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A Generalized Thermo-Elastic Diffusion Problem in a Functionally Graded Rotating Media Using Fractional Order Theory

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ABSTRACT

A generalized thermo-elastic diffusion problem in a functionally graded isotropic, unbounded, rotating elastic medium due to a periodically varying heat source in the context of fractional order theory is considered in our present work. The governing equations of the theory for a functionally graded material with GNIII model are established. Analytical solution of the problem is derived in Laplace-Fourier transform domain. Finally, numerical inversions are used to show the effect of rotation, non-homogeneity and fractional parameter on stresses, displacement, chemical potential, mass distribution, temperature, etc. and those are illustrated graphically.

Keywords: Generalized thermo-elastic diffusion; Thermo-elastic diffusion; Fractional order; Functionally graded material.

1 INTRODUCTION

THE theory of generalized thermo-elasticity studied since the nineteenth century, now developed and achieved one of the most important roles due to its diverse application in various kinds of physical problems in engineering science and physics. The first generalization proposed by Lord and Shulman [1] involves one relaxation time parameter in Fourier's law of heat conduction equation. In this theory, a flux-rate term has been introduced into Fourier's heat conduction equation to formulate it in a generalized form that involves a hyperbolic-type heat transport equation admitting finite speed of thermal signals. Another model is the temperature-rate-dependent theory of thermo-elasticity proposed by Green and Lindsay [2], which involves two relaxation time parameters. The theory obeys the Fourier law of heat conduction and asserts that heat propagates with finite speed. Three models (Models I, II and III) for generalized thermo-elasticity of homogeneous and isotropic materials have been developed later by Green and Naghdi [3-5].The linearized version of Model II reduces to the classical heat conduction theory (based on Fourier's law) and those of Models II and III permit thermal waves to propagate with finite speed. Applying the above theories of generalized thermo-elasticity, several problems have been solved on this field from which we may mention a few with references [6-13]. Functionally graded materials (FGM) as a new kind of composites were

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initially designed as thermal barrier materials for aero-space structures, in which the volume fractions of different constituents of composites vary continuously from one side to another. These novel inhomogeneous materials have excellent thermo-mechanical properties and have extensive applications in aerospace structures, tubes, nuclear reactors and overall in generalized thermo-elastic materials structures. FGMs are frequently used in generalized thermo-elasticity. Many works are done on this field with FGMs from which a few are mentioned in references [14-21]. Recently, Sur and Kanoria [22] studies on FGM with variable material properties with fractional order generalized thermoelasticity theory. In addition, Pal, Das and Kanoria [23] study on magneto-thermoelastic response in a functionally graded rotating medium due to a periodically varying heat source. The study of diffusion phenomena has aroused much interest in recent years because of its several applications in geophysics, electronics and metal oxide semiconductor (MOS) improvement in crude oil extraction from oil deposits. Diffusion can be regarded as the phenomena of random walk of an ensemble of particles from regions of high concentration to regions of low concentration until equilibrium is reached. The process of heat and mass diffusion play important roles in many engineering applications, such as satellites problems, returning space vehicles, and aircraft landing on water or land. There is now a great deal of interest in the process of diffusion in the manufacturing of integrated circuits and integrated resistors. In heat treatment of metals, the surface characteristics of metals such as wear and corrosion resistance and hardness can be improved by carburizing through diffusion. Diffusion is of fundamental importance in many disciplines of Physics, Chemistry and Biology such as sintering to produce solid materials, chemical reactor design and catalyst design in chemical industry and doping during production of semi-conductors. Thermo-elastic diffusion in an elastic solid takes place due to the coupling of the field of temperature, mass diffusion and strain. Heat and Mass exchange with the environment during the process of thermo-diffusion in an elastic solid. Nowacki [24-27] developed the theory of thermo-elastic diffusion within the context of classical coupled thermo-elasticity (CCTE) and studied some dynamical problems of diffusion in solids and then one by one several author work on this field from which some are given in references [6-8, 28-33]. Li et al [34-35] also work on that field. Youssefa and Al-Lehaibib [36] represent three dimensional generalized thermoelastic diffusion theory and apply it for a thermoelastic half-space subjected to rectangular thermal pulse. Recently, Paul and Mukhopadhyay [37] work on a two-dimensional generalized magneto thermoelastic diffusion problem for a thick plate under laser pulse heating with three-phase lag effect.

The study of fractional calculus, the generalization of the concept of derivative and integral to a non-integer order, started in the 2nd half of the 19th century has been used in recent years successfully to modify many existing models of various physical processes in various areas, e.g., chemistry, biology and modeling and identification, electronics, wave propagation, visco-elasticity and classical mechanics [38-40]. Fractional calculus is used in oil industry specially for the best finding of oil reservoirs in the ground well rather than Euclidean geometry [54]. The first application of fractional derivatives was given by Abel who applied fractional calculus in the solution of an integral equation that arises in the formulation of the tautochrone problem. Among the few works devoted on fractional calculus to thermo-elasticity, we can refer to the works of Padlubny [41] for a survey of applications of fractional calculus and Povstenko who introduced a fractional heat conduction law and found the associated thermal stresses [42]. Sherief et al. [43] introduced a new model of thermo-elasticity using fractional calculus, proved a uniqueness theorem, and derived a reciprocity relation and a variational principle. Youssef [44] introduced another new model of fractional heat conduction equation, proved a uniqueness theorem and presented one-dimensional application. Ezzat [45-46] established a new model of fractional heat conduction equation by using the new Taylor series expansion of time-fractional order which developed by Jumarie [47]. Ezzat and Mohsen [48] work on fractional order theory of thermoelastic diffusion. Recently, a problem on thermo-viscoelastic interaction subjected to fractional Fourier law with three-phase lag effect is studied by Pal et al. [49]. Abbas [50] also represents an Eigen value approach on a fractional order theory of thermoelastic diffusion problem for an infinite elastic medium with spherical cavity.

Therefore, there are many ways to work on fractional order generalized thermo-elasticity. A one-dimensional generalized thermo-elastic diffusion problem of a functionally graded isotropic, unbounded, rotating medium due to a periodically varying heat source in the context of generalized fractional order thermo-elasticity with GNIII Model is considered in our present work. The governing equations of the theory for a functionally graded material with GNIII Model are established. Analytical solution of the theory is derived in Laplace-Fourier transform domain. Finally, numerical inversions by Honig and Hirdes [51] are used to show the effect of rotation, non-homogeneity and fractional parameter on stresses, displacement, chemical potential, mass distribution, temperature, etc. and illustrated graphically.

2 BASIC EQUATIONS

The governing equations [29, 55] in the present problem based on GNIII model comprise of the following:
(A_1) Stress-displacement-temperature-chemical potential and mass concentration relations:

$$\sigma_{ij} = 2\mu e_{ij} + \delta_{ij}(\lambda_0 \Delta - \gamma_2 P) - \delta_{ij}\gamma_1 (1 + \frac{\tau_1^\alpha}{\alpha!}\frac{\partial^\alpha}{\partial t^\alpha})\theta \tag{1}$$

$$C = \gamma_2 \Delta + nP + d(1 + \frac{\tau_1^\alpha}{\alpha!}\frac{\partial^\alpha}{\partial t^\alpha})\theta \tag{2}$$

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \tag{3}$$

(B_1) Equations of motion in terms of displacement components

$$[2\mu e_{ij} + \delta_{ij}(\lambda_0 \Delta - \gamma_2 P) - \delta_{ij}\gamma_1 (1 + \frac{\tau_1^\alpha}{\alpha!}\frac{\partial^\alpha}{\partial t^\alpha})\theta]_{,j} = \rho(\ddot{u}_i + \{\omega\times(\omega\times u) + 2\omega\times\dot{u}\}_i) \tag{4}$$

(C_1) Heat conduction and diffusion equations

$$\left(K\dot{\theta}_{,i}\right)_{,i} + \left(K^*\theta_{,i}\right)_{,i} + \rho\dot{Q} = \gamma_1 T_0 \ddot{\Delta} + dT_0 \ddot{P} + l_1 T_0 (\frac{\partial^2}{\partial t^2} + \frac{\tau_1^\alpha}{\alpha!}\frac{\partial^{\alpha+2}}{\partial t^{\alpha+2}})\theta \tag{5}$$

$$\left(D\dot{P}_{,i}\right)_{,i} = \gamma_2 \ddot{\Delta} + n\ddot{P} + d(\frac{\partial^2}{\partial t^2} + \frac{\tau_1^\alpha}{\alpha!}\frac{\partial^{\alpha+2}}{\partial t^{\alpha+2}})\theta \tag{6}$$

For a functionally graded solid, the parameters $\lambda_0, \mu, K, K^*, \rho, \gamma_1, \gamma_2, l_1, D, n$ and d are no longer constants but are space dependent. Thus, we replace $\lambda_0, \mu, K, K^*, \rho, \gamma_1, \gamma_2, l_1, D, n$ and d by $\lambda'_0 f(x)$, $\mu_0 f(x)$, $K_0^* f(x)$, $K_0 f(x)$, $\rho_0 f(x)$, $\gamma_1^0 f(x)$, $\gamma_2^0 f(x)$, $l_1^0 f(x)$, $D_0 f(x)$, $n_0 f(x)$ and $d_0 f(x)$ respectively, where $\lambda'_0, \mu_0, K_0, K'_0, \rho_0, \gamma_1^0, \gamma_2^0, l_1^0$, D_0, D'_0, n_0 and d_0 assumed to be constants and $f(x)$ is a given dimensionless function of the space variable *x*= (*x*, *y*, *z*). Now, the Eqs. (1-2) and (4-6) take the following form:
(A_2) Stress-displacement-temperature-chemical potential and mass concentration relations:

$$\sigma_{ij} = [2\mu_0 e_{ij} + \delta_{ij}(\lambda'_0 \Delta - \gamma_2^0 P) - \delta_{ij}\gamma_1^0 (1 + \frac{\tau_1^\alpha}{\alpha!}\frac{\partial^\alpha}{\partial t^\alpha})\theta] f(x) \tag{7}$$

$$C = [\gamma_2^0 \Delta + n_0 P + d_0 (1 + \frac{\tau_1^\alpha}{\alpha!}\frac{\partial^\alpha}{\partial t^\alpha})\theta] f(x) \tag{8}$$

(B_2) Equations of motion in terms of displacement components

$$\begin{aligned}&\rho_0(\ddot{u}_i + \{\omega\times(\omega\times u) + 2\omega\times\dot{u}\}_i) f(x) \\ &= f(x)[2\mu_0 e_{ij} + \delta_{ij}(\lambda'_0 \Delta - \gamma_2^0 P) - \delta_{ij}\gamma_1^0 (1 + \frac{\tau_1^\alpha}{\alpha!}\frac{\partial^\alpha}{\partial t^\alpha})\theta]_{,j} + f(x)_{,j}[2\mu_0 e_{ij} + \delta_{ij}(\lambda'_0 \Delta - \gamma_2^0 P) - \delta_{ij}\gamma_1^0 (1 + \frac{\tau_1^\alpha}{\alpha!}\frac{\partial^\alpha}{\partial t^\alpha})\theta]\end{aligned} \tag{9}$$

(C_2) Heat conduction and diffusion equations

$$[K_0 f(x)\dot{\theta}_{,i} + K_0^* f(x)\theta_{,i}]_{,i} + \rho_0 \dot{Q} f(x) = [\gamma_1^0 T_0 \ddot{\Delta} + d_0 T_0 \ddot{P} + l_1^0 T_0 (\frac{\partial^2}{\partial t^2} + \frac{\tau_1^\alpha}{\alpha!}\frac{\partial^{\alpha+2}}{\partial t^{\alpha+2}})\theta] f(x) \tag{10}$$

$$[D_0 f(x)\dot{P}_{,i}]_{,i} = [\gamma_2^0 \ddot{\Delta} + n_0 \ddot{P} + d_0 (\frac{\partial^2}{\partial t^2} + \frac{\tau_1^\alpha}{\alpha!}\frac{\partial^{\alpha+2}}{\partial t^{\alpha+2}})\theta] f(x) \tag{11}$$

3 FORMULATION OF THE PROBLEM

Let us consider a functionally graded isotropic thermo-elastic medium in $-\infty < x < \infty$ whose state depends only on the variable x and time variable t which is rotating with a uniform angular velocity $\omega = \omega n$, where n is the unit vector in the direction of the axis of rotation. If we take the coordinate axes fixed in the rotating medium, the displacement equation of motion in the rotating frame of reference has two additional terms—the centripetal acceleration $\omega \times (\omega \times u)$ due to the time-varying motion only and the Coriolis acceleration $2\omega \times \dot{u}$, where u is the dynamic displacement vector measured from a steady state-deformed position, and the deformation is assumed to be very small. We shall consider the propagation of plane waves in the presence of periodically varying heat sources distributed over a plane area within the medium. Since we are dealing with an isotropic thermo-elastic medium, without any loss of generality, we may consider waves propagating in the x direction, and all the field variables are supposed to be functions of x and t only; i.e., we may assume that $u = u(x, t)$, $v = v(x, t)$, $w = 0$ and $\theta = \theta(x, t)$ where $\theta(x, t)$ denotes the temperature above the reference temperature T_0. In order to examine the effect of rotation on the propagation of plane waves, we set, $\omega = (0, 0, \omega)$, where ω is a constant. In view of the above assumptions, our problem will involve two displacement components $u(x, t)$ and $v(x, t)$. In this context we can mention the work of Sinha and Bera [52] and Rouchoudhuri [53].

It is assumed that the material properties of a functionally graded material depend only on x co-ordinate. Also, it is assumed that medium is rotating with uniform angular velocity ω. So, in the context of the linear theory of generalized thermo-elasticity Eqs. (7), (8), (9), (10) and (11) take the following form:

(A_3) Stress-displacement-temperature-chemical potential and mass concentration relations:

$$\sigma_{xx} = [(2\mu_0 + \lambda_0')\frac{\partial u}{\partial x} - \gamma_2^0 P - \gamma_1^0 (1 + \frac{\tau_1^\alpha}{\alpha!}\frac{\partial^\alpha}{\partial t^\alpha})\theta] f(x) \tag{12}$$

$$C = [\gamma_2^0 \frac{\partial u}{\partial x} + n_0 P + d_0 (1 + \frac{\tau_1^\alpha}{\alpha!}\frac{\partial^\alpha}{\partial t^\alpha})\theta] f(x) \tag{13}$$

(B_3) Equations of motion in terms of displacement components:

$$\begin{aligned} &\rho_0 [\ddot{u} - \omega^2 u - 2\omega \dot{v}] f(x) \\ &= f(x)[(2\mu_0 + \lambda_0')\frac{\partial^2 u}{\partial x^2} - \gamma_2^0 \frac{\partial P}{\partial x} - \gamma_1^0 (1 + \frac{\tau_1^\alpha}{\alpha!}\frac{\partial^\alpha}{\partial t^\alpha})\frac{\partial \theta}{\partial x}] + [(2\mu_0 + \lambda_0')\frac{\partial u}{\partial x} - \gamma_2^0 P - \gamma_1^0 (1 + \frac{\tau_1^\alpha}{\alpha!}\frac{\partial^\alpha}{\partial t^\alpha})\theta]\frac{\partial f(x)}{\partial x} \end{aligned} \tag{14}$$

$$\rho_0 [\ddot{v} - \omega^2 v + 2\omega \dot{u}] f(x) = \mu_0 f(x)\frac{\partial^2 v}{\partial x^2} + \mu_0 \frac{\partial v}{\partial x}\frac{\partial f(x)}{\partial x} \tag{15}$$

(C_3) Heat conduction and diffusion equations:

$$\frac{\partial}{\partial x}[K_0 f(x)\frac{\partial^2\theta}{\partial x\,\partial t}+K_0^* f(x)\frac{\partial\theta}{\partial x}]+\rho_0 f(x)\dot{Q}=[\grave{}\gamma_1^0 T_0\frac{\partial^3 u}{\partial t^2\partial x}+d_0 T_0\frac{\partial^2 P}{\partial t^2}+l_1^0 T_0(\frac{\partial^2}{\partial t^2}+\frac{\tau_1^\alpha}{\alpha!}\frac{\partial^{\alpha+2}}{\partial t^{\alpha+2}})\theta]f(x) \quad (16)$$

$$\frac{\partial}{\partial x}[D_0 f(x)\frac{\partial^2 P}{\partial x\,\partial t}=[\grave{}\gamma_2^0 T_0\frac{\partial^3 u}{\partial t^2\partial x}+n_0\frac{\partial^2 P}{\partial t^2}+d_0(\frac{\partial^2}{\partial t^2}+\frac{\tau_1^\alpha}{\alpha!}\frac{\partial^{\alpha+2}}{\partial t^{\alpha+2}})\theta]f(x) \quad (17)$$

Now, we introduce the following non-dimensional quantities:

$$x'=\frac{x}{l},\tau_1'=\frac{c_1}{l}\tau,u'=\frac{(2\mu_0+\lambda_0')}{\gamma_1^0 T_0 l}u,v'=\frac{(2\mu_0+\lambda_0')}{\gamma_1^0 T_0 l}v,t'=\frac{c_1}{l}t,\theta=T-T_0,\omega'=\frac{l}{c_1}\omega,\theta'=\frac{(2\mu_0+\lambda_0')}{c_1^2 T_0\rho_0}\theta,$$
$$C'=\frac{\gamma_2^0}{c_1^2\rho_0}C,\sigma_{x'x'}'=\frac{(2\mu_0+\lambda_0')}{c_1^2 T_0\rho_0\gamma_1^0}\sigma_{xx},P'=\frac{(2\mu_0+\lambda_0')}{c_1^2\rho_0\gamma_2^0}P,c_1=\sqrt{\frac{\lambda_0+2\mu_0}{\rho_0}},\ \text{l = some standard length}$$

By using non-dimensional quantities in Eqs. (12-17) and omitting primes, we obtain the following set of equations:

(A_4) Stress-displacement-temperature-chemical potential and mass concentration relations:

$$\sigma_{xx}=[C_1\frac{\partial u}{\partial x}-C_2 P-(1+K_\alpha\frac{\partial^\alpha}{\partial t^\alpha})\frac{\partial\theta}{\partial x}]f(x) \quad (18)$$

$$C=[C_u^2\frac{\partial u}{\partial x}+C_5 P+C_6(1+K_\alpha\frac{\partial^\alpha}{\partial t^\alpha})\theta]f(x) \quad (19)$$

(B_4) Equations of motion in terms of displacement components:

$$[\ddot{u}-\omega^2 u-2\omega\dot{v}]f(x)=f(x)[C_1\frac{\partial^2 u}{\partial x^2}-C_2\frac{\partial P}{\partial x}-(1+K_\alpha\frac{\partial^\alpha}{\partial t^\alpha})\frac{\partial\theta}{\partial x}]+[C_1\frac{\partial u}{\partial x}-C_2 P-(1+\frac{\tau_1^\alpha}{\alpha!}\frac{\partial^\alpha}{\partial t^\alpha})\theta]\frac{\partial f(x)}{\partial x} \quad (20)$$

$$[\ddot{v}-\omega^2 v+2\omega\dot{u}]f(x)=C_3^2[f(x)\frac{\partial^2 v}{\partial x^2}+\frac{\partial v}{\partial x}\frac{\partial f(x)}{\partial x}] \quad (21)$$

(C_4) Heat conduction and diffusion equations:

$$k_0\frac{\partial}{\partial x}[f(x)\frac{\partial^2\theta}{\partial x\,\partial t}]+C_T^2\frac{\partial}{\partial x}[f(x)\frac{\partial\theta}{\partial x}]+f(x)Q_0=[\grave{}\varepsilon_T\frac{\partial^3 u}{\partial t^2\partial x}+C_4\frac{\partial^2 P}{\partial t^2}+(\frac{\partial^2}{\partial t^2}+\frac{\tau_1^\alpha}{\alpha!}\frac{\partial^{\alpha+2}}{\partial t^{\alpha+2}})\theta]f(x) \quad (22)$$

$$\mathrm{D}_1\frac{\partial}{\partial x}[f(x)\frac{\partial^2 P}{\partial x\,\partial t}]=[\grave{}D_T\frac{\partial^3 u}{\partial t^2\partial x}+D_2\frac{\partial^2 P}{\partial t^2}+(\frac{\partial^2}{\partial t^2}+\frac{\tau_1^\alpha}{\alpha!}\frac{\partial^{\alpha+2}}{\partial t^{\alpha+2}})\theta]f(x) \quad (23)$$

where,

$$C_2=\frac{(\gamma_2^0)^2}{\gamma_1^0 T_0},C_3^2=\frac{\mu_0}{\rho_0 c_1^2},k_0=\frac{K_0}{l l_1^0 T_0 c_1},C_T^2=\frac{K_0^* l}{l_1^0 T_0 c_1^2},Q_0=\frac{l(\lambda_0'+2\mu_0)}{l_1^0 T_0 c_1^3}\dot{Q},\varepsilon_T=\frac{(\gamma_1^0)^2}{l_1^0\rho_0 c_1^2},C_4=\frac{d_0\gamma_2^0}{l_1^0 T_0},$$
$$D_1=\frac{D_0\gamma_2^0}{l_1^0 T_0 c_1 d_0},D_T=\frac{\gamma_1^0\gamma_2^0}{\rho_0 c_1^2 d_0},D_2=\frac{n_0\gamma_2^0}{T_0},C_u^2=\frac{\gamma_1^0(\gamma_2^0)^2 T_0}{(\lambda_0'+2\mu_0)\rho_0 c_1^2},C_5=\frac{n_0(\gamma_2^0)^2}{(\lambda_0'+2\mu_0)},C_6=\frac{d_0\gamma_2^0 T_0}{(\lambda_0'+2\mu_0)},$$
$$K_\alpha=\frac{(\tau_1)^\alpha}{\alpha!},C_1=1-\frac{\beta_2^2}{b(\lambda_0+2\mu_0)}.$$

We assume that the medium is initially at rest. The undisturbed state is maintained at reference temperature. Then, we have

$$u(x,0)=\dot{u}(x,0)=v(x,0)=\dot{v}(x,0)=\theta(x,0)=\dot{\theta}(x,0)=0 \tag{24}$$

Taking exponential variation of non-homogeneity as $f(x)=e^{-n_1 x}$,where, n_1 is non-dimensional constant and using it in Eqs. (18-23) we obtain the following sets of equations:

(A_5) Stress-displacement-temperature-chemical potential and mass concentration relations:

$$\sigma_{xx}=[C_1\frac{\partial u}{\partial x}-C_2P-(1+K_\alpha\frac{\partial^\alpha}{\partial t^\alpha})\frac{\partial\theta}{\partial x}]e^{-n_1x} \tag{25}$$

$$C=[C_u^2\frac{\partial u}{\partial x}+C_5P+C_6(1+K_\alpha\frac{\partial^\alpha}{\partial t^\alpha})\theta]e^{-n_1x} \tag{26}$$

(B_5) Equations of motion in terms of displacement components:

$$[\ddot{u}-\omega^2u-2\omega\dot{v}]=[C_1\frac{\partial^2u}{\partial x^2}-C_2\frac{\partial P}{\partial x}-(1+K_\alpha\frac{\partial^\alpha}{\partial t^\alpha})\frac{\partial\theta}{\partial x}]-n_1[C_1\frac{\partial u}{\partial x}-C_2P-(1+K_\alpha\frac{\partial^\alpha}{\partial t^\alpha})\theta] \tag{27}$$

$$[\ddot{v}-\omega^2v+2\omega\dot{u}]=C_3^2[\frac{\partial^2v}{\partial x^2}-n_1\frac{\partial v}{\partial x}] \tag{28}$$

(C_5) Heat conduction and diffusion equations:

$$k_0[\frac{\partial^3\theta}{\partial x^2\partial t}-n_1\frac{\partial^2\theta}{\partial x\,\partial t}]+C_T^2[\frac{\partial^2\theta}{\partial x^2}-n_1\frac{\partial\theta}{\partial x}]+Q_0=`\varepsilon_T\frac{\partial^3u}{\partial t^2\partial x}+C_4\frac{\partial^2P}{\partial t^2}+(\frac{\partial^2}{\partial t^2}+K_\alpha\frac{\partial^{\alpha+2}}{\partial t^{\alpha+2}})\theta \tag{29}$$

$$D_1[\frac{\partial^3P}{\partial x^2\partial t}-n_1\frac{\partial^2P}{\partial x\,\partial t}]=[`D_T\frac{\partial^3u}{\partial t^2\partial x}+D_2\frac{\partial^2P}{\partial t^2}+(\frac{\partial^2}{\partial t^2}+K_\alpha\frac{\partial^{\alpha+2}}{\partial t^{\alpha+2}})\theta] \tag{30}$$

4 SOLUTION OF THE PROBLEM IN LAPLACE-FOURIER TRANSFORM DOMAIN

Applying Laplace-Fourier transformation [with p and s Laplace and Fourier parameter respectively] of both sides of the Eqs. (25-30) and arranging we obtain the following set of equations:

(A_6) Stress-displacement-temperature-chemical potential and mass concentration relations:

$$\bar{\sigma}_{xx}^*=-i(s+in_1)C_1\bar{u}^*(s+in_1,p)+i(1+K_\alpha p^\alpha)\bar{\theta}^*(s+in_1,p)-C_2\bar{P}^*(s+in_1,p) \tag{31}$$

$$\bar{C}^*=-iC_u^2(s+in_1)\bar{u}^*(s+in_1,p)-i(1+K_\alpha p^\alpha)C_6\bar{\theta}^*(s+in_1,p)+C_5\bar{P}^*(s+in_1,p) \tag{32}$$

(B_6) Equations of motion in terms of displacement components:

$$[-C_1s^2-in_1C_1s-p^2+\omega^2]\bar{u}^*(s,p)+(is+n_1)[1+K_\alpha p^\alpha]\bar{\theta}^*(s,p)+[C_2(is+n_1)]\bar{P}^*(s,p)+2\omega p\bar{v}^*(s,p)=0 \tag{33}$$

$$[2\omega p]\bar{u}^*(s,p)-[C_3^2(s^2-in_1s)-p^2+\omega^2]\bar{v}^*(s,p)=0 \tag{34}$$

(C_6) Heat conduction and diffusion equations:

$$[i\,\varepsilon_T\, p^2 s]\bar{u}^*(s,p)+[(in_1 s-s^2)(pk_0+C_T^2)-p^2(1+K_\alpha p^\alpha)]\bar{\theta}^*(s,p)-[C_4 p^2]\bar{P}^*(s,p)+Q_0^*=0 \tag{35}$$

$$[iD_T P^2 s]\bar{u}^*(s,p)-p^2(1+K_\alpha p^\alpha)\bar{\theta}^*(s,p)+[-pk_0 s^2+in_1 D_1 sp+D_2 p^2]\bar{P}^*(s,p) \tag{36}$$

The above Eqs. (33-36) are system of linear algebraic equations with unknowns $\bar{u}^*,\bar{v}^*,\bar{\theta}^*,\bar{P}^*$. Solving these algebraic equations, we obtain the corresponding solutions in Laplace-Fourier transform domain and also the stress and mass concentration which are given below:

(A_7) Displacement components:

$$\bar{u}^*(s,p)=\frac{1}{R(s,p)}[-\omega p\bar{Q}_0^* B_{s,p}(1+K_\alpha p^\alpha)(is+n_1)(2D_{s,p}+C_2 p^3)] \tag{37}$$

$$\bar{v}^*(s,p)=\frac{1}{R(s,p)}[-2\omega^2 p^2\bar{Q}_0^* B_{s,p}(1+K_\alpha p^\alpha)(is+n_1)(2D_{s,p}+C_2 p^3)] \tag{38}$$

(B_7) Temperature and Chemical potential

$$\bar{\theta}^*(s,p)=\frac{1}{R(s,p)}[\omega p\bar{Q}_0^*[2D_{s,p}(A_{s,p}B_{s,p}-s^2 D_T B_{s,p}+4\omega^2 p^2)+(is+n_1)(2isD_T B_{s,p}D_{s,p}+C_2 p^3)]] \tag{39}$$

$$\bar{P}^*(s,p)=\frac{1}{R(s,p)}[\omega p^4\bar{Q}_0^*(1+K_\alpha p^\alpha)(A_{s,p}B_{s,p}-s^2 D_T B_{s,p}+4\omega^2 p^2)] \tag{40}$$

(C_7) Stress and Mass concentration

$$\begin{aligned}\bar{\sigma}_{xx}^*(s+in_1,p)=&\frac{ip\bar{Q}_0^*(1+K_\alpha p^\alpha)}{R(s+in_1,p)}[(s+in_1)(s+2in_1)(2iD_{s+in_1,p}+C_2 p^3)B_{s+in_1,p}+(2D_{s+in_1,p}+iC_2 p^3)\\&(A_{s+in_1,p}B_{s+in_1,p}-(s+in_1)^2 D_T B_{s+in_1,p}+4\omega^2 p^2)+iD_T(s+in_1)(s+2in_1)(2iD_{s+in_1,p}+C_2 p^3)]\end{aligned} \tag{41}$$

$$\begin{aligned}\bar{C}^*(s+in_1,p)=&\frac{ip\bar{Q}_0^*(1+K_\alpha p^\alpha)}{R(s+in_1,p)}[(s+in_1)(s+2in_1)(2iD_{s+in_1,p}+C_2 p^3)C_u^2 B_{s+in_1,p}+(2C_6 D_{s+in_1,p}-iC_5 p^3)\\&(A_{s+in_1,p}B_{s+in_1,p}-(s+in_1)^2 D_T B_{s+in_1,p}+4\omega^2 p^2)+iD_T C_6(s+in_1)(s+2in_1)(2iD_{s+in_1,p}+C_2 p^3)B_{s+in_1,p}]\end{aligned} \tag{42}$$

where,

$$A_{s,p}=-C_1 s(s+in_1)-p^2+\omega^2$$
$$B_{s,p}=-C_3^2 s(s-in_1)-p^2+\omega^2$$
$$C_{s,p}=p[s(pk_0+C_T^2)(in_1-s)-p^2(1+K_\alpha p^\alpha)]$$
$$D_{s,p}=p^2[D_1 s(in_1-s)+D_2]$$
$$\begin{aligned}R(s,p)=&[is(s+in_1)B_{s,p}(p^3\varepsilon_T(1+K_\alpha p^\alpha)+D_T C_{s,p})(2D_{s,p}+C_2 p^3)+(C_{s,p}D_{s,p}-C_4 p^6(1+K_\alpha p^\alpha))\\&(s^2 D_T B_{s,p}-A_{s,p}B_{s,p}-4\omega^2 p^2)]=\chi(r-r_1)(r-r_2)(r-r_3)(r-r_4)(r-r_5)(r-r_6)(r-r_7)(r-r_8),\end{aligned}$$

$$R(s+in_1,p)=[i(s+in_1)(s+2in_1)B_{s+in_1,p}(p^3\varepsilon_T(1+K_\alpha p^\alpha)+D_T C_{s+in_1,p})(2D_{s+in_1,p}+C_2p^3)+(C_{s+in_1,p}D_{s+in_1,p}$$
$$-C_4p^6(1+K_\alpha p^\alpha))((s+in_1)^2D_T B_{s+in_1,p}-A_{s+in_1,p}B_{s+in_1,p}-4\omega^2p^2)]$$
$$=\chi(s-s_1)(s-s_2)(s-s_3)(s-s_4)(s-s_5)(s-s_6)(s-s_7)(s-s_8)$$
$$\chi=D_1p^2C_3^2(pk_0+C_T^2)(C_1p+(p-2)D_T)$$

4.1 Heat source

We now take a periodic heat source term at *x*=0, so that we can write Q_0 as:

$$Q_0=\begin{cases}Q_0'\delta(x)\sin(\frac{\pi t}{\tau}),0\le t\le\tau\\0,t\ge\tau\end{cases}\tag{43}$$

In Laplace-Fourier transformation domain(43) becomes

$$\bar{Q}_0^*=\frac{Q_0'\pi\tau(1+e^{-pt})}{\sqrt{2\pi}(\pi^2+p^2\tau^2)}.\tag{44}$$

4.2 Fourier inversion

To obtain the solutions in Laplace transform domain, we take Fourier inversion of the required functions. To compute Fourier inversion, we take the help of contour integration by considering the closed contour $\Gamma=\Gamma_1+\Gamma_2$, where Γ_1 is the contour $\{\text{Re}(s):-\infty<\text{Re}(s)<\infty\}$ and Γ_2 is the upper half circle $\{|s|e^{i\theta},0\le\theta\le\pi\}$. It is observed that the singularities of the transform functions do not lie on the real line. Thus, in Laplace transform domain we get,

$$\bar{u}(x,p)=-\frac{\sqrt{\pi}\omega pQ_0'\tau(1+e^{-pt})(1+K_\alpha p^\alpha)}{\sqrt{2}(\pi^2+p^2\tau^2)}\sum_{\text{Im}(s_j)\ne0,x\ne0,j=1}^{8}A_j[B_{s_j,p}(n_1+is_j)(2D_{s_j,p}+C_2p^3)e^{-is_jx}]\tag{45}$$

$$\bar{v}(x,p)=-\frac{\sqrt{2\pi}\omega^2p^2Q_0'\tau(1+e^{-pt})(1+K_\alpha p^\alpha)}{(\pi^2+p^2\tau^2)}\sum_{\text{Im}(s_j)\ne0,x\ne0,j=1}^{8}A_j[(n_1+is_j)(2D_{s_j,p}+C_2p^3)e^{-is_jx}]\tag{46}$$

$$\bar{\theta}(x,p)=-\frac{\sqrt{2\pi}\omega pQ_0'\tau(1+e^{-pt})(1+K_\alpha p^\alpha)}{(\pi^2+p^2\tau^2)}\sum_{\text{Im}(s_j)\ne0,x\ne0,j=1}^{8}\begin{bmatrix}A_j[[is_jD_TB_{s_j,p}(n_1+is_j)(2D_{s_j,p}+C_2p^3)+2D_{s_j,p}\\(A_{s_j,p}B_{s_j,p}-s_j^2D_TB_{s_j,p}+4\omega^2p^2)]e^{-is_jx}]\end{bmatrix}\tag{47}$$

$$\bar{P}(x,p)=-\frac{\sqrt{\pi}\omega p^4Q_0'\tau(1+e^{-pt})(1+K_\alpha p^\alpha)}{\sqrt{2}(\pi^2+p^2\tau^2)}\sum_{\text{Im}(s_j)\ne0,x\ne0,j=1}^{8}A_j[(A_{s_j,p}B_{s_j,p}-s_j^2D_TB_{s_j,p}+4\omega^2p^2)]e^{-is_jx}]\tag{48}$$

$$\bar{\sigma}_{xx}(x,p)=\frac{i\sqrt{\pi}\omega pQ_0'\tau(1+e^{-pt})(1+K_\alpha p^\alpha)}{\sqrt{2}(\pi^2+p^2\tau^2)}\sum_{\text{Im}(s_j)\ne0,x\ne0,j=1}^{8}\begin{bmatrix}B_j[(s_j+in_1)(s_j+2in_1)(2iD_{s_j+in_1,p}+C_2p^3)B_{s_j+in_1,p}\\+(2D_{s_j+in_1,p}+iC_2p^3)(A_{s_j+in_1,p}B_{s_j+in_1,p}-(s_j+in_1)^2\\D_TB_{s_j+in_1,p}+4\omega^2p^2)+iD_T(s_j+in_1)(s_j+2in_1)\\(2iD_{s_j+in_1,p}+C_2p^3)]e^{-is_jx}\end{bmatrix}\tag{49}$$

$$\bar{C}(x,p)=\frac{i\sqrt{\pi}\omega p Q_0'\tau(1+e^{-pt})(1+K_\alpha p^\alpha)}{\sqrt{2}(\pi^2+p^2\tau^2)}\sum_{\mathrm{Im}(s_j)\neq 0,x\neq 0,j=1}^{8}\begin{bmatrix} B_j[(s_j+in_1)(s_j+2in_1)(2iD_{s_j+in_1,p}+C_2p^3)C_u^2B_{s_j+in_1,p} \\ +(2C_6D_{s_j+in_1,p}-iC_5p^3)(A_{s_j+in_1,p}B_{s_j+in_1,p}-(s_j+in_1)^2 \\ D_TB_{s_j+in_1,p}+4\omega^2p^2)+iD_TC_6(s_j+in_1)(s_j+2in_1) \\ (2iD_{s_j+in_1,p}+C_2p^3)B_{s_j+in_1,p}]e^{-is_jx} \end{bmatrix} \quad (50)$$

where A_j and B_j are given by

$$A_j=\prod_{\substack{n=1\\ n\neq j}}^{8}\frac{1}{(s_j-s_n)}, j=1,2,3,4,5,6,7,8.$$

$$B_j=\prod_{\substack{n=1\\ n\neq j}}^{8}\frac{1}{(r_j-r_n)}, j=1,2,3,4,5,6,7,8.$$

4.3 Laplace inversion

As the transformed functions of displacements, stress etc. are very complicated, the inverse functions can not be obtained directly as functions of *x* and *t*. We then take the help of numerical inversion of Laplace transformation. There are various methods of numerical inversion of Laplace transformation out of which we apply here the method adopted by Honig and Hirdes [51]. Let, $\bar{g}(p)$ is the Laplace transform of $g(t)$, then inverse Laplace transform can be written as $g(t)=\frac{e^{ct}}{2\pi}\int_{-\infty}^{\infty}e^{i\omega t}\bar{g}(c+i\omega)d\omega$, where c is an arbitrary number greater than real part of all the singularities of $\bar{g}(p)$. Fourier series expansion of $h(t)=e^{-ct}g(t)$ in the interval $[0,2T]$ gives the approximate formula $g_N(t)$ of $g(t)$ given by [51], $g(t)=g_\infty+E_D=g_N(t)+E_T+E_D$, where, $g_N(t)=\frac{\bar{g}(c)}{2}+\sum_{k=1}^{\infty}\mathrm{Re}[\bar{g}(c+\frac{ik\pi}{L})e^{\frac{ik\pi t}{L}}]$.
Here, E_D and E_T represents the discretization error and truncation error respectively. The values of *c* and *L* are chosen according to the criterion outlined in [51]. With the suitable choice of *L*, we have computed the values of the functions with the help of computer software and drawn the graphs accordingly.

5 NUMERICAL RESULTS AND DISCUSSION

The copper material is chosen for the purposes of numerical evaluations for which following material constants are taken [8, 31]:

$$\lambda_0=7.76\times10^{10}kgm^{-1}s^{-2},\mu_0=3.86\times0^{10}kgm^{-1}s^{-2},T_0=293K,\rho_0=8954kgm^{-2},C_E=383.1Jkg^{-1}K^{-1},Q_0'=1,$$
$$\alpha_c=1.98\times10^{-4}m^3kg^{-1},\alpha_t=1.78\times10^{-5}K^{-1},C_T^2=2,l=1,\tau_1=1,a_0=1.2\times10^4m^2s^{-2}K^{-1},b_0=9\times10^5m^5kg^{-1}s^{-2}.$$

We have shown the effect of rotation, fractional coefficient and non-homogeneity on displacements, stress, temperature, chemical potential and mass concentration with the variation of distance graphically from Fig. 1 to Fig. 13. All the associated graphs are drawn at fixed time *t* = 0.2.

Fig.1 and Fig.4 highlight the effect of fractional parameter α on displacement *u* and *v* respectively when GNIII model is considered whereas Fig.3 and Fig.4 show the effect of diffusive parameter D_1 on *u* and *v* when both the models GNII and GNIII are considered. Change of displacement u with the changing value of rotational parameter ω and non-homogeneity parameter n_1 can be obtained from Fig.2.

Fig.5, Fig.6 and Fig.7 describe the change of temperature θ with various values of the parameters.

Fig.8 and Fig.9 show the change of stress σ_{xx} with the changing values of parameter α and diffusive parameter D_1 respectively. Effect of k_0, ω and n_1 on stress σ_{xx} can be observed from Fig.10.

Fig.11 describes the effect of α on mass concentration C whereas Fig.12 and Fig.13 describe the effects of k_0, n_1 and ω on mass concentration and chemical potential respectively.

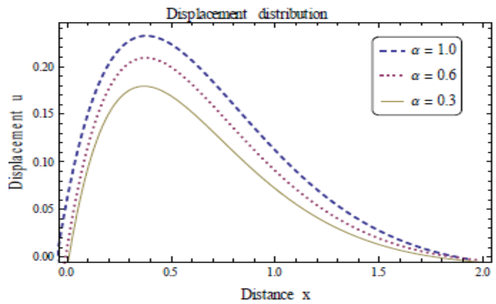

Fig.1
Displacement distribution versus x at different α .

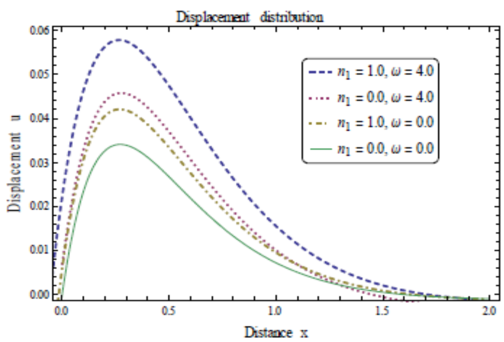

Fig.2
Displacement distribution versus x at different n_1 and ω .

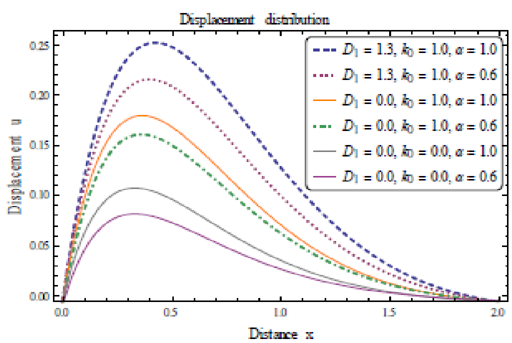

Fig.3
Displacement distribution versus x at different D_1, k_0 and α .

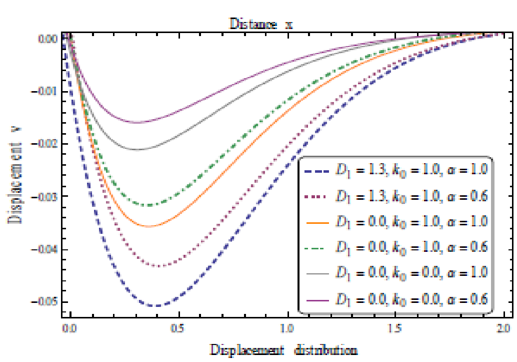

Fig.4
Displacement distribution versus x at different D_1, k_0 and α .

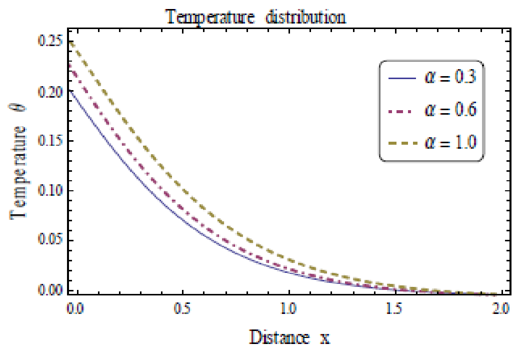

Fig.5
Temperature distribution versus x at different α .

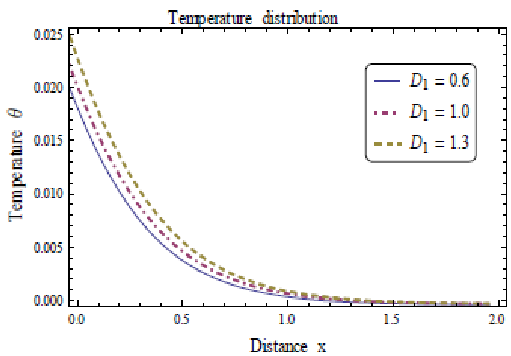

Fig.6
Temperature distribution versus x at different D_1.

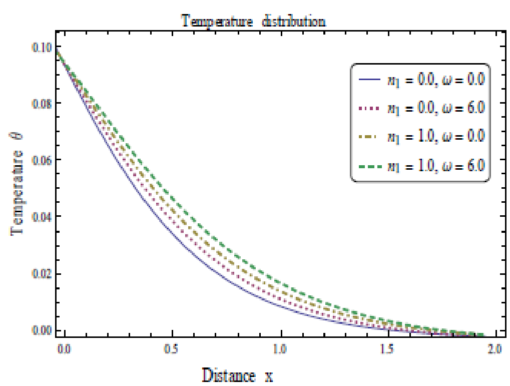

Fig.7
Temperature distribution versus x at different n_1 and ω .

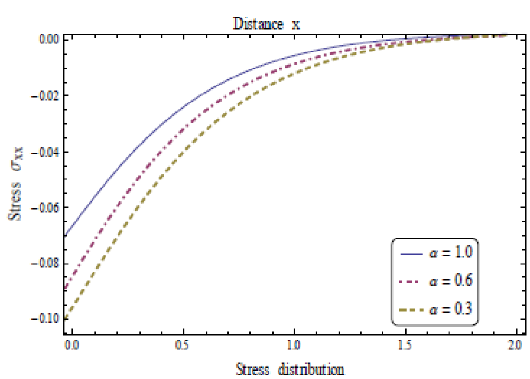

Fig.8
Stress distribution versus x at different α .

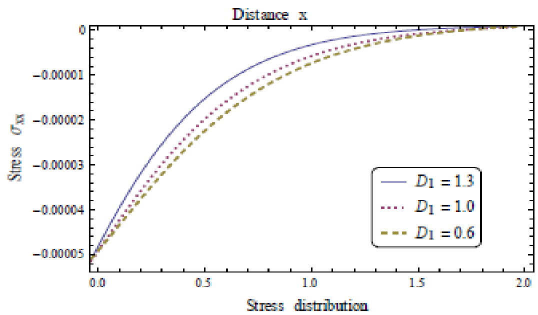

Fig.9
Stress distribution versus x at different D_1.

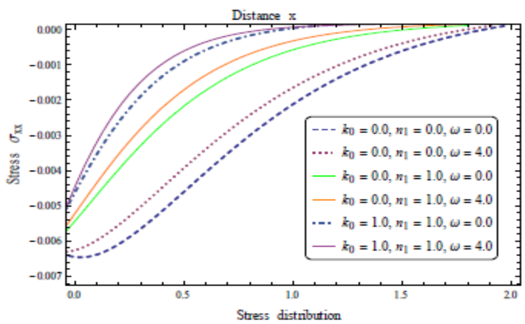

Fig.10
Stress distribution versus x at different k_0 ,n_1 and ω .

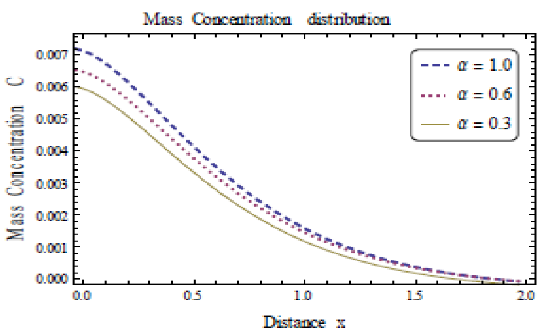

Fig.11
Mass concentration distribution versus x at different α .

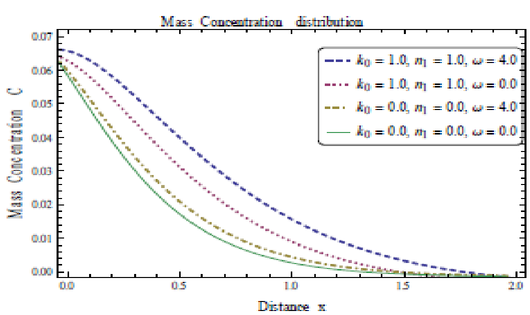

Fig.12
Mass concentration distribution versus x at different k_0 ,n_1 and ω .

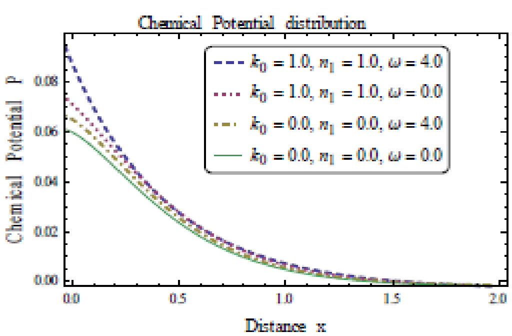

Fig.13
Chemical potential distribution versus x at different k_0 ,n_1 and ω .

5.1 Observations from the graphs

1. It is seen from Fig.1 to Fig.4 that in each case magnitude of displacement increases with increasing value of x, reaches a peak value and then decay gradually and finally converges to the zero value, which is expected.
2. It is also seen from the graphs that effect of all the parameters are significantly present.
3. It is clear from the Fig.1 that the peak of the displacement u occurs at $x = 0.36$ for GNIII model for different values of α and the peak value is increasing with increase of α .
4. Fig.2 clarifies the effect of non-homogeneity parameter n_1 on displacement u with and without rotation for GNIII model. The peak of the displacement occurs at $x = 0.3$ and the peak increases in present of n_1 in both the cases.
5. Fig.4 classifies the effect of fractional parameter α in presence and absence of diffusive parameter D_1 for both GNII and GNIII models. Modulus of the peak of displacement u with the variation of α occurs at x = 0.33 when GNII model is considered and 0.35, 0.4 when GNIII model is considered. Significantly, the peak is increasing with the increase of α and in the presence of D_1 .
6. It is also observed that two different models yield two different peak points.
7. The nature of the graphs of temperature, stress, chemical potential and mass concentration is similar. All the variables occurs maximum magnitude at $x = 0$ and then decay gradually with the increasing values of x and finally converges to zero. Effect of all parameters α, n_1, D_1 and ω clearly observed from the figures.

6 CONCLUSIONS

This article studies the fractional order thermoelastic interaction in a functionally graded isotropic unbounded rotating medium in the context of the linear theory of generalized thermoelastic diffusion with energy dissipation (GNIII) and without energy dissipation (GNII). The material properties under consideration are assumed to vary exponentially with distance. The analysis of the results permits some concluding remarks:

1. The diffusive parameter, non-homogeneity parameter, fractional parameter has a significant effect on the solutions of displacements, temperature, stress, mass concentration and chemical potential, which can be visualized from the figures.
2. From the graph, it is clear that the magnitude of the displacement u always take greater value for a rotating medium than that for a non-rotating medium.
3. Temperature, stress, chemical potential and mass concentration approaches to zero after traversing some distance, which is expected for physical problems.
4. The problem reduces to a problem for elastically homogeneous medium for zero value of the non-homogeneity parameter.

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