

Influences of Heterogeneities and Initial Stresses on the Propagation of Love-Type Waves in a Transversely Isotropic Layer Over an Inhomogeneous Half-Space

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ABSTRACT

In the present paper, we are contemplating the influences of heterogeneities and pre-stresses on the propagation of Love-type waves in an initially stressed heterogeneous transversely isotropic layer of finite thickness lying over an inhomogeneous half space. The material constants and pre-stress have been taken as space dependent and arbitrary functions of depth in the respective media. To simplify the problem, we have used Whittaker's function and separation of variables method. We present a general dispersion relation to describe the impacts on the propagation of Love-type waves in the structure. The present dispersion relation is analyzed case wise and also validated by comparison of the standard Love wave equation. Further, numerical computations are demonstrated graphically for the set of dimensionless parameters between dimensionless phase velocity and dimensionless wave number of the wave.

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Keywords : Transversely; Isotropic; Heterogeneity; Phase velocity; Initial stress.

1 INTRODUCTION

IN this work, we are studying a theoretical problem of seismology, in which we analysed seismic wave propagation in two dissimilar types of characteristics media. Mainly two kinds of seismic waves are generated by Earthquakes or explosions, first one is body wave which propagates within the Earth and second one is surface wave that propagates along its surface. Love-type wave is a type of surface seismic waves and the existence of these waves were predicted by A.E.H. Love in 1911. They have a decreasing trend as the depth of the layer increases from the surface of the propagation. The present Earth's model (Fig.1) of this work describes the Love-type wave propagation in the composite structure of two different types of media, which may help us to examine the behaviour and distinctiveness of seismic Love-type surface waves in the Earth's interior. The basic information related to fundamental of seismic waves and elastic materials are well documented in the books of Love [1], Ewing et al. [2], Biot [3], Gubbins [4] and Ding et al. [5] etc.

The Earth's interior is the composition of dissimilar types of materials with different type of properties like isotropic, anisotropic, orthotropic, transversely isotropic, homogeneous and heterogeneous etc. The propagation of seismic waves is influenced by these materials. In this way the study of seismic wave propagation in the composite structure of these aforementioned layered media has their own importance. Transversely isotropic materials are the special class of orthotropic materials. Geophysically, the rock formations of crust are locally polar anisotropic that is

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transversely isotropic. Recently, Singh et al. [6] developed a problem to find the effect of semi-infinite smooth moving punch in an initially stressed magnetoelastic transversely isotropic medium due to shear wave propagation. The effect of the transverse isotropy and magnetic field on the interface waves in a conducting medium was investigated by Acharya et al. [7]. Ahmad and Khan [8] studied rotational effects on the wave plane propagation in a transversely isotropic unbounded medium rotating about its axis. Baljeet [9] outlined the plane wave propagation in a rotating, two-temperature thermo-elastic transversely isotropic solid half-space. Kundu et al. [10], Zhu et al. [11] and Kakar [12] established dispersion equations for different types of seismic surface wave propagation in the isotropic layered structure.

There may exist varying stress inside the Earth because of atmospheric pressure, gravitational pull, slow creep process, manufacturing process and pressure due to overburden etc. (Dey and De, [13]). Thus, the Earth is considered to be initially stressed. Many authors and researchers have theoretically considered the seismic surface wave propagation in pre-stressed medium. Dhua and Chattopadhyay [14], Kundu et al. [15] and Chattaraj et al. [16] developed dispersion relation to examine the surface waves propagation in various types of pre-stressed media. The propagation of seismic surface waves are also influenced by pre-stressed media through which they propagate. Dey and Addy [17], Mahmoud [18], Kepceler [19] and Biot [20] discussed how the elastic waves are influenced by initially stress medium.

Geophysical studies have revealed the fact that the earth medium is heterogeneous throughout and different types of heterogeneities (e.g. linear, quadratic, exponential, etc.) may exist in the earth's medium. According to Birch [21], as we move along the depth of the earth the rate of change in rigidity and density vary rapidly and also reported that for different layers of the Earth medium, rigidity varies at different rates. These variations may arise due to heterogeneity. Bullen [22] approximated density law inside the Earth as a quadratic polynomial in depth parameter for 413–984 km depth. In the recent years, sufficient interest has arisen in the problem connected with bodies whose mechanical properties are functions of space, i.e. heterogeneous bodies. Dey et al. [23] have shown the possibility of torsional surface wave propagation in different types of heterogeneous elastic media. Gupta et al. [24, 25] studied Love wave propagation in heterogeneous layered media with rigid and free boundary surfaces.

With the view of above, in the present study we have assumed quadratic variation of heterogeneities. This study visualizes the impacts on the propagation of Love-type waves in a heterogeneous half-space overloaded by a heterogeneous transversely isotropic layer under initial stress. The effect of heterogeneities, pre-stresses (compressive and tensile) on the non-dimensional phase velocity (c / β_1) of Love-type waves are demonstrated graphically with respect to the non-dimensional wave number (kH). All graphs are plotted by using MATLAB software. The graphical results of this problem are discussed in numerical computation and discussion section.

2 FORMULATION OF THE PROBLEM

Consider a composite structure consisting of an initially stressed heterogeneous transversely isotropic layer (M_1) of finite thickness H lying over a heterogeneous half-space (M_2). The assumed model of the problem is shown in Fig.1. The Cartesian co-ordinate system has been considered such that the x -axis is in the direction of Love-type wave propagation and z -axis is vertically downwards. Therefore, the particle displacements of both medium take place along y -axis only *i.e.*, displacements in the direction of x and z axes vanish. Also, the rate of change along y -axis vanishes *i.e.* $\frac{\partial}{\partial y} \equiv 0$. The rigidities, densities and pre-stress of the layer and half-space are taken as:

For layer

$$\left. \begin{aligned} A &= A_0 [1 + \alpha z]^2, N = N_0 [1 + \alpha z]^2, F = F_0 [1 + \alpha z]^2, \\ C &= C_0 [1 + \alpha z]^2, G = G_0 [1 + \alpha z]^2, \\ \rho_1 &= \rho_{01} [1 + \alpha z]^2, P = P_0 [1 + \alpha z]^2, \end{aligned} \right\} \tag{1}$$

where, A, N, G, F and C are material constants, ρ_1 is density and P is initial stress of the layer.

For half-space

$$\left. \begin{aligned} \mu_2 &= \mu_{02} [1 + \beta z]^2, \\ \rho_2 &= \rho_{02} [1 + \gamma z]^2, \end{aligned} \right\} \quad (2)$$

where, μ_2 is rigidity, ρ_2 is density of the half-space and $\beta \neq \gamma$.

The arbitrary constants α, β and γ have inverse dimension of length.

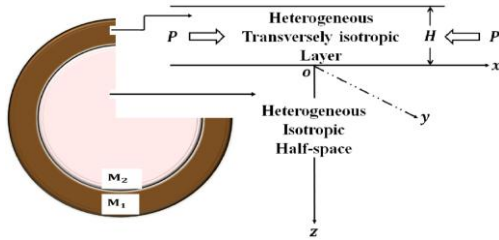


Fig. 1
Geometry of the problem.

3 SOLUTION FOR THE HETEROGENEOUS TRANSVERSELY ISOTROPIC LAYER (M_I)

Let (u_1, v_1, w_1) is the component set of displacement vectors of the layer. Therefore, the dynamical equations of equilibrium for initially stressed medium are given by Biot [3]

$$\begin{aligned} \frac{\partial \tau_{11}}{\partial x} + \frac{\partial \tau_{12}}{\partial y} + \frac{\partial \tau_{13}}{\partial z} - P \left(\frac{\partial \omega_z}{\partial y} - \frac{\partial \omega_y}{\partial z} \right) &= \rho_1 \frac{\partial^2 u_1}{\partial t^2}, \\ \frac{\partial \tau_{12}}{\partial x} + \frac{\partial \tau_{22}}{\partial y} + \frac{\partial \tau_{23}}{\partial z} - P \frac{\partial \omega_z}{\partial x} &= \rho_1 \frac{\partial^2 v_1}{\partial t^2}, \\ \frac{\partial \tau_{13}}{\partial x} + \frac{\partial \tau_{23}}{\partial y} + \frac{\partial \tau_{33}}{\partial z} + P \frac{\partial \omega_y}{\partial x} &= \rho_1 \frac{\partial^2 w_1}{\partial t^2}, \end{aligned} \quad (3)$$

where τ_{ij} ($i, j = 1, 2, 3$) are the component of stresses and given by

$$\begin{aligned} \tau_{11} &= A \frac{\partial u_1}{\partial x} + (A - 2N) \frac{\partial v_1}{\partial y} + F \frac{\partial w_1}{\partial z}, \quad \tau_{12} = N \left(\frac{\partial u_1}{\partial y} + \frac{\partial v_1}{\partial x} \right), \\ \tau_{22} &= (A - 2N) \frac{\partial u_1}{\partial x} + A \frac{\partial v_1}{\partial y} + F \frac{\partial w_1}{\partial z}, \quad \tau_{23} = G \left(\frac{\partial w_1}{\partial y} + \frac{\partial v_1}{\partial z} \right), \\ \tau_{33} &= F \frac{\partial u_1}{\partial x} + F \frac{\partial v_1}{\partial y} + C \frac{\partial w_1}{\partial z}, \quad \tau_{13} = G \left(\frac{\partial u_1}{\partial z} + \frac{\partial w_1}{\partial x} \right) \end{aligned} \quad (4)$$

and $\omega_x, \omega_y, \omega_z$ are the rotational components given by

$$\omega = \frac{1}{2} \left(\frac{\partial w_1}{\partial y} + \frac{\partial v_1}{\partial z} \right), \quad \omega_x = \frac{1}{2} \left(\frac{\partial u_1}{\partial z} + \frac{\partial w_1}{\partial x} \right), \quad \omega_y = \frac{1}{2} \left(\frac{\partial v_1}{\partial x} + \frac{\partial u_1}{\partial y} \right). \quad (5)$$

But, for the Love-type waves propagation

$$v = v_1(x, z, t) \quad \text{and} \quad u_1 = 0 = w_1 \quad (6)$$

By using of Eqs.(1), (4), (5) and (6) in Eq.(3), the only non-vanished dynamic equation of motion is

$$\frac{\partial^2 v_1}{\partial z^2} + \frac{2\alpha}{(1+\alpha z)} \frac{\partial v_1}{\partial z} + \left(\frac{N_0}{G_0} - \frac{P_0}{2G_0} \right) \frac{\partial^2 v_1}{\partial x^2} = \frac{\rho_{01}}{G_0} \frac{\partial^2 v_1}{\partial t^2}. \tag{7}$$

Assuming the solution of resulting Eq. (7) as $v_1 = f(z_1)e^{ik(x-ct)}$ and substituting in it, we get

$$\frac{d^2 f}{dz^2} + \frac{2\alpha}{(1+\alpha z)} \frac{df}{dz} + k^2 \left(\frac{c^2}{c_1^2} + \eta - \frac{N_0}{G_0} \right) f = 0, \tag{8}$$

where, $c_1 = \sqrt{\frac{G_0}{\rho_{01}}}$ and $\eta = \frac{P_0}{2G_0}$.

To simplify the previous Eq. (8), we use the following substitution: $f = \frac{\phi_1(z)}{(1+\alpha z)}$. Therefore Eq. (8) takes the form

$$\frac{d^2 \phi_1}{dz^2} + \xi^2 \phi_1 = 0, \tag{9}$$

where, $\xi^2 = k^2 \left(\frac{c^2}{c_1^2} + \eta - \frac{N_0}{G_0} \right)$.

Solution of Eq. (9) is found as:

$$\phi_1(z_1) = A_1 \cos \xi z + A_2 \sin \xi z, \tag{10}$$

where, A_1 and A_2 are the unknown coefficients.

Finally, the solution of component v_1 is

$$v_1 = \frac{1}{(1+\alpha z)} (A_1 \cos \xi z + A_2 \sin \xi z) e^{ik(x-ct)}. \tag{11}$$

4 SOLUTION FOR THE INHOMOGENEOUS HALF-SPACE (M_2)

Let us consider (u_2, v_2, w_2) is the component set of displacement vectors of half-space and then the dynamics equation of motion for the half-space are (Biot, [3])

$$\begin{aligned} \frac{\partial \sigma_{11}}{\partial x} + \frac{\partial \sigma_{12}}{\partial y} + \frac{\partial \sigma_{13}}{\partial z} &= \rho_2 \frac{\partial^2 u_2}{\partial t^2}, \\ \frac{\partial \sigma_{12}}{\partial x} + \frac{\partial \sigma_{22}}{\partial y} + \frac{\partial \sigma_{23}}{\partial z} &= \rho_2 \frac{\partial^2 v_2}{\partial t^2}, \\ \frac{\partial \sigma_{13}}{\partial x} + \frac{\partial \sigma_{23}}{\partial y} + \frac{\partial \sigma_{33}}{\partial z} &= \rho_2 \frac{\partial^2 w_2}{\partial t^2}. \end{aligned} \tag{12}$$

The components of stresses are

$$\sigma_{ij} = \lambda \Delta \delta_{ij} + 2\mu_2 \varepsilon_{ij}, \quad (i, j = 1, 2, 3) \quad (13)$$

where, $\varepsilon_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$, λ, μ_2 are Lamé's constants.

Now, for the love-type waves

$$v_2 = v_2(x, z, t) \quad \text{and} \quad u_2 = 0 = w_2 \quad (14)$$

Eqs.(12) together with the Eqs.(2), (13) and (14) gives only one non-vanish equation

$$\frac{\partial^2 v_2}{\partial x^2} + \frac{2\beta}{(1+\beta z)} \frac{\partial v_2}{\partial z} + \frac{\partial^2 v_2}{\partial z^2} = \frac{\rho_{02}(1+\gamma z)^2}{\mu_{02}(1+\beta z)^2} \frac{\partial^2 v_2}{\partial t^2}, \quad (15)$$

Assuming the solution of previous Eq.(14) as $v_2 = g(z) e^{ik(x-ct)}$ and on substituting one gets

$$\frac{d^2 g}{dz^2} + \frac{2\beta}{(1+\beta z)} \frac{dg}{dz} + k^2 \left(\frac{c^2}{c_2^2} \frac{(1+\gamma z)^2}{(1+\beta z)^2} - 1 \right) g = 0, \quad (16)$$

where, $c_2 = \sqrt{\frac{\mu_{02}}{\rho_{02}}}$.

For the simplification of Eq. (16), we take the following substitution: $g(z) = \frac{\phi_2(z)}{(1+\beta z)}$. Therefore, Eq. (16) takes the form

$$\frac{d^2 \phi_2}{dz^2} + k^2 \left(\frac{c^2}{c_2^2} \frac{(1+\gamma z)^2}{(1+\beta z)^2} - 1 \right) \phi_2 = 0. \quad (17)$$

To find the solution of Eq. (17), we introduce some dimensionless quantities

$$\left. \begin{aligned} \psi(z) &= \frac{2gk}{\beta} (1+\beta z), \quad r = \frac{c^2}{c_2^2} \frac{k\gamma(\beta-\gamma)}{g\beta^3}, \\ s &= -\frac{1}{2} \sqrt{1 - \frac{c^2}{c_2^2} \frac{4k^2(\beta-\gamma)^2}{\beta^4}} \quad \text{and} \quad g = \sqrt{1 - \frac{\gamma^2 c^2}{\beta^2 c_2^2}}. \end{aligned} \right\} \quad (18)$$

On using above quantities of Eq. (18) in Eq. (17), the Eq. (17) takes the form

$$\frac{d^2 \phi_2(\psi)}{d\psi^2} + k^2 \left(-\frac{1}{4} + \frac{r}{\psi} + \frac{1-s^2}{\psi^2} \right) \phi_2(\psi) = 0. \quad (19)$$

Eq. (18) is Whittaker's equation (Whittaker and Watson, [26]) and its solution is given by

$$\phi_2(\psi) = B_1 W_{r,s}(\psi) + B_2 W_{-r,s}(-\psi). \quad (20)$$

where, B_1 and B_2 are the unknown coefficients of Whittaker's functions $W_{r,s}(\psi)$ and $W_{-r,s}(-\psi)$.

For $\lim_{z \rightarrow \infty} g(z) \rightarrow 0$, i.e., $\lim_{z \rightarrow \infty} \phi_2(\psi) \rightarrow 0$, the appropriate solution for the required displacement component of half-space is

$$v_2(z) = \frac{1}{(1 + \beta z)} B_1 W_{r,s}(\psi) e^{ik(x-ct)}. \tag{21}$$

5 CONDITIONS OF CONTINUITY

At $z = 0$,

$$\begin{aligned} a) & v_1 = v_2, \\ b) & G_0 \frac{\partial v_1}{\partial z} = \mu_{02} \frac{\partial v_2}{\partial z}. \end{aligned}$$

At $z = -H$,

$$\frac{\partial v_1}{\partial z} = 0.$$

6 DISPERSION EQUATION

On using of above boundary conditions with the help of Eqs.(11) and (21), we get a composite system of three homogeneous linear equations with three unknowns A_1, A_2 and B_2

$$A_1 - B_1 M = 0, \tag{22}$$

$$\alpha G_0 A_1 - \xi G_0 A_2 - \mu_{02} (\beta M - M') B_1 = 0, \tag{23}$$

$$(\xi \sin \xi H - \alpha \cos \xi H - \alpha \xi H \sin \xi H) A_1 + (\xi \cos \xi H - \alpha \sin \xi H - \alpha \xi H \cos \xi H) B_1 = 0. \tag{24}$$

On eliminating A_1, A_2 and B_1 from Eqs. (22), (23) and (24) with linear expansion of $W_{r,s}(\psi)$, the dispersion equation of Love-type waves is obtained as:

$$\tan \xi H = \xi \frac{\alpha^2 H G_0 + \mu_{02} (1 - \alpha H) \left(\beta - \frac{M'}{M} \right)}{\left[\xi^2 (1 - \alpha H) + \alpha^2 \right] G_0 - \alpha \mu_{02} \left(\beta - \frac{M'}{M} \right)}, \tag{25}$$

where, $\frac{M'}{M} = \beta s - 9k + \frac{\beta}{2} \left(\frac{1 + 3Q}{1 + Q} \right)$ and $Q = \frac{9k}{\beta} \left(\frac{1 + 2s - 2r}{2s + 1} \right)$.

7 PARTICULAR CASES

Several cases of dispersive Eq. (25) are given below

Case I

If the layer is free from initial stress *i.e.*, $P = 0$, therefore Eq. (25) reduces to

$$\tan \xi_1 H = \xi_1 \frac{\alpha^2 H G_0 + \mu_{02} (1 - \alpha H) \left(\beta - \frac{M'}{M} \right)}{\left[\xi_1^2 (1 - \alpha H) + \alpha^2 \right] G_0 - \alpha \mu_{02} \left(\beta - \frac{M'}{M} \right)}, \quad (26)$$

$$\text{where, } \xi_1 = k \sqrt{\frac{c^2}{c_1^2} - \frac{N_0}{G_0}}.$$

Case II

If we consider the upper layer is homogeneous *i.e.*, $\alpha = 0$, then Eq. (26) becomes

$$\tan \xi_1 H = \frac{\mu_{02}}{\xi_1 G_0} \left(\beta - \frac{M'}{M} \right). \quad (27)$$

Case III

If the half-space is also homogeneous *i.e.*, $\beta \rightarrow 0$ and $\gamma \rightarrow 0$, which implies that $\frac{M'}{M} = -\sqrt{1 - \frac{c^2}{c_2^2}}$. Therefore Eq. (27) reduces to

$$\tan \xi_1 H = \frac{\mu_{02}}{\xi_1 G_0} \sqrt{1 - \frac{c^2}{c_2^2}}. \quad (28)$$

Case IV

When the layer is isotropic *i.e.*, $N_0 = G_0 = \mu_1$ (say). Then Eq. (28) takes the form of the classical form of Love waves (Love, [1])

$$\tan kH \sqrt{\left(\frac{c^2}{c_1^2} - 1 \right)} = \frac{\mu_{02}}{\mu_1} \frac{\sqrt{1 - \frac{c^2}{c_2^2}}}{\sqrt{\left(\frac{c^2}{c_1^2} - 1 \right)}}. \quad (29)$$

8 NUMERICAL COMPUTATIONS AND DISCUSSION

With the purpose of showing influences of initial stress and heterogeneities on Love-type wave propagation, based on dispersion Eq. (25), we are using following data of material constants

For the layer (Acharya et al. [7])

$$G_0 = 4 \times 10^{10} \text{ N / m}^2, N_0 = 6.3 \times 10^{10} \text{ N / m}^2 \text{ and } \rho_{01} = 7100 \text{ kg / m}^3.$$

For the half-space (Gubbins, [4])

$$\mu_{02} = 78.4 \times 10^9 \text{ N / m}^2 \text{ and } \rho_{01} = 3535 \text{ kg / m}^3,$$

The above data are in good agreement with the condition $c_1 < c < c_2$ of Love-type waves. By using of above numerical data, we have shown the impacts of dimensionless heterogeneous parameters ($\alpha/k, \beta/k, \gamma/k$), dimensionless initial compressive stress ($\eta > 0$) and dimensionless initial tensile stress ($\eta < 0$) on the propagation of Love-type wave in the following figures (Figs. 2-6). All graphs have been plotted for the dimensionless phase velocity (c/c_1) with respect to the dimensionless wave number (kH) on the Love-type wave propagation. The phase velocity curves in all figures follow same decreasing trend with respect to the kH . The numerical values of the dimensionless parameters for figures are given in Table 1. Moreover, we have shown some graphs for the appropriate cases.

Table 1
Values of parameters for figures.

Parameters	α/k	β/k	γ/k	$\eta = P_0 / 2\mu_{01}$
Fig. 2	-	0.1	0.4	0.1
Fig. 3	0.2	-	0.4	0.1
Fig. 4	0.2	0.1	-	0.1
Fig. 5	0.2	0.1	0.4	-
Fig. 6	0.0	0.1	0.4	-

- stands for the variation of parameter in the corresponding figure.

Fig. 2 manifests the impact of dimensionless heterogeneity parameter α/k . In this figure, the values of α/k for curves 1, 2, 3, 4 and 5 have been taken as 0.10, 0.12, 0.14, 0.16 and 0.18 respectively. It has been observed that the curves accumulate at $kH = 1.5$, which shows that, at $kH = 1.5$, α/k has no effect on the phase velocity of Love-type wave. It has also been revealed from the figure that as the value of α/k increases, the phase velocity of Love-type waves decreases for $kH < 1.5$ and increases for $kH > 1.5$.

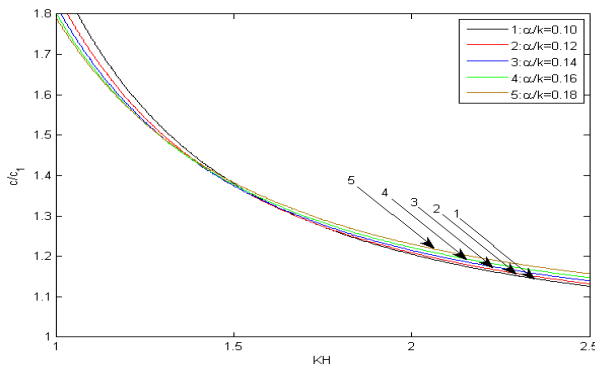


Fig. 2
Variation of phase velocity (c/c_1) with wave number kH for different values of heterogeneity parameter α/k .

Impacts of dimensionless heterogeneity parameter β/k have been elucidated in Fig. 3. In this figure, the values of β/k have been taken as 0.10, 0.12, 0.14, 0.16 and 0.18 for curves 1, 2, 3, 4 and 5 respectively. It has been noticed from the figure that, the phase velocity (c/c_1) also increases as the value of β/k increases. The figure also suggests that β/k has very significant effect on the phase velocity in lower frequency region as compare to the higher frequency region.

The curves in Fig. 4 have been plotted to get a better understanding of the impact of dimensionless heterogeneity parameter γ/k . The values of γ/k in this figure have been taken as 0.20, 0.22, 0.24, 0.26 and 0.28 for curves 1, 2,

3, 4 and 5 respectively. It has been seen from the figure that, the phase velocity (c/c_1) decreases as the value of γ/k increases. Same as Fig. 3, γ/k has very significant effect on the phase velocity in lower frequency region as compare to the higher frequency region.

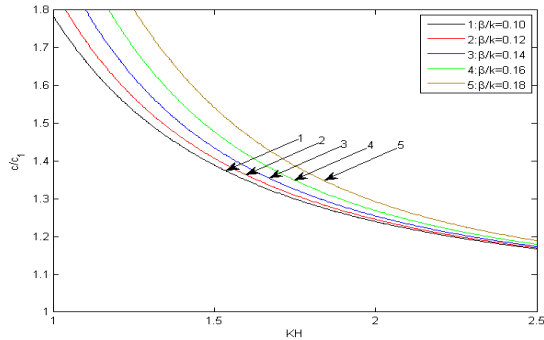


Fig. 3

Variation of phase velocity (c/c_1) with wave number kH for different values of heterogeneity parameter β/k .

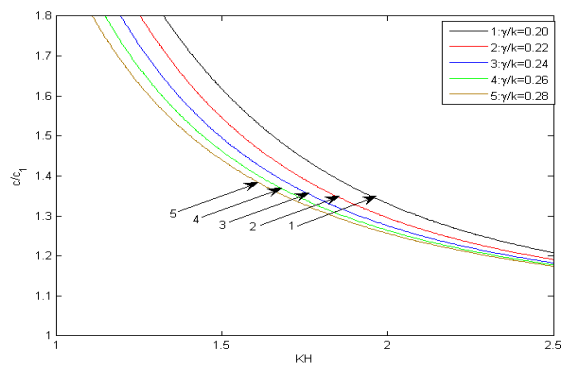


Fig. 4

Variation of phase velocity (c/c_1) with wave number kH for different values of heterogeneity parameter γ/k .

In Fig. 5, the study has been made to know the effect of initial compressive stress ($\eta > 0$). In this figure the values of η for curves 1, 2, 3, 4 and 5 have been taken as 0.0, 0.1, 0.2, 0.3 and 0.4 respectively. It has been followed from this figure that, the compressive initial stress η has decreasing effect on the phase velocity (c/c_1). The curve no. 1 of Fig. 5 represents the case I in the absence of compressive initial stress.

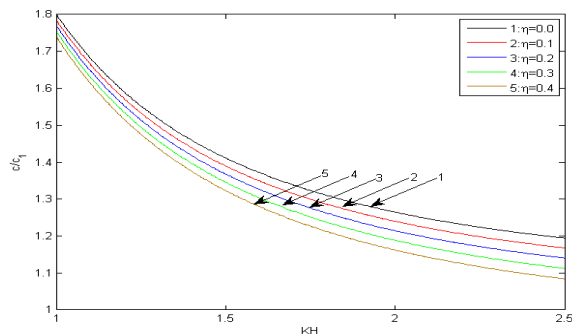


Fig. 5

Variation of phase velocity (c/c_1) with wave number kH for different values of initial compressive stress parameter η .

The impact of dimensionless initial tensile stress ($\eta < 0$) has been inferred in Fig. 6. The curves 1, 2, 3, 4 and 5 have been plotted for the values 0.0, -0.1, -0.2, -0.3 and -0.4 of η respectively. Clearly, Fig. 6 shows that the phase velocity (c/c_1) increases as the value of initial tensile initial stress η decreases. Also, the curve no. 1 of this figure

corresponds to the case I in the absence tensile initial stress. Both figures 5 and 6 have similar but opposite effect on the phase velocity of Love-type waves.

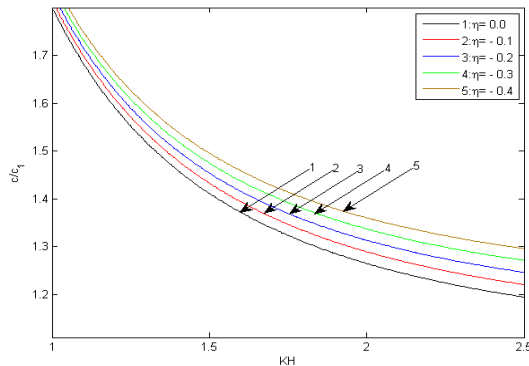


Fig. 6
Variation profile of phase velocity (c / c_1) with wave number kH for different values of initial tensile stress parameter η .

9 CONCLUSIONS

In the present study, we have conducted a theoretical analysis with some numerical examples of parameters to understand the effect of initial stresses and heterogeneities on the propagation of Love-type waves through a mathematical model. The study reveals that the presence of heterogeneities in both media and initial stress present in the layer affects the propagation of Love-type waves significantly.

From the overall study we have following conclusions:

1. It is well-known fact that the velocity of love-type waves decays with the increase of depth and here all figures also suggest that the phase velocity curves of Love-type waves follow same decreasing trend with respect to the kH , which is justifying the results of the study. The higher values of kH are ignored, as the velocity of Love-type wave decays with respect to depth and finally will be diminished.
2. The initial tensile stress ($\eta < 0$) and heterogeneity parameter ($-\beta / k$) have proportional impacts on the phase velocity of Love-type waves.
3. The initial compressive stress ($\eta > 0$) and heterogeneity parameter ($-\gamma / k$) have inverse impacts on the phase velocity of Love-type waves.
4. The heterogeneity parameter (α / k) has inverse impact on the phase velocity of wave for $kH < 1.5$, whereas it has proportional impact on the phase velocity for $kH > 1.5$.

The above conclusion indicates that the heterogeneities, initial stresses and isotropy of the proposed Earth model have remarkable effect on the Love-type surface wave propagation. Also we have shown the validation of this problem by comparison of standard wave equation of Love [1]. Hence, the results of the present theoretical study may helpful to seismologists in the analysis of Earth's interior, and to know the cause and assessment of damage due to earthquakes. Also, the results can be used for the practical application of seismic waves in the heterogeneous layered earth structure.

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