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Effect of Thermal Gradient on Vibration of Non-Homogeneous Square Plate with Exponentially Varying Thickness

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ABSTRACT

Vibrations of plate and plate type structures made up of composite materials have a significant role in various industrial mechanical structures, aerospace industries and other engineering applications. The main aim of the present paper is to study the two dimensional thermal effect on the vibration of non-homogeneous square plate of variable thickness having clamped boundary. It is assumed that temperature varies bi-parabolic i.e. parabolic in x-direction & parabolic in y-direction and thickness is considered to vary exponentially in x direction. Also, density is taken as the function of "x" due to non-homogeneity present in the plate's material. Rayleigh Ritz technique is used to calculate the natural frequency for both the modes of vibration for the various values of taper parameter, non-homogeneity constant and thermal gradient. All the calculations are carried out for an alloy of Aluminum, Duralumin, by using mathematica.

Keywords : Vibration; Frequency; Thermal gradient; Taper constant; Non-homogeneity constant.

1 INTRODUCTION

VIBRATION phenomenon is commonly used in science and engineering applications. Vibration is profitably used in musical instruments, testing equipments etc. but undesirable in many cases such as machine tools, resonance etc. With the development of technology, especially in space technology, composite materials now became the necessity to reduce or optimize the effects of vibration. In these days, plates made up of composite materials with various geometrical shapes are commonly used as primary component in many structural and machinery applications. Plates of different shape and size with variable thickness are extensively used in various engineering structures and machines in aerospace industry, marine industry, automotive manufacturing industry, earth-quake resistant structures, defence etc. These plates have considerably greater capability for vibration as compared to the plates of uniform thickness.

Since most of the machines and mechanical structures operate under the influence of high temperature, the effect of elevated temperature cannot be neglected. Also, non-homogeneity develops in material due to variation in temperature. Therefore, scientists and researchers are interested to know about the effects of variation in temperature on these plates.

Practicing engineers and scientists from different industries are always try to get the knowledge about first few modes of vibration before finalizing any design of machine or structure. But sometimes, they have neither the

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analytical capability of solving the problem nor the money and time needed for experimental program. Therefore, they are forced to drop their project/problems at this point. Hence, it becomes the need of the hour to study the effect of temperature variations on the vibrational behavior of plates and to provide a wide range of numerical data within the realm of them (practicing engineers, researchers, scientists etc.) for the fulfillment of their requirement. In available literature, a lot of work has been carried out to examine the effect of one dimensional temperature variation on the vibration of plates whose thickness varies in one or two directions. But, almost negligible work is done in two dimensional temperature variations for non-homogeneous plate.

A.K. Gupta & A. Khanna [1] discussed the vibration of visco elastic rectangular plate with linearly thickness variations in both directions having four sided clamped boundary conditions. A.K. Gupta & P. Singhal [2] studied the thermal effect on free vibration of non-homogeneous orthotropic visco–elastic rectangular plate with parabolic thickness variation. A. Khanna & A.K. Sharma [3] considered the mechanical vibration of homogeneous square plate with linearly thickness variation. They considered the parabolic temperature variation in two directions. A. Khanna & M. Bhatia [4] studied the free vibration of square plate of variable thickness with thermal effect. They considered bi-parabolic variation in thickness under the influence of bi-directional variation in temperature i.e. linearly in x-direction and parabolic in y-direction. Leissa [5] provided an excellent data on vibration of plates in his monograph. Huang & Leissa [6] discussed the vibration analysis of rectangular plates with side cracks via the Ritz method. B. Singh & V. Saxena [7] investigated the transverse vibration of rectangular plate with bi-directional thickness variation. G. Fauconneau & R.D. Marangoni [8] evaluated the effect of a thermal gradient on the natural frequencies of a rectangular plate. Free vibration analysis of rectangular plates with internal columns and uniform elastic edge supports is given by Wu & Lu [9]. H.P. Lee, S.P. Lim & T. Chow [10] worked on free vibration of composite rectangular plates with rectangular cutouts. They worked on simply-supported rectangular plates having central rectangular cutouts and double square cutouts. J.R. Kuttler & V.G. Sigillit [11] discussed the vibrational frequencies of clamped plates of variable thickness. M. Daleh & A.D. Keer [12] worked on the natural vibration analysis of clamped rectangular orthotropic plate. R.K. Jain & S.R. Soni [13] discussed the free vibration of rectangular plates with parabolic varying thickness at two parallel simply supported edges boundary condition. R. Lal [14] studied the transverse vibration of orthotropic non-uniform rectangular plates with continuously varying density. S.K. Malhotra, N. Ganesan & M.A. Veluswami [15] discussed the vibration of orthotropic square plates having variable thickness (linear variation) with the help of Rayleigh-Ritz method for various boundary conditions. Recently, Alijani & Amabili [16] analyzed nonlinear vibrations of imperfect rectangular plates with free edges. T. Johri & I. Johri [17] studied exponential thermal effect on vibration of non-homogeneous orthotropic rectangular plate having bi-linear variation in thickness. T. Sakata & K. Hosokawa [18] studied the vibration of clamped orthotropic rectangular plates with C-C-C-C boundary conditions. Recently, free vibration analysis of thick trapezoidal and triangular laminated plates resting on elastic supports is presented by Quintana & Nallim [19]. T. Sakiyama & M. Huang [20] investigated the free vibration analysis of rectangular plates with variable thickness. Y.F. Xing & B. Liu [21] gave new exact solutions for free vibration of thin orthotropic rectangular plates with all combinations of simply supported and clamped boundary conditions.

The objective of the present study is twofold. First is to find the magnitude of frequency for both the modes of vibration at different values of varying parameters i.e. thermal gradient, non-homogeneity constant and taper constant and second is to analyze how these variations affect the frequency. Rayleigh Ritz technique has been applied to find both the modes of frequency. Numeric values of frequency are given in tabular form and variation in frequency with respect to varying parameters is shown in graphical manner.

2 DIFFERENTIAL EQUATION OF MOTION AND ITS SOLUTION

Differential equation of motion for square plate of variable thickness is [1]

$$\begin{aligned}&[D_1(W_{,xxxx}+2W_{,xxyy}+W_{,yyyy})+2D_{1,x}(W_{,xxx}+W_{,xyy})+2D_{1,y}(W_{,yyy}+W_{,yxx})\\&+D_{1,xx}(W_{,xx}+\nu W_{,yy})+D_{1,yy}(W_{,yy}+\nu W_{,xx})+2(1-\nu)D_{1,xy}W_{,xy}]-\rho g p^2 W=0\end{aligned} \tag{1}$$

A comma followed by a suffix denotes partial differentiation with respect to that variable. Here, D_1 is the flexural rigidity of plate i.e.

$$D_1 = Eg^3 / 12(1-\nu^2) \tag{2}$$

The thickness of the plate is assumed to vary exponentially in *x*- direction i.e.

$$g = g_0 e^{\frac{\beta x}{a}} \tag{3}$$

where β is taper constant in *x*-direction and $g = g_0$ at $x = y = 0$.

To make easy and convenient calculation, first six terms are considered in the expansion of $\exp(\beta x / a)$. Now Eq. (3) can be written as:

$$g = g_0[1 + \frac{(\beta x / a)}{1!} + \frac{(\beta x / a)^2}{2!} + \frac{(\beta x / a)^3}{3!} + \frac{(\beta x / a)^4}{4!} + \frac{(\beta x / a)^5}{5!}] \tag{4}$$

Due to non-homogeneity present in the plate's material, it is considered that density of the plate's material varies parabolic in one direction i.e.

$$\rho = \rho_0(1 + \alpha_1 x^2 / a^2) \tag{5}$$

where, α_1 i.e. $(0 \le \alpha_1 < 1)$, is the non-homogeneity constant.

Also, it is assumed that square plate of engineering material has a steady two dimensional i.e. bi-parabolic temperature variations as:

$$\tau = \tau_0(1 - (x/a)^2)(1 - (y/a)^2) \tag{6}$$

where, τ denotes the temperature excess above the reference temperature at any point on the plate and τ_0 denotes the temperature at any point on the boundary of plate. The temperature dependence of the modulus of elasticity for most of engineering materials can be expressed as:

$$E = E_0(1 - \gamma\tau) \tag{7}$$

where, E_0 is the value of the young modulus at reference temperature i.e. $\tau = 0$ and γ is the slope of the variation of *E* with τ. On using Eq. (6) in Eq. (7), one gets:

$$E = E_0\{1 - \alpha(1 - (x/a)^2)(1 - (y/a)^2)\} \tag{8}$$

where, $\alpha = \gamma\tau_0, (0 \le \alpha < 1)$ is thermal gradient.

Now, put the value of *E* and *g* from Eqs. (8) and (4) in Eq. (2), one obtains

$$D_1 = \frac{[E_0\{1 - \alpha(1 - (x/a)^2)(1 - (y/a)^2)\}g_0{}^3\{1 + \frac{(3\beta x / a)}{1!} + \frac{(3\beta x / a)^2}{2!} + \frac{(3\beta x / a)^3}{3!} + \frac{(3\beta x / a)^4}{4!} + \frac{(3\beta x / a)^5}{5!}\}]}{12(1-\nu^2)} \tag{9}$$

3 SOLUTION OF DIFFERENTIAL EQUATION OF MOTION

To find the solution of equation of motion, Rayleigh Ritz technique is applied. In this method, one requires maximum strain energy must be equal to the maximum kinetic energy. So it is necessary for the problem under consideration that

$$\delta(S-K)=0 \tag{10}$$

The expression for kinetic energy (*K*) and strain energy (*S*) are [4]

$$K=\frac{1}{2}\rho p^2\int_0^a\int_0^a gW^2 dydx \tag{11}$$

$$S=\frac{1}{2}\int_0^a\int_0^a D_1[(W_{,xx})^2+(W_{,yy})^2+2\nu W_{,xx}W_{,yy}+2(1-\nu)(W_{,xy})^2]dydx \tag{12}$$

Since the plate is assumed to be clamped at all the four edges, the boundary conditions for square plate are

$$\begin{aligned} W=W_{,x}=0 \quad & at \quad x=0,a \\ W=W_{,y}=0 \quad & at \quad y=0,a \end{aligned} \tag{13}$$

To satisfy equation (13), the corresponding two term deflection function is taken as [3]

$$W=[(x/a)(y/a)(1-(x/a))(1-(y/a))]^2(A_1+A_2(x/a)(y/a)(1-(x/a))(1-(y/a))] \tag{14}$$

Assuming the non-dimensional variable as:

$$X=x/a, Y=y/a, \overline{W}=W/a, \overline{g}=g/a \tag{15}$$

On using Eqs. (4), (5), (9) & (15) in Eqs. (11) and (12), one obtains

$$K=\frac{1}{2}\rho_0 p^2\overline{g_0}a^5\int_0^1\int_0^1(1+\alpha_1X^2)[1+\frac{(\beta X)}{1!}+\frac{(\beta X)^2}{2!}+\frac{(\beta X)^3}{3!}+\frac{(\beta X)^4}{4!}+\frac{(\beta X)^5}{5!}]\overline{W}^2 dYdX \tag{16}$$

$$\begin{aligned} S=L\int_0^1\int_0^1&[1-\alpha(1-(X)^2)(1-(Y)^2)][1+\frac{(3\beta X)}{1!}+\frac{(3\beta X)^2}{2!}+\frac{(3\beta X)^3}{3!}+\frac{(3\beta X)^4}{4!}+ \\ &\frac{(3\beta X)^5}{5!}][(W_{,XX})^2+(W_{,YY})^2+2\nu W_{,XX}W_{,YY}+2(1-\nu)(W_{,XX})^2]dYdX \end{aligned} \tag{17}$$

where, $L=E_0\overline{g_0}^3a^3/12(1-\nu^2)$

On using Eqs. (16) & (17) in Eq. (10), one gets

$$\delta(S^*-\lambda^2K^*)=0 \tag{18}$$

where,

$$S^* = \int_0^1 \int_0^1 [1-\alpha(1-(X)^2)(1-(Y)^2)][1+\frac{(3\beta X)}{1!}+\frac{(3\beta X)^2}{2!}+\frac{(3\beta X)^3}{3!}+\frac{(3\beta X)^4}{4!}+\frac{(3\beta X)^5}{5!}][(W_{,XX})^2+(W_{,YY})^2+2\nu W_{,XX}W_{,YY}+2(1-\nu)(W_{,XX})^2]dYdX$$

$$K^* = \int_0^1 \int_0^1 (1+\alpha_1 X^2)[1+\frac{(\beta X)}{1!}+\frac{(\beta X)^2}{2!}+\frac{(\beta X)^3}{3!}+\frac{(\beta X)^4}{4!}+\frac{(\beta X)^5}{5!}]\overline{W}^2 dYdX$$

and $\lambda^2 = 12\rho_0 p^2(1-\nu^2)/E_0 g_0^{\,2}$, is the frequency parameter.

Eq. (18) consist two unknown constants i.e. A_1 & A_2 arising due to the substitution of W from Eq. (14). These two constants are to be determined as follows

$$\partial(S^*-\lambda^2 K^*)/\partial A_n = 0 \quad n=1,2 \tag{19}$$

On simplifying (19), one gets

$$b_{n1}A_1 + b_{n2}A_2 = 0, \quad n=1,2 \tag{20}$$

where, b_{n1} & $b_{n2} (n=1,2)$ involve varying parameters and the frequency parameter i.e. λ^2.

To obtain the non-trivial solution of Eq. (20), the determinant of the coefficients of Eq. (20) must be zero. So one gets,

$$\begin{aligned} &E_0 = 7.08*10^{10} N/M^2 \quad , \quad g_0 = 0.01M \\ &\rho_0 = 2.80*10^3 Kg/M^3 \quad , \quad \upsilon = 0.345 \end{aligned} \tag{21}$$

Eq. (21) is a quadratic equation in λ^2 from which one can obtain λ at various values of varying parameters i.e. taper constant, non-homogeneity constant and thermal gradient.

4 RESULTS AND DISCUSSION

Computations have been made for the frequency parameter λ of square plate for different values of taper constant (β), non-homogeneity constant (α_1) and thermal gradient (α) for the first two modes of vibration. The following material (for duralumin) parameters are used in calculations:

$$E_0 = 7.08*10^{10} N/M^2 \quad , \quad g_0 = 0.01M \quad \rho_0 = 2.80*10^3 Kg/M^3 \quad , \quad \upsilon = 0.345$$

All the numeric values of frequency for first two modes of vibration are tabulated. For better understanding, variation in both the modes of frequency with varying parameters is shown in graphs.

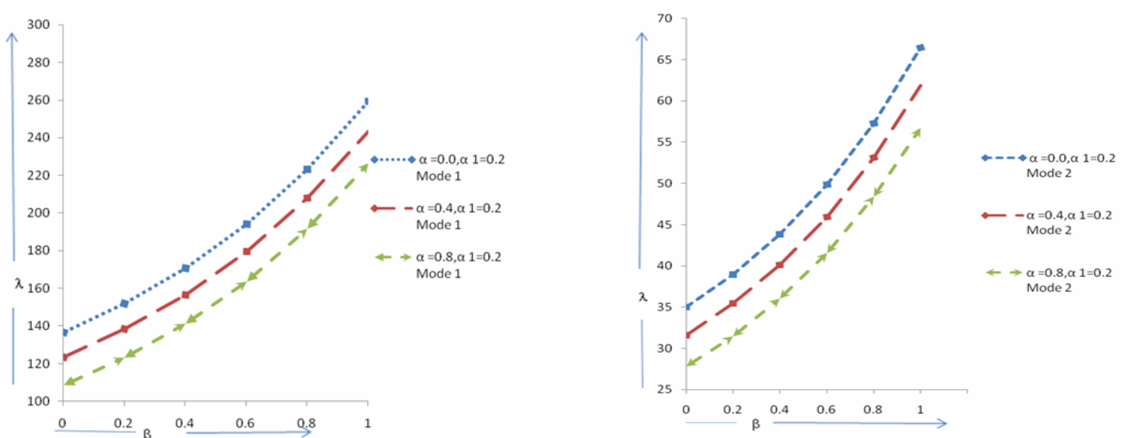
Table1. includes numeric values of frequency for both the modes of vibration at different values of taper constant (β) for the following combinations of thermal gradient (α) & non-homogeneity constant (α_1) i.e.

Case 1: $\alpha = 0.0$, $\alpha_1 = 0.2$ Case 2: $\alpha = 0.4$, $\alpha_1 = 0.2$ Case 3: $\alpha = 0.8$, $\alpha_1 = 0.2$.

It is observed that as taper constant 'β' increases, frequency parameter 'λ' also increases for both the modes of vibration in case (1), (2) and (3). It is interesting to note that value of frequency for both the modes of vibration decreases as thermal gradient increases from case (1) to case (3). Variation in frequency for Mode1 and Mode2 in Table 1. is shown in Fig. 1(a) and Fig. 1(b) respectively for case (1), (2) and (3).

Table 1
Variation of Frequency parameter (λ) for different values of Taper constant (β) & Thermal gradient (α) at fixed value of Non-homogeneity constant (α_1 =0.2)

β	α =0.0		α = 0.4		α = 0.8	
	Mode1	Mode2	Mode1	Mode2	Mode1	Mode2
0.0	136.88	35.06	123.72	31.68	108.99	27.86
0.2	152.05	38.97	138.48	35.47	123.43	31.52
0.4	170.93	43.85	156.86	40.18	141.42	36.04
0.6	194.38	49.90	179.74	46.00	163.83	41.61
0.8	223.51	57.35	208.21	53.15	191.75	48.41
1.0	259.67	66.48	243.62	61.88	226.49	56.71



(a) (b)

Fig.1
(a) Frequency (Mode1) Vs taper parameter (β). (b) Frequency (Mode2) Vs taper parameter (β).

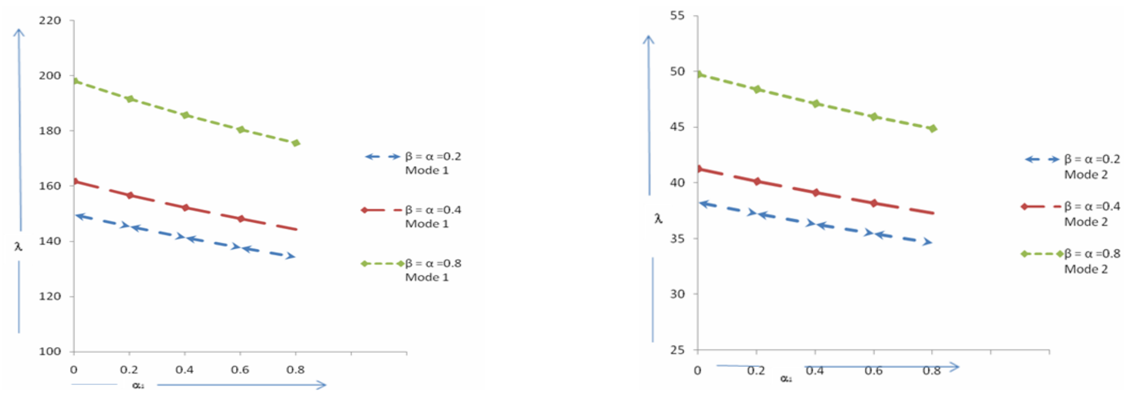
In Table 2., numeric values of frequency for both the modes of vibration at different values of non-homogeneity constant (α_1) are given for the following combinations of taper constant (β) & thermal gradient (α) i.e.

Case 4: $\alpha = \beta = 0.2$ Case 5: $\alpha = \beta = 0.4$ Case 6: $\alpha = \beta = 0.8$.

Here, author observed that as non-homogeneity constant α_1 increases, frequency parameter ‘ λ ’ decreases for both the modes of vibration for case (4), (5) and (6). It is evident to note that the values of frequency for both the modes of vibration increase as the combined values of thermal gradient & taper constant increase from case (4) to case (6). Variation in frequency for Mode1 and Mode2 in Table 2. is shown in Fig. 2(a) and Fig. 2(b) respectively for case (4), (5) & (6).

Table 2
Variation of Frequency parameter (λ) for different values of Non-homogeneity constant (α_1), Thermal gradient (α and Taper constant (β)

α_1	$\alpha = \beta = 0.2$		$\alpha = \beta = 0.4$		$\alpha = \beta$ =0.8	
	Mode1	Mode2	Mode1	Mode2	Mode1	Mode2
0.0	149.81	38.28	161.74	41.29	198.11	49.79
0.2	145.42	37.26	156.86	40.18	191.75	48.41
0.4	141.40	36.32	152.40	39.15	185.96	47.15
0.6	137.70	35.45	148.30	38.19	180.68	45.98
0.8	134.28	34.63	144.51	37.31	175.82	44.89



(a) (b)

Fig.2
(a) Frequency (Mode1) Vs non homogeneity constant (α_1). (b) Frequency (Mode2) Vs non homogeneity constant (α_1).

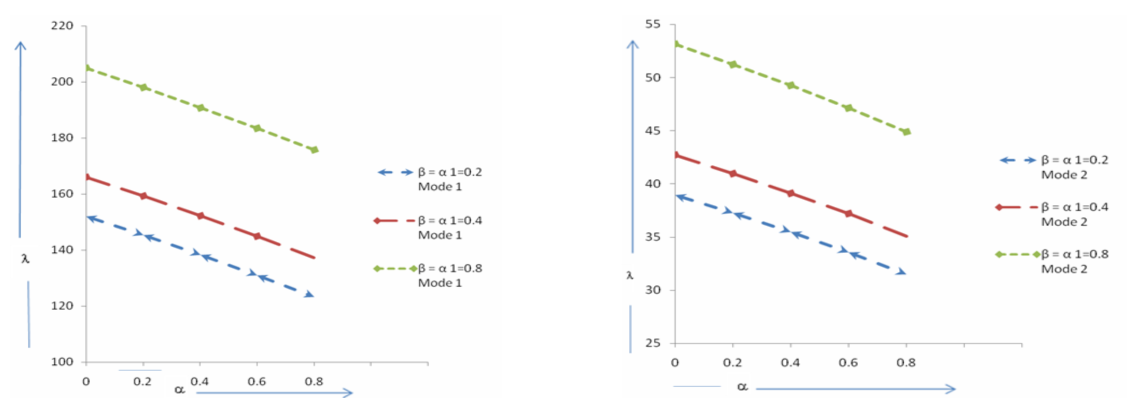
Table 3. shows numeric values of frequency (λ) for both the modes of vibration at different values of thermal gradient (α) for the following combinations of taper constant (β) & non-homogeneity constant (α_1) i.e.

Case 7: $\beta = \alpha_1 = 0.2$ Case 8: $\beta = \alpha_1 = 0.4$ Case 9: $\beta = \alpha_1 = 0.8$

It is obvious from the Table 3. that as thermal gradient α increases, frequency parameter 'λ' decreases continuously for both the modes of vibration in case (7), (8) and (9). Also, it is clearly seen that the values of frequency for both the modes of vibration increase as the combined values of non-homogeneity constant and taper constant increase from case (7) to case (9). Further, variation of frequency for Mode1 and Mode2 in Table 3. is shown in Fig. 3(a) and Fig. 3(b) respectively for case (7), (8) and (9).

Table 3
Variation of Frequency parameter (λ) for different values of Thermal gradient (α), Taper constant (β) and Non-homogeneity constant (α_1)

α	$\beta = \alpha_1 = 0.2$		$\beta = \alpha_1 = 0.4$		$\beta = \alpha_1 = 0.8$	
	Mode1	Mode2	Mode1	Mode2	Mode1	Mode2
0.0	152.05	38.97	166.07	42.72	205.04	53.15
0.2	145.42	37.26	159.38	40.98	198.12	51.26
0.4	138.48	35.47	152.40	39.15	190.97	49.26
0.6	131.17	33.56	145.08	37.20	183.55	47.15
0.8	123.43	31.52	137.39	35.12	175.82	44.89



(a) (b)

Fig.3
(a) Frequency (Mode 1) Vs thermal gradient (α). (b) Frequency (Mode 2) Vs thermal gradient (α).

5 CONCLUSIONS

Conclusion from the case study is that the variation in varying parameters of the plate strongly affects the frequency of the plate's vibration. By choosing appropriate values of varying parameters, desired or required values of frequencies can be obtained. Therefore, authors suggest the industrial scientists and design engineers to go through the findings of the present paper in order to provide much better authentic structures and machines with more strength, durability and efficiency.

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