g_0^3\frac{df_0}{dr}+Ag_0^4f_0-k_sg_0\frac{df_0}{dr}+k_pf_0^3;\\
&A_0^{'}=g_0^2;\\
&A_1=2\left(g_0\frac{d^3f_1}{dr^3}+g_1\frac{d^3f_0}{dr^3}\right)-\mathrm{B}\left(g_0^2\frac{d^2f_1}{dr^2}+2g_0g_1\frac{d^2f_0}{dr^2}\right)+\mathrm{B}\left(g_0^3\frac{df_1}{dr}+3g_0^2g_1\frac{df_0}{dr}\right)\\
&+A\left(g_0^4f_1+4g_0^3g_1f_0\right)-k_s\left(g_0\frac{df_1}{dr}+g_1\frac{df_0}{dr}\right)+k_p\left(3f_0^3f_1\right);\\
&A_0^{'}=2g_0\,\mathrm{g}_1;\\
&A_2=2\left(g_0\frac{d^3f_2}{dr^3}+g_1\frac{d^3f_1}{dr^3}+g_2\frac{d^3f_0}{dr^3}\right)-\mathrm{B}\left(g_0^2\frac{d^2f_2}{dr^2}+2g_0g_1\frac{d^2f_1}{dr^2}+2g_0g_2\frac{d^2f_0}{dr^2}+g_1^2\frac{d^3f_0}{dr^3}\right)\\
&+\mathrm{B}\left(g_0^3\frac{df_2}{dr}+3g_0^2g_1\frac{df_1}{dr}+3g_0g_1^2\frac{df_0}{dr}+3g_0^2g_2\frac{df_0}{dr}\right)+A\left(g_0^4f_2+6g_0^2g_1^2f_0+4g_0^3g_2f_0+4g_0^3g_1f_1\right)\\
&-k_s\left(g_0\frac{df_2}{dr}+g_1\frac{df_1}{dr}+g_0\frac{df_0}{dr}\right)+k_p\left(3f_0^2f_2+3f_0f_1^{\,2}\right);\\
&A_2^{'}=2g_0g_2+g_1^2;
\end{aligned} \tag{54}$$

The other polynomials are generated in similar way. According to Eq. (52) and (53) the first term of the series solutions is:

$$f_0=\alpha+\frac{\beta r^2}{2}, \tag{55}$$

$$g_0=g_0r, \tag{56}$$

First order equations are:

$$f_1=\mathcal{L}^{-1}\frac{1}{s^4}\mathcal{L}\left[k_{w}f_0-k_s\frac{d^2f_0}{dr^2}+A_0\right], \tag{57}$$

$$g_1=\mathcal{L}^{-1}\left[\frac{1}{s}\mathcal{L}\left[A_0^{'}\right]\right], \tag{58}$$

and second order equation are;

$$f_2=\mathcal{L}^{-1}\frac{1}{s^4}\mathcal{L}\left[k_{w}f_1-k_s\frac{d^2f_1}{dr^2}+A_1\right], \tag{59}$$

$$g_2 = \mathcal{L}^{-1}\left[\frac{1}{s}\mathcal{L}\left[A_1^{'}\right]\right], \tag{60}$$

Third order equations are:

$$f_3 = \mathcal{L}^{-1}\frac{1}{s^4}\mathcal{L}\left[k_w f_2 - k_s\frac{d^2 f_2}{dr^2} + A_2\right], \tag{61}$$

$$g_3 = \mathcal{L}^{-1}\left[\frac{1}{s}\mathcal{L}\left[A_2^{'}\right]\right], \tag{62}$$

Solving Eqs. (55) - (63) the following expression may be obtained successively,

$$F(r) = f_0 + f_1 + f_2 + f_3 + \cdots \tag{63}$$

where the constant α and β are found using the boundary at $r=1$ condition in Eqs. (7-9) Clamped, Free and simply supported.

Setting the controlling parameter as zero and substitute into Eq. (63) then imposing the boundary conditions Eqs. (7) - (9) on Eq. (63) leads to the following simultaneous expression:

$$\begin{aligned} &\psi_{11}^{(n)}(\Omega)f_0 + \psi_{12}^{(n)}(\Omega)f_2 = 0 \\ &\psi_{21}^{(n)}(\Omega)f_0 + \psi_{22}^{(n)}(\Omega)f_2 = 0 \end{aligned} \tag{64}$$

The polynomials $\psi_{11}, \psi_{12}, \psi_{21}$ and ψ_{22} are represented in terms of the natural frequency Ω, meanwhile $\psi_{11}, \psi_{12}, \psi_{21}$ and ψ_{22} are representing a series expression obtained from Eq. (63). Therefore, Eq. (64) may be written in matrix form as:

$$\begin{bmatrix} \psi_{11}^{(n)}(\Omega) & \psi_{12}^{(n)}(\Omega) \\ \psi_{21}^{(n)}(\Omega) & \psi_{22}^{(n)}(\Omega) \end{bmatrix} \begin{Bmatrix} f_0 \\ f_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix},$$

The following Characteristic determinant is obtained applying the non-trivial condition

$$\begin{bmatrix} \psi_{11}^{(n)}(\Omega) & \psi_{12}^{(n)}(\Omega) \\ \psi_{21}^{(n)}(\Omega) & \psi_{22}^{(n)}(\Omega) \end{bmatrix} = 0, \tag{65}$$

Solving Eq. (65) gives the natural frequencies.

$$\frac{\left|\Omega_j^{(i)} - \Omega_j^{(i-1)}\right|}{\left|\Omega_j^{(i)}\right|} \le \varepsilon, j = 1,2,3,\ldots n \tag{66}$$

where the iteration counter is represented by i, the estimated value of the jth dimensionless natural frequency is $\Omega_j^{(i)}$ and small number chosen is ε. For this study $\varepsilon = 0.0001$. From the results its shows that few iterations the solution has converged.

3.5 The stress-deflection expression

$$\sigma_{rr} = \frac{E(r,z)z}{1-\nu^2}\left(\frac{d^2 f}{dr^2} + \frac{\nu}{r}\frac{df}{dr}\right),$$
$$\sigma_{\theta\theta} = \frac{E(r,z)z}{1-\nu^2}\left(\frac{1}{r}\frac{df}{dr} + \nu\frac{d^2 f}{dr^2}\right), \quad (67)$$

4 RESULTS AND DISCUSSION

The analytical solution of governing equation of motion of the circular plate under various boundary conditions with differential transform method is hereby presented. The material properties for the thin uniform thickness, homogenous circular plate used are: E=207Gpa, material density $\rho = 7850kg/m^3$, and thickness of the plate $h = 0.01m$ respectively. The analytical also compared with already reported results as reported in literature [11] and presented in Table 3 and 4. Good agreement of result is observed along the entire values under different boundary and regularity conditions. Generally, the natural frequency is expreesed in dimensionless form Ω. Since dimensionless analysis is carried out, the results is valid for all thickness of radius. The parametric studies of the controlling factors are presented in both tabular and graphical form. The values of the natural frequency is a dependent upon the value of initial value choosing g_0.

The number of iterations needed to obtain convergence in relation to natural frequency differs. For instance, fundamental mode requires 2 iterations for LH-ADM while the higher mode requires more iterations. This behaviour is attributed to more complex series functions combination. Results shown in Table 1., illustrate that, fundamental natural frequency gives a reasonable prediction of the circular plate but more iterations still required to give other higher mode natural frequencies and also increase the It is interesting to note that, present results with LH-ADM agree very well with past results.

Table 1
Validation of fundamental natural frequency for symmetric condition.

Edge Condition/Dimensionless Natural frequency	Simply Supported		Clamped		Free	
	Leissa [13]	Present	Leissa [13]	Present	Wu et al. [14]	Present
Ω_1	4.977	4.9351	10.2158	10.2158	9.003	9.0032

4.1 Effect of foundation Parameter on natural frequency

This section investigates the variation effect of the elastic foundation on the first two natural frequencies of the thin uniform thickness circular plate under different regularity and boundary conditions discussed earlier. Table 2-5., presents the effects of foundation Parameter on natural frequency. The analysis is performed on the three boundary conditions discussed earlier and two conditions at the center. In this study, Consideration is given to:

a) Elastic Winkler type foundation ($k_p = 0, g_s = 0, k_w = 0,50,100,150,$)
b) Elastic Pasternak type foundation ($k_p = 0, k_w = 0, g_s = 10,50,100,$)
c) Two-parameters elastic foundation ($k_p = 0, k_w = 50, g_s = 10,50,100$)

Although, it a known character of plate to be affected by characteristic of elastic foundation, comparing Tables 2-5., to Table 1., indicates that for both plate and foundation stiffness to be comparable there is a need to properly study the foundation stiffness to be chosen.

As it is expected in all cases, increasing the foundation stiffness results into higher value of natural frequencies. Moreover, it is also observed that, effect of the difference in natural frequencies is more significant for higher mode of the circular plate. Figs. 2 and 3 confirms the directly proportional relationship between stiffness and natural frequency. Increasing stiffness results into increases in natural frequency.

Table 2
Shear Pasternak parameter variation on natural frequency.

Edge Condition	Natural frequency	(k_w =0, m=0)		
	Mode	g_s =10	g_s =50	g_s =100
Simply Supported	Ω_1	9.0731916	17.71499	24.363131
	Ω_2	33.981698	48.72694	63.242599
Clamped Support	Ω_1	13.145639	20.90975	27.613639
	Ω_2	42.409585	56.33333	69.906398
Free Support	Ω_1	11.550058	18.29656	24.158648
	Ω_2	43.233944	59.74169	75.901677

Table 3
Combine Winkler and Pasternak variation effect on natural frequency.

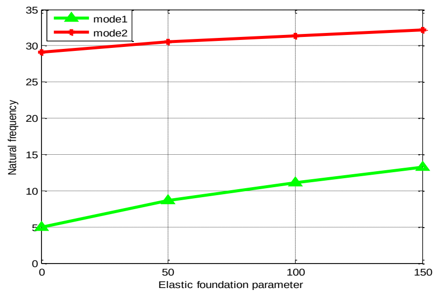
Edge Condition	Natural frequency	(k_w =0, m=0)		
	Mode	g_s =10	g_s =50	g_s =100
Simply Supported	Ω_1	9.0731916	17.71499	24.363131
	Ω_2	33.981698	48.72694	63.242599
Clamped Support	Ω_1	13.145639	20.90975	27.613639
	Ω_2	42.409585	56.33333	69.906398
Free Support	Ω_1	11.550058	18.29656	24.158648
	Ω_2	43.233944	59.74169	75.901677

Table 4
Showing variation of elastic foundation on natural frequency symmetric case m=0.

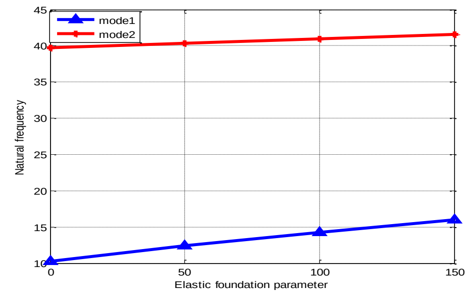
Edge Condition	Natural frequency	Elastic foundation support (m=0)			
	Mode	k_w =0	k_w =50	k_w =100	k_w =150
Simply Supported	Ω_1	4.93514	8.62297	11.15149	13.20438
	Ω_2	29.71931	30.54229	31.35016	32.1377
Clamped Support	Ω_1	10.21583	12.4243	14.29556	15.94877
	Ω_2	39.77117	40.35467	40.9695	41.57523
Free Support	Ω_1	9.00323	11.44799	13.45572	15.20054
	Ω_2	38.43915	39.35556	39.98574	40.60615

Table 5
Showing variation of elastic foundation on natural frequency asymmetric case m=1.

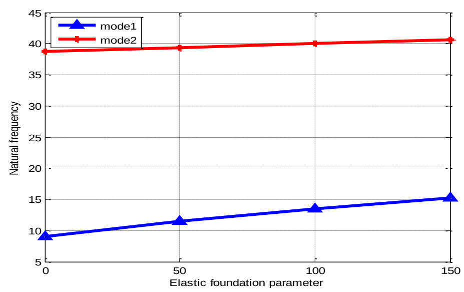
Edge Condition	Natural frequency	Elastic foundation support (m=1)			
	Mode	k_w =0	k_w =50	k_w =100	k_w =150
Simply Supported	Ω_1	13.8981	15.5935	17.1218	18.5245
	Ω_2	48.4797	48.9961	49.5032	50.0023
Clamped Support	Ω_1	21.2604	22.4055	23.4948	24.5358
	Ω_2	60.8302	61.2398	61.6467	62.0509
Free Support	Ω_1	20.7549	21.9265	23.0385	24.0993
	Ω_2	59.9668	60.3687	60.751	61.1956



(a)

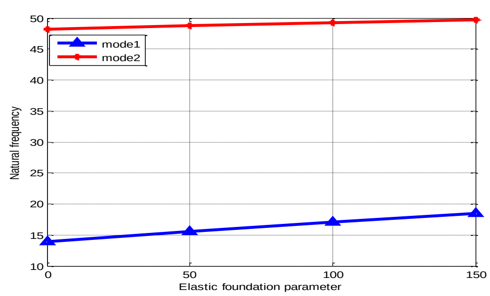


(b)

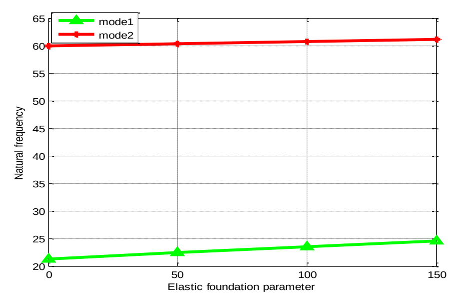


(c)

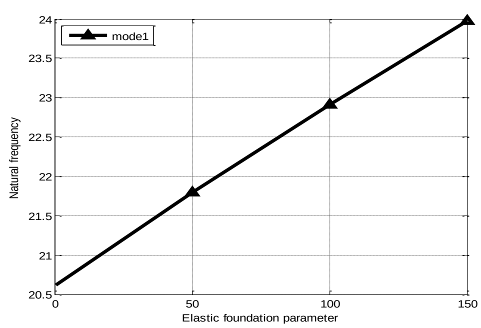
Fig.2
Variation of elastic foundation parameter on natural frequency for free, simply supported and clamped edge symmetric case.



(a)



(b)



(c)

Fig.3
Variation of elastic foundation parameter on natural frequency for free, simply supported and clamped edge asymmetric case.

4.2 Mode shapes

Homogenous circular plate is hereby considered. According to Shariyat and Alipour [12], for transient stress investigation, the response is normally based on modal superposition principle and the modal stress which to certain level, will expose the characteristics and content of the whole response of the plate. Based on that, study of the non-dimension radial and circumferential stress is determined using Eq. (67) and results illustrated in Figs. 4 - 6. The mode shape for the first two natural frequencies are shown in Figs. 4-6 respectively. It is essential to note that, the mode shape obey the classical theory of vibration. For radial and circumferential stresses, location of the vibrating node and antinodes are in away different due to the vanishing mode of the boundary condition. Figs.7 and 8 shows mode shape due to the bending moment and surface stress, it is clearly shown that, the location of node and antinodes of the vibrating plate changes. Figs.7 and 8 when compared to Figs. 4-6 different in the mode shape is clearly shown. Invariably, the extrema mode shapes location differs based on the boundary conditions.

It is clearly observed from Figs. 4-6 that, the symmetric case is presenting half modal shape, while the asymmetric is presenting the full mode shape.

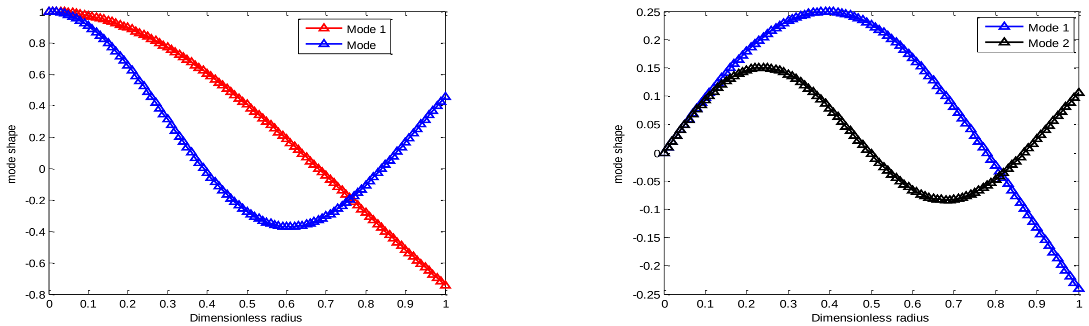


Fig.4
Symmetric and axisymmetric modes shape of free edge condition.

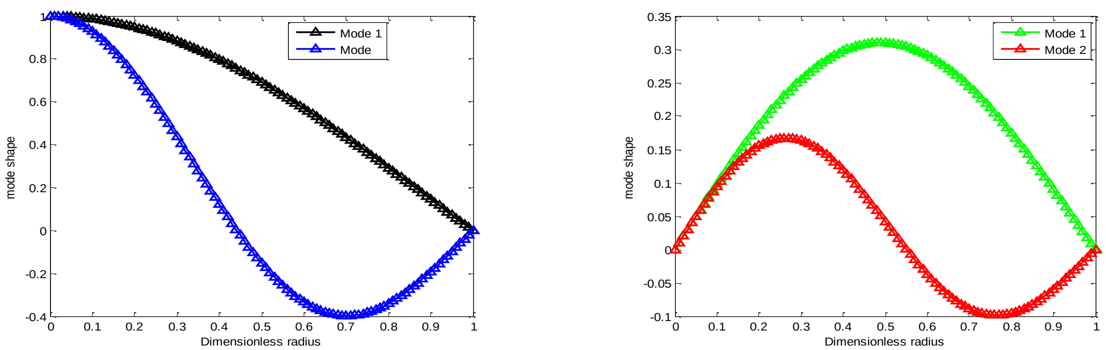


Fig.5
Symmetric and axisymmetric modes shape of simply-supported edge condition.

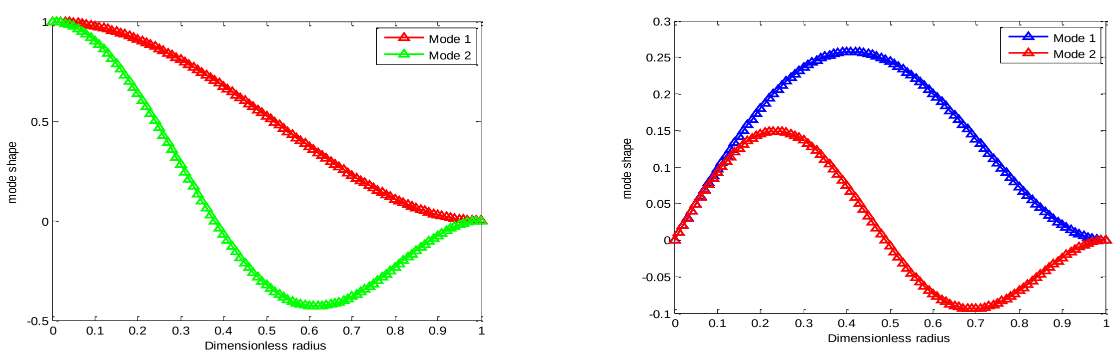


Fig.6
Symmetric and axisymmetric modes shape of clamped edge condition.

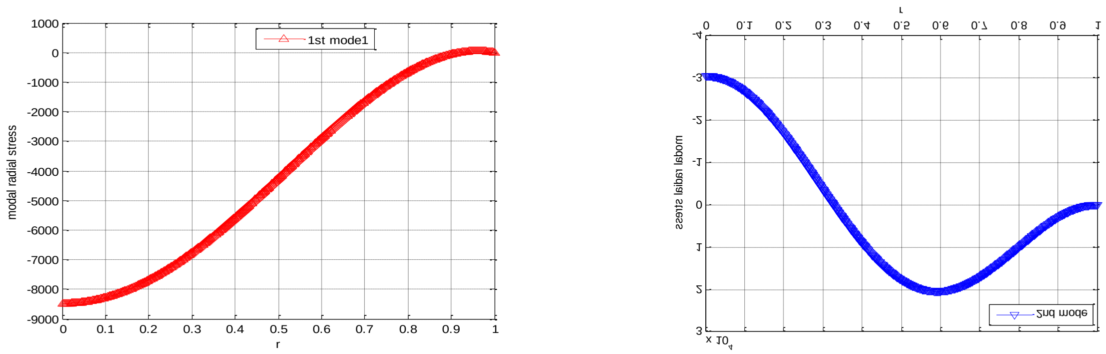


Fig.7
Radial stress for free edge first and second mode symmetric case.

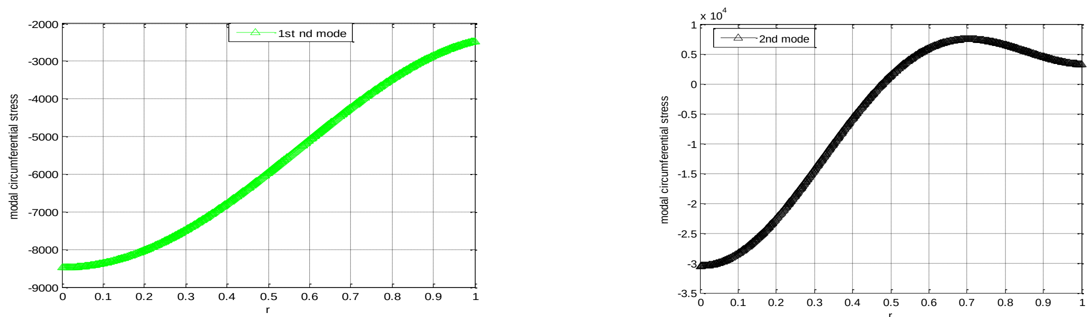


Fig.8
Circumferential stress for free edge first and second mode symmetric case.

5 CONCLUSION

In this study, nonlinear analysis of circular plates resting on Winkler and Pasternak foundations is presented. The nonlinear ordinary differential equations is solved hybrid Adomian decomposition method. The accuracies of the obtained analytical solutions are ascertained with experimental results obtained by some researchers in the past works. The obtained solutions were used to examine the effects of foundation parameter. From the parametric studies, the following observations were established. Increases in elastic foundation parameter increases the natural frequency. Extrema mode is disturbed due to the presence of radial and circumferential stress. The present study exposes the effect of elastic foundation parameters on dynamic behaviour of thin circular plate. It is expected that the present study will contribute to the understanding of the study of dynamic behaviour of circular plate under various parameters.

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